## Material Balances for Batch and Semi-Batch Processes <br> كمية ثـانـة

- A batch process is used to process a fixed amount of material each time it is operated. Initially, the material to be processed is charged into the system. After processing of the material is complete, the products are removed.
- Batch processes are used industrially for speciality processing applications (e.g., producing pharmaceutical products), which typically operate at relatively low production rates.
- Look at Figure11a that illustrates what occurs at the start of a batch process, and after thorough mixing, the final solution remains in the system (Figure 11b).



Figure 11b The final state of a batch mixing process.

Figure 11a The initial state of a batch mixing process.

- We can summarize the hypothetical operation of the batch as a flow system (open system) as follows (Figure 12):

Final conditions: All values $=0$

Initial conditions: All value $=0$

Flows out:

$$
\begin{aligned}
& \mathrm{NaOH}=1,000 \mathrm{lb} \\
& \underline{\mathrm{H}_{2} \mathrm{O}=9,000 \mathrm{lb}} \\
& \text { Total }=10,000 \mathrm{lb}
\end{aligned}
$$

Flows in:

$$
\begin{aligned}
& \mathrm{NaOH}=1,000 \mathrm{lb} \\
& \underline{\mathrm{H}}_{2} \underline{\mathrm{O}}=9,000 \mathrm{lb} \\
& \text { Total } 10,000 \mathrm{lb}
\end{aligned}
$$



Figure 12 The batch process in Figure 11 represented as an open system.

المو اد تدخل اثناء التثظغيل
区 In a semi-batch process material enters the process during its operation, but does not leave. Instead mass is allowed to accumulate in the process vessel. Product is withdrawn only after the process is over. لا تخرج الىى بعد انتهاء العطلية
® A figure 13 illustrates a semi-batch mixing process. Initially the vessel is empty (Figure 13a). Figure13b shows the semi-batch system after 1 hour of operation. Semi-batch processes are open and unsteady state.
® Only flows enter the systems, and none leave, hence the system is an unsteady state - one that you can treat as having continuous flows, as follows:

Final conditions:
Flows out: All values $=0$

$$
\begin{aligned}
& \mathrm{NaOH}=1,000 \mathrm{lb} \\
& \underline{\mathrm{H}_{2} \mathrm{O}}=9,000 \mathrm{lb}
\end{aligned}
$$

$$
\text { Total }=10,000 \mathrm{lb}
$$

Flows in:

$$
\begin{aligned}
& \mathrm{NaOH}=1,000 \mathrm{lb} \\
& \underline{\mathrm{H}}_{2} \underline{\mathrm{O}=9,000 \mathrm{lb}} \\
& \text { Total }=10,000 \mathrm{lb}
\end{aligned}
$$

Initial conditions: All values $=0$


Figure 6.13 Initial condition for the semi-batch mixing process. Vessel is empty.


Figure 13b Condition of a semi-batch mixing process after 1 hour of operation.

## Example 3

A measurement for water flushing of a steel tank originally containing motor oil showed that 0.15 percent by weight of the original contents remained on the interior tank surface. What is the fractional loss of oil before flushing with water, and the pounds of discharge of motor oil into the environment during of a $10,000 \mathrm{gal}$ tank truck that carried motor oil? (The density of motor oil is about $0.80 \mathrm{~g} / \mathrm{cm}^{3}$ ).

## Solution

Basis: 10,000 gal motor oil at an assumed $77^{\circ} \mathrm{F}$

The initial mass of the motor oil in the tank was

$$
(10000 \mathrm{gal})(3.785 \mathrm{lit} / 1 \mathrm{gal})\left(1000 \mathrm{~cm}^{3} / 1 \mathrm{lit}\right)\left(0.8 \mathrm{~g} / \mathrm{cm}^{3}\right)(1 \mathrm{lb} / 454 \mathrm{~g})=66700 \mathrm{lb}
$$

The mass fractional loss is $\mathbf{0 . 0 0 1 5}$. The oil material balance is
$\underline{\text { Initial }} \frac{\text { unloaded }}{66,700}=66,700(0.9985) \quad+\quad \frac{\text { residual discharged on cleaning }}{66,700(0.0015)}$

Thus, the discharge on flushing is $\mathbf{6 6 , 7 0 0}(\mathbf{0 . 0 0} \mathbf{1 5})=\mathbf{1 0 0} \mathbf{l b}$.

## Questions

1. Is it true that if no material crosses the boundary of a system, the system is a closed system?
2. Is mass conserved within an open process?
3. Can an accumulation be negative? What does a negative accumulation mean?
4. Under what circumstances can the accumulation term in the material balance be zero for a process?
5. Distinguish between a steady-state and an unsteady-state process.
6. What is a transient process? Is it different than an unsteady-state process?
7. Does Equation 6.4 apply to a system involving more than one component?
8. When a chemical plant or refinery uses various feeds and produces various products, does Equation 6.4 apply to each component in the plant?
9. What terms of the general material balance, Equation (6.5), can be deleted if
a. The process is known to be a steady-state process.
b. The process is carried out inside a closed vessel.
c. The process does not involve a chemical reaction.
10. What is the difference between a batch process and a closed process?
11. What is the difference between a semi-batch process and a closed process?
12. What is the difference between a semi-batch process and an open process?

## Answers:

1. Yes
2. Not necessarily - accumulation can occur
3. Yes; depletion الاستنز اف
4. No reaction (a) closed system, or (b) flow of a component in and out are equal.
5. In an unsteady-state system, the state of the system changes with time, whereas with a steady-state system, it does not.
6. A transient process is an unsteady-state process.
7. Yes
8. Yes
9. (a) Accumulation; (b) flow in and out; (c) generation and consumption
10. None
11. A flow in occurs
12. None, except in a flow process, usually flows occur both in and out

## Problems

1. Here is a report from a catalytic polymerization unit:

## Charge:

Propanes and butanes
Production:
Propane and lighter
Butane
Polymer

## Pounds per hour

15,500

What is the production in pounds per hour of the polymer?
2. A plant discharges $4,000 \mathrm{gal} / \mathrm{min}$ of treated wastewater that contains $0.25 \mathrm{mg} / \mathrm{L}$ of PCB , (polychloronated biphenyls) into a river that contains no measurable PCBs upstream of the discharge. If the river flow rate is 1,500 cubic feet per second, after the discharged water has thoroughly mixed with the river water, what is the concentration of PCBs in the river in $\mathrm{mg} / \mathrm{L}$ ?

## Answers:

1. $7,740 \mathrm{lb} / \mathrm{hr}$

## ( $3.785 \mathrm{lit} / 1 \mathrm{gal}$ ) <br> $\frac{1 \mathrm{~L}}{3.531 \times 10^{-2} \mathrm{ft}^{3}}$

2. $1.49 * 10^{-3} \mathrm{mg} / \mathrm{L}$.

### 2.2 General Strategy for Solving Material Balance Problems

## Problem Solving

An orderly method of analyzing problems and presenting their solutions represents training in logical thinking that is of considerably greater value than mere knowledge of how to solve a particular type of problem.

## The Strategy for Solving Problems

1. Read and understand the problem statement.
2. Draw a sketch of the process and specify the system boundary.
3. Place labels for unknown variables and values for known variables on the sketch.
4. Obtain any missing needed data.
5. Choose a basis.
6. Determine the number of unknowns.
7. Determine the number of independent equations, and carry out a degree of freedom analysis.
8. Write down the equations to be solved.
9. Solve the equations and calculate the quantities asked for.
10. Check your answer.

## Example 4

A thickener in a waste disposal unit of a plant removes water from wet sewage sludge as shown in Figure 10. How many kilograms of water leave the thickener per 100 kg of wet sludge that enter the thickener? The process is in the steady state.


Figure 10

## Solution

## Basis: 100 kg wet sludge

The system is the thickener (an open system). No accumulation, generation, or consumption occurs. The total mass balance is

$$
\underline{\text { In }}=\frac{\text { Out }}{100 \mathrm{~kg}=70 \mathrm{~kg}+\mathrm{kg} \text { of water }}
$$

Consequently, the water amounts to 30 kg .

## Example 5

A continuous mixer mixes NaOH with $\mathrm{H}_{2} \mathrm{O}$ to produce an aqueous solution of NaOH . Determine the composition and flow rate of the product if the flow rate of NaOH is $1000 \mathrm{~kg} / \mathrm{hr}$, and the ratio of the flow rate of the $\mathrm{H}_{2} \mathrm{O}$ to the product solution is 0.9 . For this process,

1. Sketch of the process is required.
2. Place the known information on the diagram of the process.
3. What basis would you choose for the problem?
4. How many unknowns exist?
5. Determine the number of independent equations.
6. Write the equations to be solved.
7. Solve the equations.
8. Check your answer.

## Solution

1. The process is an open one, and we assume it to be steady state.


Figure E7.2
2. Because no contrary information is provided about the composition of the $\mathrm{H}_{2} \mathrm{O}$ and NaOH streams, we will assume that they are $100 \% \mathrm{H}_{2} \mathrm{O}$ and NaOH , respectively.

3. Basis ( 1000 kg or 1 hour or $1000 \mathrm{~kg} / \mathrm{hr}$ ) (all are equivalent)
4. We do not know the values of four variables: $\mathrm{W}, \mathrm{P}, \mathrm{P}_{\mathrm{NaOH}}$ and $\mathrm{P}_{\mathrm{H} 2 \mathrm{O}}$.
5. You can write three material balances:

- one for the NaOH
- one for the $\mathrm{H}_{2} \mathrm{O}$
- one total balance (the sum of the two component balances)

Only two are independent.

Note: You can write as many independent material balances as there are species involved in the system.
6. Material balance: in $=$ out or in - out $=0$

$$
\begin{array}{lccc}
\mathrm{NaOH} \text { balance: } & 1000=P_{\mathrm{NaOH}} & \text { or } & 1000-P_{\mathrm{NaOH}}=0 \\
\mathrm{H}_{2} \mathrm{O} \text { balance: } & W=P_{\mathrm{H}_{2} \mathrm{O}} & \text { or } & W-P_{\mathrm{H}_{2} \mathrm{O}}=0 \\
\text { Given ratio: } & W=0.9 P & \text { or } & W-0.9 P=0 \\
\text { Sum of components in } P: P_{\mathrm{NaOH}}+P_{\mathrm{H}_{2} \mathrm{O}}=P \text { or } P_{\mathrm{NaOH}}+P_{\mathrm{H}_{2} \mathrm{O}}-P=0 \tag{4}
\end{array}
$$

Could you substitute the total mass balance $1000+\mathrm{W}=\mathrm{P}$ for one of the two component mass balances? Of course In fact, you could calculate $P$ by solving just two equations:

$$
\begin{aligned}
\text { Total balance: } & & 1000+W & =P \\
\text { Given ratio: } & & W & =0.9 P
\end{aligned}
$$

## 7. Solve equations:

$W=0.9 \mathrm{P}$ substitute in total balance $1000+0.9 \mathrm{P}=\mathrm{P}$
$\therefore \mathrm{P}=10000 \mathrm{~kg} \& \mathrm{~W}=0.9 * 10000=9000 \mathrm{~kg} \quad$ (The basis is still $1 \mathrm{hr}\left(\mathrm{F}_{\mathrm{NaOH}}=1000 \mathrm{~kg}\right)$ )
From these two values you can calculate the amount of $\mathrm{H}_{2} \mathrm{O}$ and NaOH in the product

$$
\text { From the }\left\{\begin{array} { l } 
{ \mathrm { NaOH } \text { balance: } } \\
{ \mathrm { H } _ { 2 } \mathrm { O } \text { balance: } }
\end{array} \text { you get } \left\{\begin{array}{l}
P_{\mathrm{NaOH}}=1000 \mathrm{~kg} \\
P_{\mathrm{H}_{2} \mathrm{O}}=9000 \mathrm{~kg}
\end{array}\right.\right.
$$

Then

$$
\begin{aligned}
\omega_{\mathrm{NaOH}}^{P} & =\frac{1000 \mathrm{~kg} \mathrm{NaOH}}{10,000 \mathrm{~kg} \text { Total }}=0.1 \\
\omega_{\mathrm{H}_{2} \mathrm{O}}^{P} & =\frac{9,000 \mathrm{~kg} \mathrm{H}_{2} \mathrm{O}}{10,000 \mathrm{~kg} \text { Total }}=0.9
\end{aligned}
$$

Note

$$
\omega_{\mathrm{NaOH}}^{P}+\omega_{\mathrm{H}_{2} \mathrm{O}}^{P}=1
$$

8. The total balance would have been a redundant balance, and could be used to check the answers

$$
\begin{aligned}
& P_{\mathrm{NaOH}}+P_{\mathrm{H} 2 \mathrm{O}}=P \\
& 1,000+9,000=10,000
\end{aligned}
$$

Note: After solving a problem, use a redundant equation to check your values.

## Degree of Freedom Analysis

The phrase degrees of freedom have evolved from the design of plants in which fewer independent equations than unknowns exist. The difference is called the degrees of freedom available to the designer to specify flow rates, equipment sizes, and so on. You calculate the number of degrees of freedom $\left(\mathrm{N}_{\mathrm{D}}\right)$ as follows:

> Degrees of freedom = number of unknowns - number of independent equations

$$
\mathbf{N}_{\mathbf{D}}=\mathbf{N}_{\mathbf{U}}-\mathbf{N}_{\mathbf{E}}
$$

* When you calculate the number of degrees of freedom you ascertain the solve ability of a problem. Three outcomes exist:

| Case | $\mathbf{N}_{\mathbf{D}}$ | Possibility of Solution |
| :---: | :---: | :---: |
| $\mathrm{N}_{\mathrm{U}}=\mathrm{N}_{\mathrm{E}}$ | 0 | Exactly specified (determined); a solution exists |

