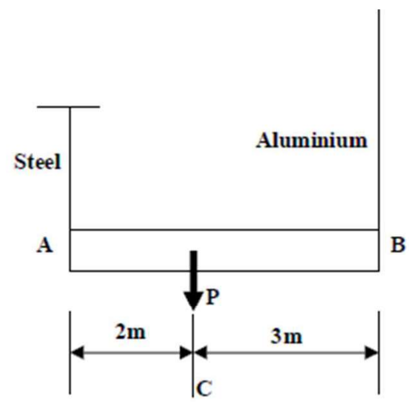
Ex: The rigid bar (AB) attached to two vertical rods as shown in figure, horizontally before the load (P) applied. If the load (P=50kN), determine its vertical movement.

	St.	**Al.**
L (m)	**3**	**4**
Area (mm^2)	**300**	**500**
E (GPa)	**200**	**70**

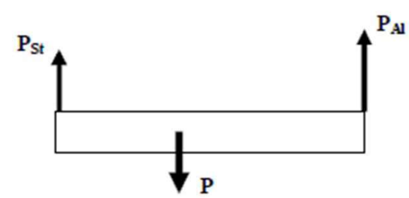


Sol :

From F.B.D

$\therefore \sum F_y = 0 \Rightarrow P_{St} + P_{Al} = P = 50kN$ **------(1)**

$\therefore \sum M_C = 0 \Rightarrow 2 * P_{St} = 3 * P_{Al} \Rightarrow P_{St} = 1.5 P_{Al}$ **-----(2)**



F.B.D

Sub. (1) into (2)

$\therefore 1.5 P_{Al} + P_{Al} = 50kN \Rightarrow P_{Al} = 20kN$

$\therefore P_{St} = 30kN$

$$\therefore \delta_{St} = \frac{P_{St} * L_{St}}{A_{St} * E_{St}} = \frac{30 * 10^3 * 3}{300 * 10^{-6} * 200 * 10^9} = 1.5 * 10^{-3} m = 1.5mm$$

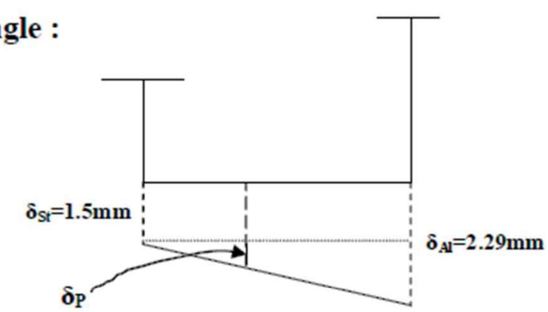
$$\therefore \delta_{Al} = \frac{P_{Al} * L_{Al}}{A_{Al} * E_{Al}} = \frac{20 * 10^3 * 4}{500 * 10^{-6} * 70 * 10^9} = 2.29 * 10^{-3} m = 2.29mm$$

For triangle shown by similarity of triangle :

$$\therefore \frac{\delta_P - 1.5}{2} = \frac{2.29 - 1.5}{5}$$

$$\delta_P = \frac{2(0.79)}{5} + 1.5$$

$$\delta_P = 1.816 \ mm$$



Poisson's Ratio: Biaxial and Triaxial Deformations

The ratio of strain in the lateral direction to the linear strain in the axial direction and it's denoted by the Greek letter υ (nu) and can be expressed as:

$$\upsilon = \frac{Lateral\ Strain}{Longitudinal\ Strain} = -\frac{\varepsilon y}{\varepsilon x}$$

υ : *Poison's Ratio*

$$\upsilon = -\frac{\varepsilon y}{\varepsilon x}$$

$$\varepsilon_y = -\upsilon \times \varepsilon_x = -\upsilon \frac{\sigma x}{E}$$

for biaxial stress state:

$\sigma_x=0 \quad \sigma_y \neq 0 \ \sigma_z=0$

In the x- direction resulting from

σ_x ,$\epsilon_x=\sigma_x/E$

In the y-direction resulting from

σ_y ,$\epsilon_y=\sigma_y/E$

In the x-direction resulting from

σ_y ,$\epsilon_x=-\upsilon(\sigma x E)$

In the y-direction resulting from the

σ_x , $\epsilon_y=-\upsilon(\sigma x E)$

The total strain in the x-direction will be:

$\epsilon_x= \sigma_x/E - \upsilon\ \sigma_y/E$

$\epsilon_x= \sigma_y/E - \upsilon\ \sigma_x/E$

$\epsilon_x= -\upsilon\ \sigma_x/E - \upsilon\ \sigma_y/E$

Triaxial tensile stresses:

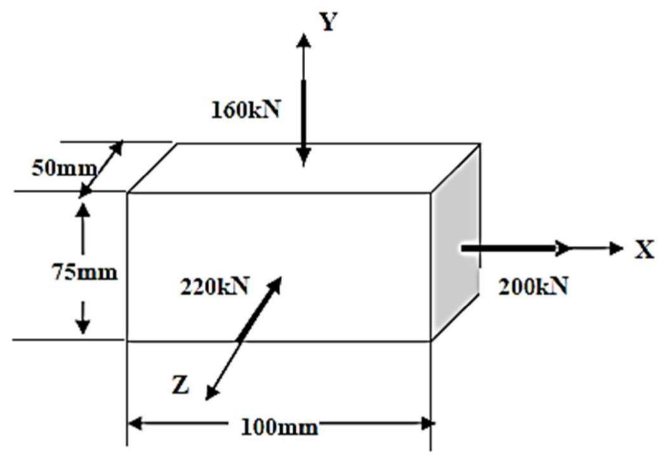
$\sigma_x \neq 0 \quad \sigma_y \neq 0 \; \sigma_z \neq 0$

$\epsilon_x = \sigma_x/E - \upsilon\, \sigma_y/E - \upsilon\, \sigma_z/E$

$\epsilon_y = \sigma_y/E - \upsilon\, \sigma_x/E - \upsilon\, \sigma_z/E$

$\epsilon_z = \sigma_z/E - \upsilon\, \sigma_x/E - \upsilon\, \sigma_y/E$

Ex: -9- A rectangular Aluminum block is (100mm) long in X-direction , (75mm) wide in Y-direction and (50mm) thick in Z-direction . It is subjected to try axial loading consisting of uniformly distributed tensile force of (200kN) in the X-direction and uniformly distributed compressive forces of 160kN in Y-direction and (220kN) in Z-direction. If the Poisson's ratio (υ =0.333) and (E=70GPa). Determine a single distributed load that must applied in Xdirection that would produce the same deformation in Z-direction as original loading.



$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

$$\sigma_x = \frac{P_x}{A} = \frac{200*10^3}{0.05*0.075} = 53.3MPa\ (tension)$$

$$\sigma_y = \frac{160*10^3}{0.05*0.1} = 32MPa\ (compression)$$

$$\sigma_z = \frac{220*10^3}{0.075*0.1} = 24.34MPa\ (compression)$$

$$\therefore \varepsilon_z = \frac{1}{70*10^9}[-24.34 - 0.333(-32+53.3)]*10^6$$

$$\varepsilon_z = -0.52*10^{-3}$$

$$\varepsilon_z = -\upsilon\frac{\sigma_x}{E} = -\upsilon\frac{P_x}{A*E} \Rightarrow -0.52*10^{-3} = -0.333\frac{P_x}{0.075*0.05*70*10^9}$$

$$\therefore P_x = 410\ kN\ (Tension)$$