1.3.2. Pressure Drop

As the fluid flows inside the tubes through a condenser or evaporator, a pressure drop occurs both in the straight tubes and in the U-bends or heads of the heat exchanger. Some drop in pressure is also attributable to entrance and exit losses.

$$\Delta p = f \frac{L}{D} \frac{V^2}{2} \rho \qquad (1\text{-}10)$$

where Δp = pressure drop, Pa
f = friction factor, dimensionless

Since the pressure drop in the straight tubes in an evaporator or condenser may represent only 50 to 80 percent of the total pressure drop, experimental or catalog data on the pressure drop as a function of flow rate are desirable. If the pressure drop at one flow rate is known, it is possible to predict the pressure drop at other flow rates. The expression applicable to straight tubes, Eq. (1 -10), indicates that the pressure drop is proportional to the square of the velocity and thus the square of the flow rate.

The other contributors to pressure drop resulting from changes in flow area and direction are also almost exactly proportional to the square of the flow rate, so if the pressure drop and flow rate Δp_1 and w_1 are known, the pressure drop Δp_2 at a different flow rate w_2 can be predicted:

$$\Delta p_2 = \Delta p_1 \left(\frac{w_2}{w_1}\right)^2 \qquad (1\text{-}11)$$

1.4. Liquid in shell; heat transfer and pressure drop

In shell-and-tube evaporators, where refrigerant boils inside tubes, the liquid being cooled flows in the shell across bundles of tubes, as shown schematically in Fig. 4. The liquid is directed by baffles so that it flows across the tube bundle many times and does not short-circuit from the inlet to the outlet. The analytical prediction of the heat-transfer coefficient of liquid flowing normal to a tube is complicated in itself, and the complex flow pattern over a bundle of tubes makes the prediction even more difficult. In order to proceed with the business of designing heat exchangers, engineers resort to correlations that relate the Nusselt, Reynolds, and Prandtl numbers to the geometric configuration of the tubes and baffles. Such an equation by Emerson can be modified to the form

$$\frac{hD}{k} = (\text{terms controlled by geometry})\,(\text{Re}^{0.6})\,(\text{Pr}^{0.3})\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad (1\text{-}12)$$

where μ = viscosity of fluid at bulk temperature, Pa · s
μ_w = viscosity of fluid at tube-wall temperature, Pa · s

Although in this text we shall delve no deeper into the complexities of designing a shell-and-tube heat exchanger, one important but simple realization emerges from Eq. (1 -12): for a given evaporator or condenser when water flows in the shell outside the tubes

$$\text{Water-side heat-transfer coefficient} = (\text{const})\,(\text{flow rate})^{0.6} \qquad (1\text{-}13)$$

The convection coefficient varies as the 0.6 power of the flow rate compared with the 0.8 power for flow inside tubes, as indicated by Eq. (1 -9).

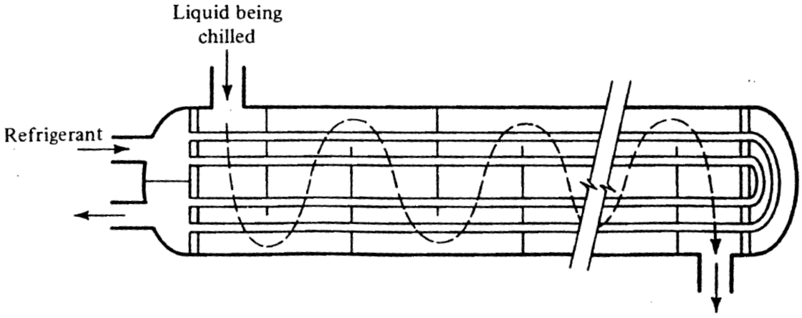


Fig. 4. Shell flow of liquid across tube bundles.

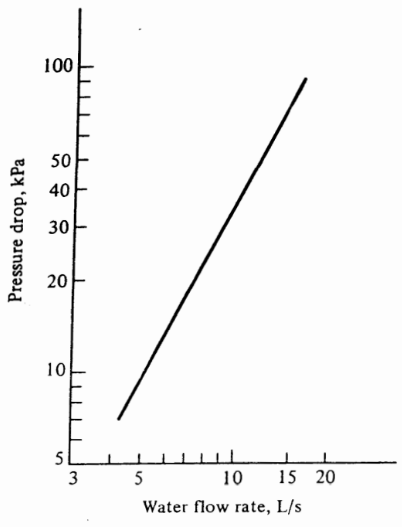


Figure 12-5 Pressure drop of water flowing in the shell of an evaporator. *(York Division of Borg Warner.)*

The pressure drop of liquid flowing through the shell across tube bundles is also difficult to predict analytically, but when an experimental point is available for one flow rate, predictions of the pressure drop at other flow rates can be made quite accurately.

Figure 12-5 shows the water pressure drop taken from catalog data of a water-chilling evaporator. The applicable exponent in the pressure-drop-flow-rate relationship here is 1.9

1.5. Extended surface; fins

Extended surface; fins: Equation (1-8) expresses the resistances a heat exchanger encounters when transferring heat from one fluid to another. Suppose that in Eq. (1-8) $1/h_oA_o$ is 80 percent of the total resistance to heat transfer. Efforts to improve the U value by increasing h_i provide only modest benefits. If, for example, h_i were doubled so that $1/h_iA_i$ is cut in half, the decrease in the total resistance could at best be reduced by 10 percent. The resistance on the outside of the tube, $1/h_oA_o$, is said to be the *controlling resistance.*

When one of the fluids in a condenser or evaporator is a gas (hereafter considered to be air), the properties of the air compared with those of the liquid, such as water, result in heat-transfer coefficients of the order of one-tenth to one-twentieth that of the water. The air-side resistance in a configuration such as shown in Fig. 2 would provide the controlling resistance. In order to decrease *1/hA*, the area *A* is usually increased by using fins.

The bar fin, shown in Fig. 1-3 6 rudimentary fin whose performance can be predicted analytically and will be used to illustrate some important characteristics. The fins are of length L and thickness $2y$ m. The conductivity of the metal is k W/m · K, and the air-side coefficient is h_f W/m^2 · K. To solve for the temperature distribution through the fin, a heat balance can be written about an element of thickness dx m. The heat balance states that the rate of heat flow entering the element at position 1

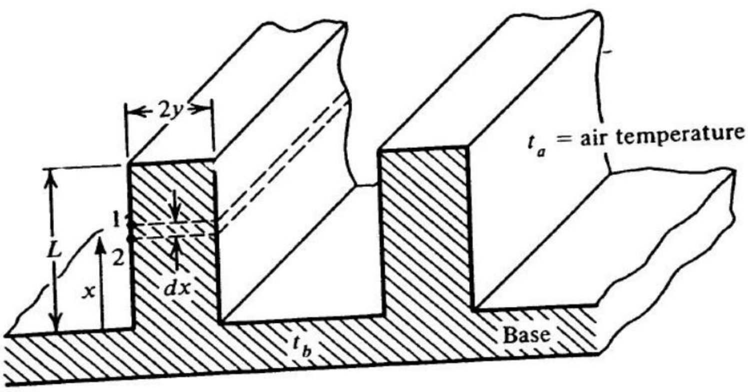


Figure 6 Bar fin.

from the end of the fin plus that transferred to the element from the air equals the rate of heat transferred out of the element at position 2 toward the base. For one-half a fin width and a fin depth of Z m, the heat balance in symbols is

$$kyZ\left(\frac{dt}{dx}\right)_1 + Z\,dx\,h_f(t_a - t) = kyZ\left(\frac{dt}{dx}\right)_2 \qquad (1\text{-}14)$$

where t_a = temperature of air
t = temperature of fin

Canceling Z and factoring gives

$$ky\left[\left(\frac{dt}{dx}\right)_2 - \left(\frac{dt}{dx}\right)_1\right] = dx\,h_f(t_a - t) \qquad (1\text{-}15)$$

For the differential length dx the change in the temperature gradient is

$$\left(\frac{dt}{dx}\right)_1 - \left(\frac{dt}{dx}\right)_2 = \frac{d}{dx}\left(\frac{dt}{dx}\right)dx = \frac{d^2t}{dx^2}\,dx \qquad (1\text{-}16)$$

Substituting into Eq. 1-15, we get

$$\frac{d^2t}{dx^2} = \frac{h_f(t_a - t)}{ky} \qquad (1\text{-}17)$$

By solving the second-order differential equation (1-17) the temperature distribution throughout the fin can be shown to be

$$\frac{t - t_b}{t_a - t_b} = \frac{\cosh M(L - x)}{\cosh ML} \qquad (1\text{-}18)$$

where t_b = temperature of base of fin, °C

$$M = \sqrt{\frac{h_f}{ky}}$$

When a finned coil cools air, points in the fin farther away from the base are higher

Fin effectiveness

The ratio of the actual rate of heat transfer to that which would be transferred if the fin were at temperature t_b is called the *fin effectiveness*

$$\text{Fin effectiveness} = \eta = \frac{\text{actual } q}{q \text{ if fin were at base temperature}} \qquad (1\text{-}19)$$

Harper and Brown found that the fin effectiveness for the bar fin at Fig. 6 can be represented by

$$\eta = \frac{\tanh ML}{ML}$$

- The bar fin is not a common shape but the dominant type of finned surface is the rectangular plate fin mounted on cylindrical tubes. The net result is a rectangular or square fin mounted on a circular base, one section of which is shown in Fig. 7 *a*. The fin effectiveness of the rectangular plate fin is often calculated by using properties of the corresponding annular fin (Fig. 7 *b*), for which a graph of the fin effectiveness is available, as in Fig. 8. The corresponding annular fin has the same area and thickness as the plate fin it represents.

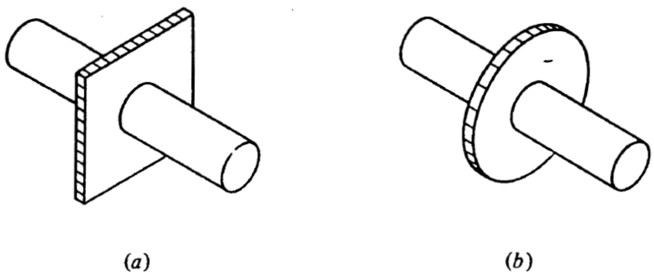


Figure 7 Determining fin effectiveness of a rectangular plate fin (a) by treating it as an (b) annular fin of the same area.

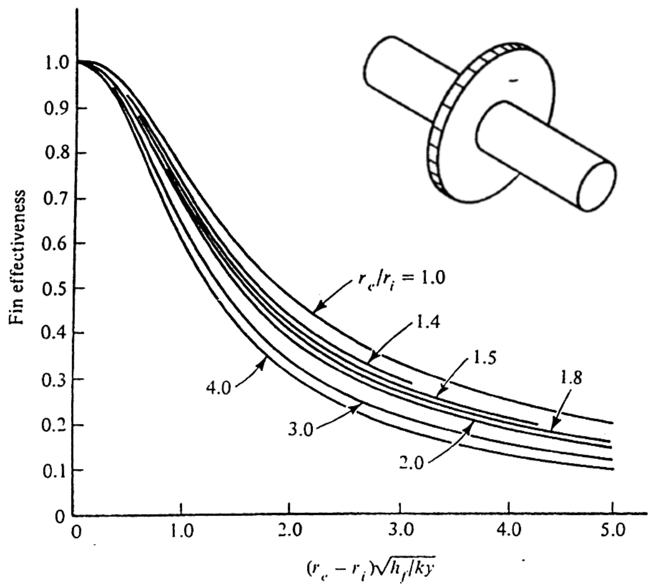
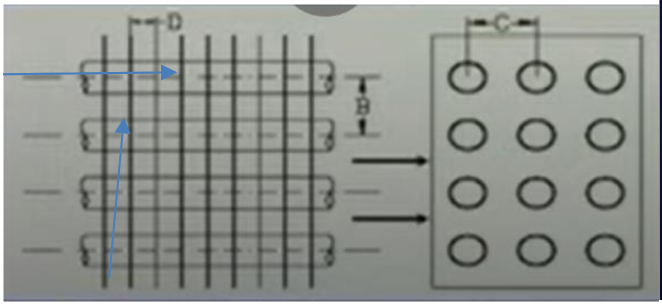
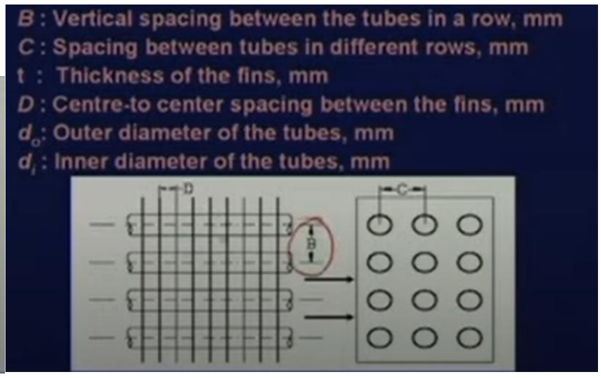


Figure 8 Fin effectiveness of an annular fin.





Illustrative Example: What is the fin effectiveness of a rectangular plate fin made of aluminum 0.3 mm thick mounted on a 16-mm-OD tube if the vertical tube spacing is 50 mm and the horizontal spacing is 40 mm? The air-side heat-transfer coefficient is 65 W/m^{2}K, and the conductivity of aluminum is 202 W/mK.

Solution The annular fin having the same area as the plate fin (Fig. 8) has an external radius of 25.2 mm. The half-thickness of the fin y = 0.15 mm
2y=0.3
Y=0.3/2=0.15 mm=0.00015 m

$$M = \sqrt{\frac{65}{202(0.00015)}} = 46.3 \text{ m}^{-1}$$

$$(r_e - r_i)M = (0.0252 - 0.008)(46.3) = 0.8$$

From Fig. 12-8 for $(r_e - r_i)M = 0.8$ and $r_e/r_i = 25.2/8 = 3.15$ the fin effectiveness η is 0.72.

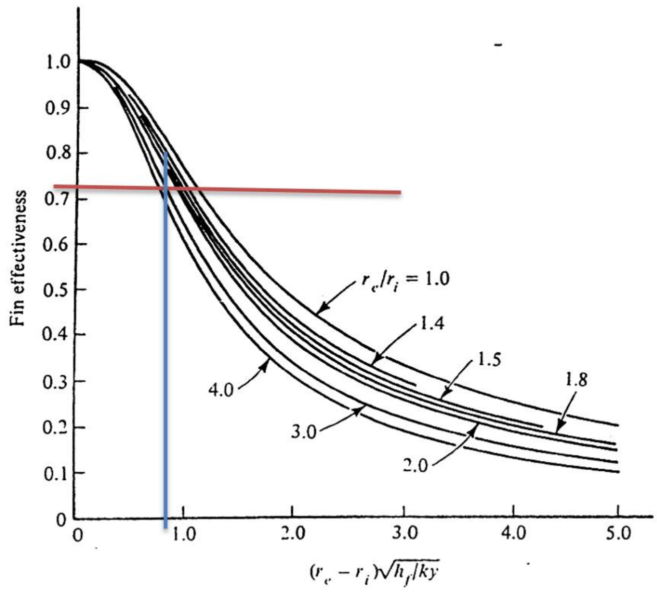


Figure 8 Fin effectiveness of an annular fin.

Case Study: The air-side area of a finned condenser or evaporator is composed of two portions, the prime area and the extended area. The *prime* area A_P is that of the tube between the fins, and the *extended* area A_e is that of the fin. Since the prime area is at the base temperature, it has a fin effectiveness of 1.0. It is to the extended surface that the fin effectiveness less than 1.0 applies. Equation (8) for the overall heat-transfer coefficient can be revised to read

$$\frac{1}{U_oA_o} = \frac{1}{U_iA_i} = \frac{1}{h_oA_o} + \frac{x}{kA_m} + \frac{1}{h_iA_i}$$

$$\frac{1}{U_oA_o} = \frac{1}{U_iA_i} = \frac{1}{h_f(A_p + \eta A_e)} + \frac{x}{kA_m} + \frac{1}{h_iA_i} \qquad (1\ \text{-}20)$$

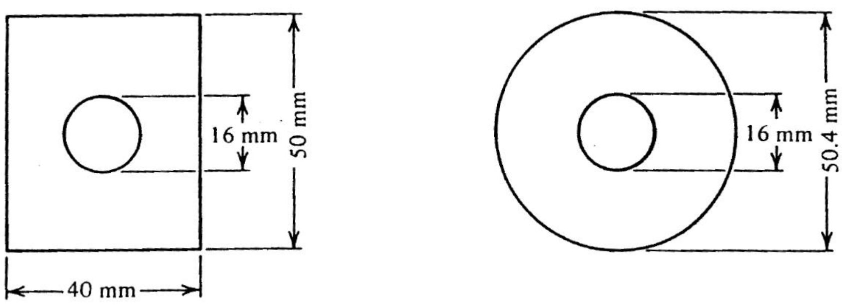


Figure 9 Annular fin of same area as rectangular plate fin.

Problem. 3: (a) Compute the fin effectiveness of a bar fin made of aluminum that is 0.12 mm thick and 20 mm long when h_f = 28 W/m^2.K, the base temperature is 4 C, and the air temperature is 20 C.

(b) If you are permitted to use twice as much metal for the fin as originally specified in part (a) and you can either double the thickness or double the length, which choice would be preferable in order to transfer the highest rate of heat flow. Why?

Solution:

(a) Aluminum fins

k = 202 W/m.K

2y = 0.12 mm = 0.00012 m

y = 0.00006 m

L = 20 mm = 0.020 m

$$M = \sqrt{\frac{h_f}{ky}}$$

$$M = \sqrt{\frac{28}{(202)(0.00006)}}$$

M = 48.1 m^{-1}

$$\eta = \frac{\tanh ML}{ML}$$

ML = (48.1 m^{-1})(0.020 m) = 0.962

$$\eta = \frac{\tanh(0.962)}{0.962}$$

η = 0.7746 - - - - Ans.

(b) If the fin thickness is doubled.

2y = 0.24 m = 0.00024 m

y = 0.00012 m

$$M = \sqrt{\frac{28}{(202)(0.00012)}}$$

M = 33.99 m^{-1}

$$\eta = \frac{\tanh ML}{ML}$$

ML = (33.99 m^{-1})(0.020 m) = 0.6798

$$\eta = \frac{\tanh(0.6798)}{0.6798}$$

η = 0.87 > 0.7746

If the length L is doubled

L = 40 mm = 0.040 m

$$M = \sqrt{\frac{28}{(202)(0.00006)}}$$

M = 48.1 m^{-1}

$$\eta = \frac{\tanh ML}{ML}$$

ML = (48.1 m^{-1})(0.040 m) = 1.924

$$\eta = \frac{\tanh(1.924)}{1.924}$$

η = 0.498 < 0.7746

Ans. Therefore double the fin thickness to improve rate of heat flow with an efficiency of 87 % compared to 77.46 %.

Problem. 4: Compute the fin effectiveness of an aluminum rectangular plate fin of a finned air-cooling evaporator if the fins are 0.18 mm thick and mounted on a 16-mm-OD tubes. The tube spacing is 40 mm in the direction of air flow and 45 mm vertically. The air-side coefficient is 55 W/m^2.K.

Solution

$h_f = 55$ W/m^2.K

Alumimum Fins, k = 202 W/m.K

$2y = 0.00018$ mm

$y = 0.00009$ mm

$$M = \sqrt{\frac{h_f}{ky}}$$

$$M = \sqrt{\frac{55}{(202)(0.00009)}}$$

$M = 55$ m^{-1}.

Equivalent external radius.

$$\pi\left[(r_e)^2 - \left(\frac{16}{2}\right)^2\right] = (40)(45) - \pi\left(\frac{16}{2}\right)^2$$

$r_e = 23.94$ mm $= 0.02394$ m

$r_i = 8$ mm $= 0.008$ m

$(r_e - r_i)M = (0.02394 - 0.008)(55) - 0.88$

$r_e/r_i = 23.94$ mm / 8 mm = 3

From Fig. 12-8/

Fin Effectiveness = 0.68 - - - Ans.

Problem. 5: A shell-and-tube condenser has a U value of 800 W/m2 .K based on the water-side are and a water pressure drop of 50 kPa. Under this operating condition 40 percent of the heat-transfer resistance is on the water side. If the water-flow rate is doubled, what will the new U value and the new pressure drop be?

U_1 = 800 W/m2.K

h_1 = Water-side coefficient

$$h_1 = \frac{1}{(0.40)\left(\frac{1}{800}\right)} = 2{,}000$$

Eq. 12-13, replace 0.6 by 0.8 for condenser.

Water-side coefficient = (const)(flow rate)$^{0.8}$

For $w_2 / w_1 = 2$

$$\frac{h_2}{h_1} = \left(\frac{w_2}{w_1}\right)^{0.8}$$

$h_2 = (2000)(2)^{0.8} = 3482.2$ W/m^2.K

Remaining resistance = (0.60)(1 / 800) = 0.00075

New U-Value:

$$\frac{1}{U_2} = \frac{1}{3482.2} + 0.00075$$

U_2 = 964 W/m^2.K - - - Ans.

New Pressure Drop:

$$\Delta p_2 = \Delta p_1 \left(\frac{w_2}{w_1}\right)^2$$

$$\Delta p_2 = (50)(2)^2$$

Δp_2 = 200 kPa - - - Ans.

1.6. Gas flowing over finned tubes; heat transfer and pressure drop

A precise prediction of the air-side heat-transfer coefficient when the air flows over finned tubes is complicated because the value is a function of geometric factors, e.g., the fin spacing, the spacing and diameter of the tubes, and the number of rows of tubes deep. Usually the coefficient varies approximately as the square root of the face velocity of the air. A rough estimate of the air-side coefficient h_f can be computed from the equation derived from illustrative data in the ARI standard

$$h_f = 38V^{0.5}$$

Rich[7] conducted tests of coils of various fin spacings and correlated the dimensionless heat-transfer numbers with specially defined Reynolds numbers.

The drop in pressure of the air flowing through a finned coil is also dependent upon the geometry of the coil. Figure 12-10 shows the pressure drop of a commercial cooling coil when the finned surfaces are dry. As expected, the pressure drop is higher for coils with a large number of fins per meter of tube length. The ordinate is the pressure drop per number of rows of tubes deep, so the values would be multiplied by 6 for a six-row coil, for example.

For the coil series whose pressure drops are shown in Fig. 12-10 the pressure drop for a given coil varies as the face velocity to the 1.56 power. That exponent is fairly typical of commercial plate-fin coils.

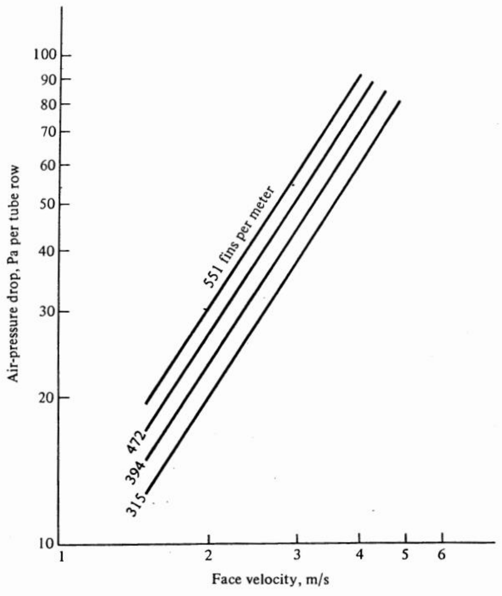


Figure 12-10 Pressure drop of air flowing through a finned coil *(Bohn Heat Transfer Division of Gulf & Western Manufacturing Company.)*