Contents

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4th CLASS
LECTURE No. 4

CHAPTER

13

Keys and Coupling

13.1 Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

13.2 Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys, and **5.** Splines.

We shall now discuss the above types of keys, in detail, in the following pages.

Top

13.3 Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. *Rectangular sunk key.* A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

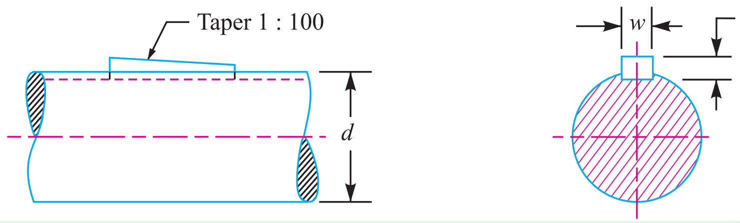

Fig. 13.1. Rectangular sunk key.

2. *Square sunk key.* The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. *Parallel sunk key.* The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. *Gib-head key.* It is a rectangular sunk key with a head at one end known as ***gib head.*** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 13.2 (*a*) and its use in shown in Fig. 13.2 (*b*).

Helicopter driveline couplings.

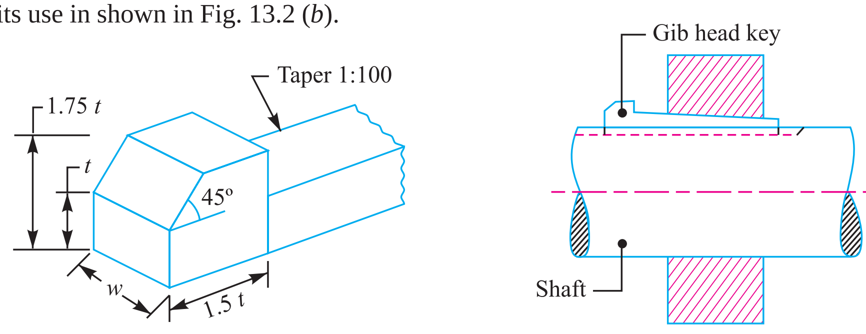

Fig. 13.2. Gib-head key.

The usual proportions of the gib head key are :

Width, $w = d / 4$;

and thickness at large end, $t = 2w / 3 = d / 6$

5. *Feather key.* A key attached to one member of a pair and which permits relative axial movement is known as ***feather key.*** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

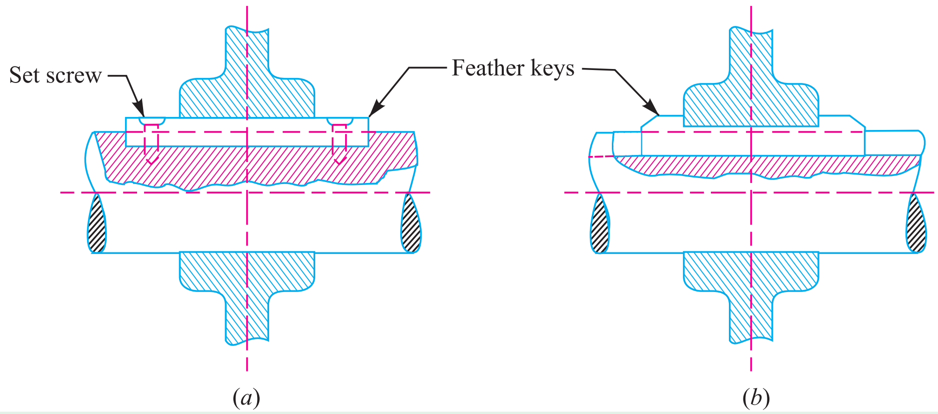

Fig. 13.3. Feather key.

The feather key may be screwed to the shaft as shown in Fig. 13.3 (*a*) or it may have double gib heads as shown in Fig. 13.3 (*b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

The following table shows the proportions of standard parallel, tapered and gib head keys, according to IS : 2292 and 2293-1974 (Reaffirmed 1992).

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

6. *Woodruff key.* The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 13.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

Contents

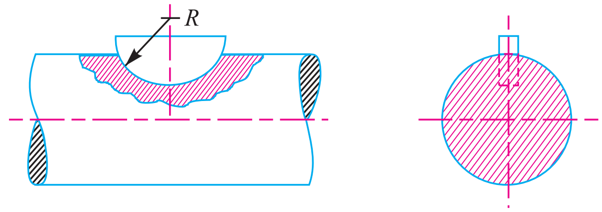

Fig. 13.4. Woodruff key.

The main advantages of a woodruff key are as follows :

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.

13.4 Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and **2.** Hollow saddle key.

A ***flat saddle key*** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. 13.5. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

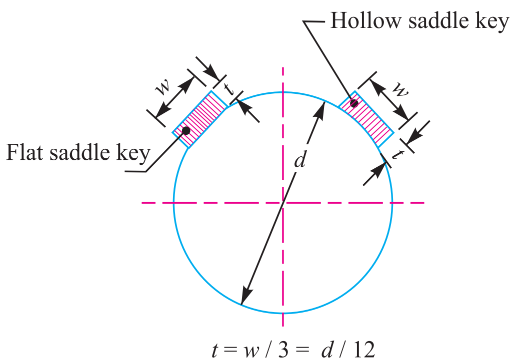

Fig. 13.5. Saddle key.

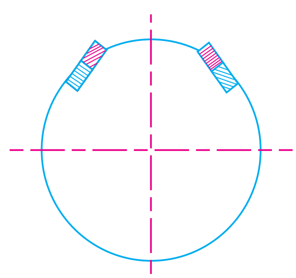

Fig. 13.6. Tangent key.

A ***hollow saddle key*** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

13.5 Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. 13.6. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

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* The usual form of rectangular sunk key is very likely to turn over in its keyway unless well fitted as its sides.

Top

Contents

13.6 Round Keys

The round keys, as shown in Fig. 13.7(*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

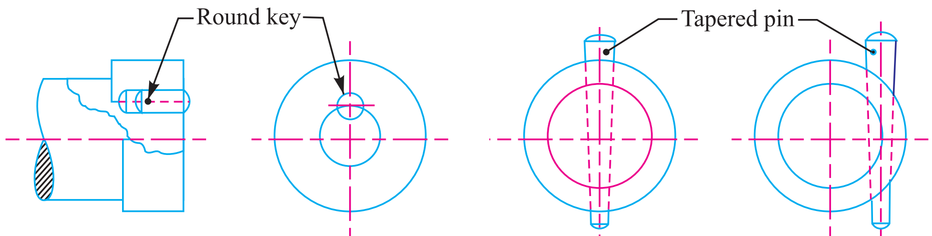
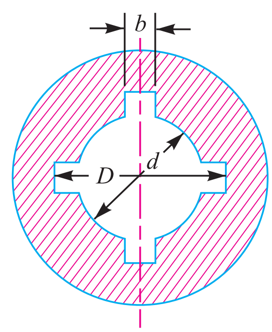


Fig. 13.7. Round keys.

Sometimes the tapered pin, as shown in Fig. 13.7 (*b*), is held in place by the friction between the pin and the reamed tapered holes.

13.7 Splines



$D = 1.25\ d$ and $b = 0.25\ D$

Fig. 13.8. Splines.

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. 13.8. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.

13.8 Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 13.9.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

Top

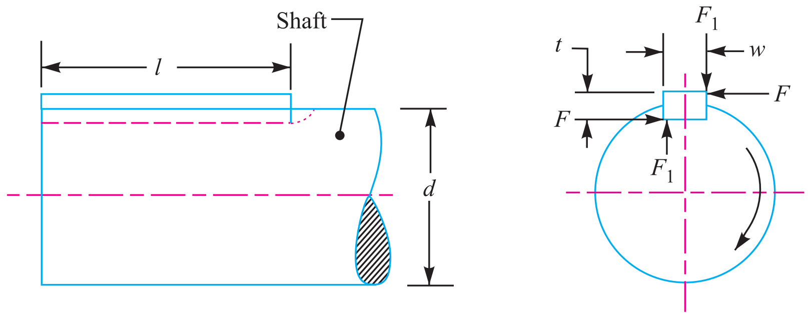


Fig. 13.9. Forces acting on a sunk key.

13.9 Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 13.9.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...[Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad ...(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation (*iii*), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

Contents

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{...}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{...}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \quad \text{... (Taking } w = d/4) \quad \text{...}(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \quad \text{... [From equation } (vi)]$$

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : d = 50 mm ; τ = 42 MPa = 42 N/mm^2 ; σ_c = 70 MPa = 70 N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, w = 16 mm **Ans.**

and thickness of key, t = 10 mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \quad \text{...}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\, (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \quad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

l = 117.7 say 120 mm **Ans.**

Top

Contents

Example 13.2. *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\ (45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\ l$$

$\therefore$ l = 1.8 × 10^6 / 26 775 = 67.2 mm

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\ l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$\therefore$ l = 1.8 × 10^6 / 17 213 = 104.6 mm

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

13.10 Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

Top

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Example 13.3. *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ; τ = 56 MPa = 56 N/mm^2 ; σ_c = 112 MPa = 112 N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$\therefore$ $w = 149 \times 10^3 / 84 \times 10^3 = 1.8$ mm

This width of keyway is too small. The width of keyway should be at least d / 4.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm} \quad \textbf{Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\;0.8125 = 571\,844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47 \quad \textbf{Ans.}$$

13.11 Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Contents

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

Couplings

Note : A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator.

13.12 Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

13.13 Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows :

1. ***Rigid coupling.*** It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view :

(*a*) Sleeve or muff coupling.

(*b*) Clamp or split-muff or compression coupling, and

(*c*) Flange coupling.

2. ***Flexible coupling.*** It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :

(*a*) Bushed pin type coupling,

(*b*) Universal coupling, and

(*c*) Oldham coupling.

We shall now discuss the above types of couplings, in detail, in the following pages.

Flexible PVC (non-metallic) coupling

Note : This picture is given as additional information and is not a direct example of the current chapter.

Top

Contents

13.14 Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. 13.10. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, $D = 2d + 13$ mm

and length of the sleeve, $L = 3.5\ d$

where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. *Design for sleeve*

The sleeve is designed by considering it as a hollow shaft.

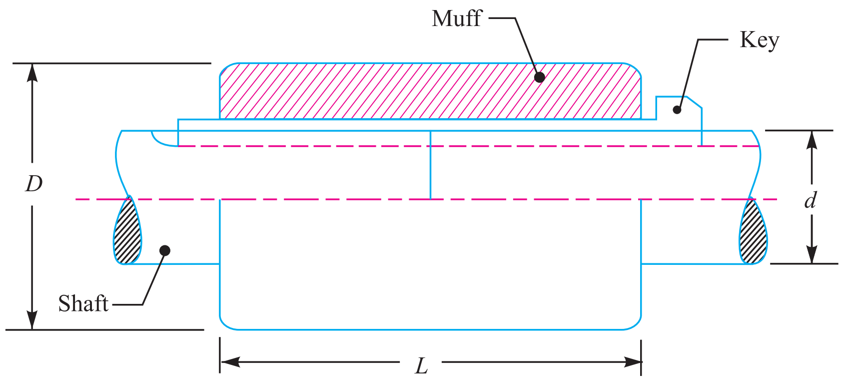


Fig. 13.10. Sleeve or muff coupling.

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \text{... } (\because\ k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. *Design for key*

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 d). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\ d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

Top

Contents

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : P = 40 kW = 40 × 10^3 W; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm^2; σ_{cs} = 80 MPa = 80 N/mm^2; τ_c = 15 MPa = 15 N/mm^2

A type of muff couplings.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ d^3 = 1100 × 10^3/7.86 = 140 × 10^3 or d = 52 say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm} \quad \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$\therefore$ τ_c = 1100 × 10^3/370 × 10^3 = 2.97 N/mm^2

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

Top

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

Spilt-sleeve coupling.

Note : This picture is given as additional information and is not a direct example of the current chapter.

13.15 Clamp or Compression Coupling

It is also known as ***split muff coupling.*** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. 13.11. The halves of the muff are made of cast iron. The shaft ends are made to abutt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the

(a) Heavy duty flex-flex coupling.

(b) Heavy duty flex-rigid coupling.

coupling. The usual proportions of the muff for the clamp or compression coupling are :

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\, d$

where d = Diameter of the shaft.

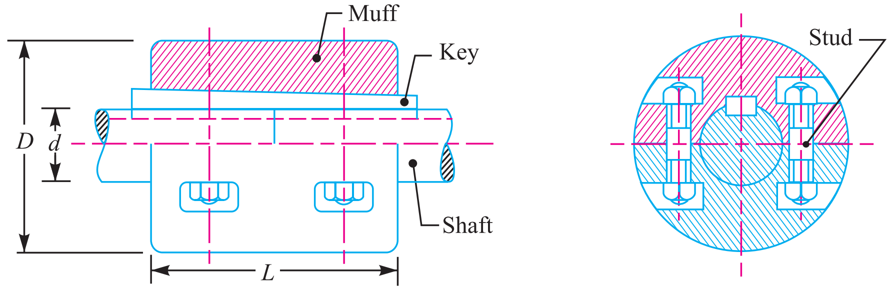


Fig. 13.11. Clamp or compression coupling.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling (Art. 13.14).

2. *Design of clamping bolts*

Let T = Torque transmited by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2\,\sigma_t$$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2\,\sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2\,\sigma_t \times \frac{n}{2}}{\frac{1}{2}L \times d}$$

∴ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2\,\sigma_t \times \frac{n}{2}}{\frac{1}{2}L \times d} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\ (d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 13.5. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : P = 30 kW = 30 × 10^3 W ; N = 100 r.p.m. ; τ = 40 MPa = 40 N/mm^2 ; n = 6 ; σ_t = 70 MPa = 70 N/mm^2 ; μ = 0.3

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm}$$ **Ans.**

2. *Design for muff*

We know that diameter of muff,

D = 2d + 13 mm = 2 × 75 + 13 = 163 say 165 mm **Ans.**

and total length of the muff,

L = 3.5 d = 3.5 × 75 = 262.5 mm **Ans.**

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\ 70 \times 6 \times 75 = 5830(d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

13.16 Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess.

Contents

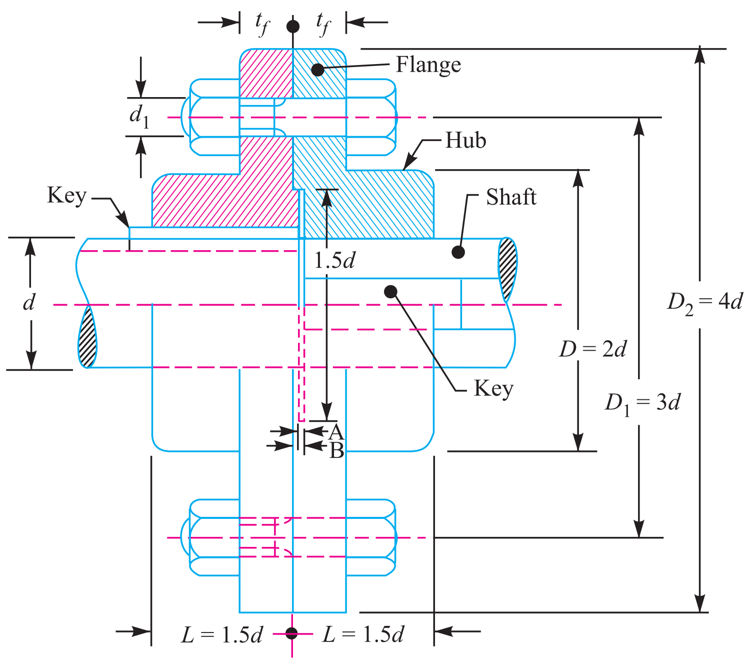


Fig. 13.12. Unprotected type flange coupling.

This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting. The flange couplings are of the following three types :

Flange Couplings.

1. *Unprotected type flange coupling.* In an unprotected type flange coupling, as shown in Fig. 13.12, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways.

The usual proportions for an unprotected type cast iron flange couplings, as shown in Fig. 13.12, are as follows :

If d is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub,

$$D = 2\,d$$

Top

Length of hub, $L = 1.5\ d$

Pitch circle diameter of bolts,

$$D_1 = 3d$$

Outside diameter of flange,

$$D_2 = D_1 + (D_1 - D) = 2\ D_1 - D = 4\ d$$

Thickness of flange, $t_f = 0.5\ d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

2. *Protected type flange coupling.* In a protected type flange coupling, as shown in Fig. 13.13, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

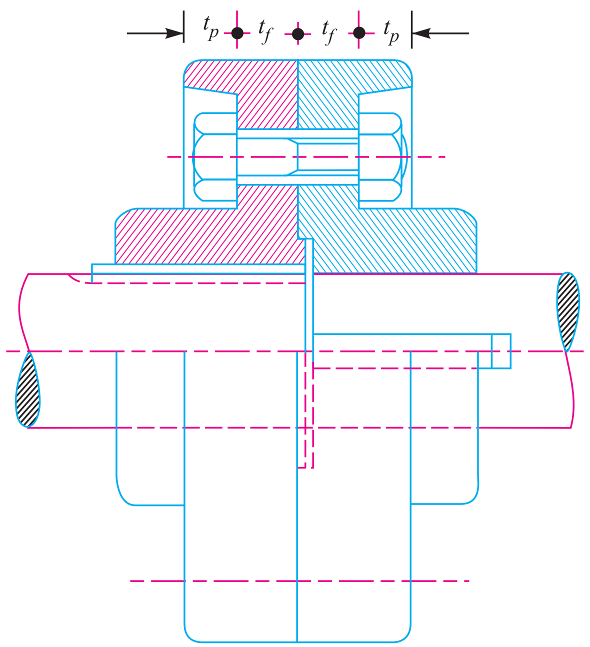


Fig. 13.13. Protective type flange coupling.

The thickness of the protective circumferential flange (t_p) is taken as 0.25 *d*. The other proportions of the coupling are same as for unprotected type flange coupling.

3. *Marine type flange coupling.* In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. 13.14. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.

The number of bolts may be choosen from the following table.

Table 13.2. Number of bolts for marine type flange coupling.
(According to IS : 3653 – 1966 (Reaffirmed 1990))

Shaft diameter (mm)	35 to 55	56 to 150	151 to 230	231 to 390	Above 390
No. of bolts	4	6	8	10	12

Contents

The other proportions for the marine type flange coupling are taken as follows :

Thickness of flange $= d / 3$

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, $D_1 = 1.6\ d$

Outside diameter of flange, $D_2 = 2.2\ d$

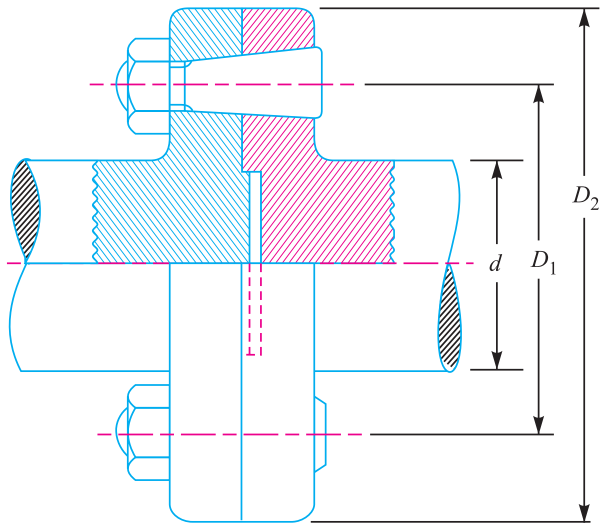


Fig. 13.14. Marine type flange coupling.

13.17 Design of Flange Coupling

Consider a flange coupling as shown in Fig. 13.12 and Fig. 13.13.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.* cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below :

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\therefore \qquad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses.

Top

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

$$T = \text{Circumference of hub} \times \text{Thickness of flange} \times \text{Shear stress of flange} \times \text{Radius of hub}$$

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

$$\text{Load on each bolt} = \frac{\pi}{4}(d_1)^2 \tau_b$$

∴ Total load on all the bolts

$$= \frac{\pi}{4}(d_1)^2 \tau_b \times n$$

and torque transmitted,
$$T = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f)\,\sigma_{cb}$$

∴ Torque,
$$T = (n \times d_1 \times t_f \times \sigma_{cb})\,\frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Example 13.6. *Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :*

Shear stress for shaft, bolt and key material	=	*40 MPa*
Crushing stress for bolt and key	=	*80 MPa*
Shear stress for cast iron	=	*8 MPa*

Draw a neat sketch of the coupling.

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2

The protective type flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm } \textbf{Ans.}$$

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3 / 63\,147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, $w = 12$ mm **Ans.**

and thickness of key, $t = w = 12$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$$\therefore \quad l = L = 52.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\, \tau_k$$

$$\therefore \quad \tau_k = 215 \times 10^3 / 11\,025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\, \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 215 \times 10^3 / 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\, d = 0.5 \times 35 = 17.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

$\therefore \quad \tau_c = 215 \times 10^3/134\ 713 = 1.6 \text{ N/mm}^2 = 1.6 \text{ MPa}$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$$n = 3$$

and pitch circle diameter of bolts,

$$D_1 = 3d = 3 \times 35 = 105 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$\therefore \quad (d_1)^2 = 215 \times 10^3/4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\ d = 4 \times 35 = 140 \text{ mm}$$ **Ans.**

Thickness of the protective circumferential flange,

$$t_p = 0.25\ d = 0.25 \times 35 = 8.75 \text{ say } 10 \text{ mm}$$ **Ans.**

Example 13.7. *Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 r.p.m. and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_s = 40$ MPa $= 40$ N/mm^2 ; $\tau_b = 30$ MPa $= 30$ N/mm^2 ; $\sigma_{ck} = 2\tau_k$; $T_{max} = 1.25\ T_{mean}$; $\tau_c = 14$ MPa $= 14$ N/mm^2

The protective type of cast iron flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of shaft (d). We know that the full load or mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\ \pi N} = \frac{15 \times 10^3 \times 60}{2\ \pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

and maximum torque transmitted,

$$T_{max} = 1.25\ T_{mean} = 1.25 \times 716 \times 10^3 = 895 \times 10^3 \text{ N-mm}$$

We also know that maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

Contents

∴ $d^3 = 895 \times 10^3 / 7.86 = 113\ 868$ or $d = 48.4$ say 50 mm **Ans.**

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 50 = 100 \text{ mm } \textbf{Ans.}$$

and length of the hub, $L = 1.5\,d = 1.5 \times 50 = 75$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron, by considering it as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left(\frac{(100)^4 - (50)^2}{100}\right) = 184\ 100\ \tau_c$$

∴ $\tau_c = 895 \times 10^3 / 184\ 100 = 4.86 \text{ N/mm}^2 = 4.86$ MPa

Since the induced shear stress in the hub is less than the permissible value of 14 MPa, therefore the design for hub is safe.

(a) Miniature flexible coupling

(b) Miniature rigid coupling

(c) Rigid coupling.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress, therefore a square key may be used.

From Table 13.1, we find that for a 50 mm diameter shaft,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = w = 16$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

∴ $l = L = 75$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 75 \times 16 \times \tau_k \times \frac{50}{2} = 30 \times 10^3\ \tau_k$$

∴ $\tau_k = 895 \times 10^3 / 30 \times 10^3 = 29.8 \text{ N/mm}^2 = 29.8$ MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 75 \times \frac{16}{2} \times \sigma_{ck} \times \frac{50}{2} = 15 \times 10^3\ \sigma_{ck}$$

∴ $\sigma_{ck} = 895 \times 10^3 / 15 \times 10^3 = 59.6 \text{ N/mm}^2 = 59.6$ MPa

Since the induced shear and crushing stresses in key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

Top

Contents

$\therefore \quad t_f = 0.5 \times 50 = 25$ mm **Ans.**

Let us now check the induced shear stress in the flange, by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (100)^2}{2} \times \tau_c \times 25 = 392\ 750\ \tau_c$$

$\therefore \quad \tau_c = 895 \times 10^3 / 392\ 750 = 2.5$ N/mm^2 = 2.5 MPa

Since the induced shear stress in the flange is less than the permissible value of 14 MPa, therefore the design for flange is safe.

4. Design for bolts

Let $\quad d_1$ = Nominal diameter of bolts.

Since the diameter of shaft is 50 mm, therefore let us take the number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 50 = 150 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\, 30 \times 4 \times \frac{150}{4} = 7070\ (d_1)^2$$

$\therefore \quad (d_1)^2 = 895 \times 10^3 / 7070 = 126.6 \quad$ or $\quad d_1 = 11.25$ mm

Assuming coarse threads, the nearest standard diameter of the bolt is 12 mm (M 12). **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 50 = 200 \text{ mm } \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 50 = 12.5 \text{ mm } \textbf{Ans.}$$

Example 13.8. *Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa.*

Solution. Given : $P = 90$ kW = 90×10^3 W ; $N = 250$ r.p.m. ; $\tau_s = 40$ MPa = 40 N/mm^2 ; $\theta = 1° = \pi / 180 = 0.0175$ rad ; $\tau_b = 30$ MPa = 30 N/mm^2

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{90 \times 10^3 \times 60}{2\,\pi \times 250} = 3440 \text{ N-m} = 3440 \times 10^3 \text{ N-mm}$$

Considering strength of the shaft, we know that

$$\frac{T}{J} = \frac{\tau_s}{d/2}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{80}{d} \qquad \ldots \left(\because J = \frac{\pi}{32} \times d^4\right)$$

$\therefore \quad d^3 = 35 \times 10^6 / 80 = 0.438 \times 10^6 \quad$ or $\quad d = 76$ mm

Top

Contents

Considering rigidity of the shaft, we know that

$$\frac{T}{J} = \frac{C \times \theta}{l}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20\,d} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{73.5}{d} \qquad \text{... (Taking } C = 84 \text{ kN/mm}^2\text{)}$$

$$\therefore \quad d^3 = 35 \times 10^6 / 73.5 = 0.476 \times 10^6 \quad \text{or} \quad d = 78 \text{ mm}$$

Taking the larger of the two values, we have

d = 78 say 80 mm **Ans.**

Let us now design the cast iron flange coupling of the protective type as discussed below :

1. *Design for hub*

We know that the outer diameter of hub,

$D = 2d = 2 \times 80 = 160$ mm **Ans.**

and length of hub, $L = 1.5\,d = 1.5 \times 80 = 120$ mm **Ans.**

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. The shear stress for the hub material (which is cast iron) is usually 14 MPa. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(160)^4 - (80)^4}{160}\right] = 754 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 3440 \times 10^3 / 754 \times 10^3 = 4.56 \text{ N/mm}^2 = 4.56 \text{ MPa}$$

Since the induced shear stress for the hub material is less than 14 MPa, therefore the design for hub is safe.

2. *Design for key*

From Table 13.1, we find that the proportions of key for a 80 mm diameter shaft are :

Width of key, w = 25 mm **Ans.**

and thickness of key, t = 14 mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

$\therefore \quad l = L = 120$ mm **Ans.**

Assuming that the shaft and key are of the same material. Let us now check the induced shear stress in key. We know that the torque transmitted (T),

$$3440 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 120 \times 25 \times \tau_k \times \frac{80}{2} = 120 \times 10^3\, \tau_k$$

$$\tau_k = 3440 \times 10^3 / 120 \times 10^3 = 28.7 \text{ N/mm}^2 = 28.7 \text{ MPa}$$

Since the induced shear stress in the key is less than 40 MPa, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

$\therefore \quad t_f = 0.5\,d = 0.5 \times 80 = 40$ mm **Ans.**

Let us now check the induced shear stress in the cast iron flange by considering the flange at the junction of the hub under shear. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi D^2}{2} \times t_f \times \tau_c = \frac{\pi (160)^2}{2} \times 40 \times \tau_c = 1608 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 3440 \times 10^3 / 1608 \times 10^3 = 2.14 \text{ N/mm}^2 = 2.14 \text{ MPa}$$

Since the induced shear stress in the flange is less than 14 MPa, therefore the design for flange is safe.

Top

Contents

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 80 mm, therefore let us take number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 80 = 240 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{4}(d_1)^2\, n \times \tau_b \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \times 4 \times 30 \times \frac{240}{2} = 11\,311\,(d_1)^2$$

$$\therefore \quad (d_1)^2 = 3440 \times 10^3 / 11\,311 = 304 \quad \text{or } d_1 = 17.4 \text{ mm}$$

Assuming coarse threads, the standard nominal diameter of bolt is 18 mm. **Ans.**

The other proportions are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 80 = 320 \text{ mm} \quad \textbf{Ans.}$$

Thickness of protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 80 = 20 \text{ mm} \quad \textbf{Ans.}$$

Example 13.9. *Design a rigid flange coupling to transmit a torque of 250 N-m between two co-axial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel. Four bolts are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below:*

Shear stress on shaft	*= 100 MPa*
Bearing or crushing stress on shaft	*= 250 MPa*
Shear stress on keys	*= 100 MPa*
Bearing stress on keys	*= 250 MPa*
Shearing stress on cast iron	*= 200 MPa*
Shear stress on bolts	*= 100 MPa*

After designing the various elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in the various members may be checked if thumb rules are used for fixing the dimensions.

Soution. Given : T = 250 N-m = 250 × 10^3 N-mm ; n = 4; τ_s = 100 MPa = 100 N/mm^2 ; σ_{cs} = 250 MPa = 250 N/mm^2 ; τ_k = 100 MPa = 100 N/mm^2 ; σ_{ck} = 250 MPa = 250 N/mm^2 ; τ_c = 200 MPa = 200 N/mm^2 ; τ_b = 100 MPa = 100 N/mm^2

The cast iron flange coupling of the protective type is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 100 \times d^3 = 19.64\, d^3$$

$$\therefore \quad d^3 = 250 \times 10^3 / 19.64 = 12\,729 \quad \text{or } d = 23.35 \text{ say } 25 \text{ mm} \quad \textbf{Ans.}$$

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 25 = 50 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 25 = 37.5$ mm

Top

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(50)^4 - (25)^4}{50} \right] = 23\ 013\ \tau_c$$

$$\therefore \qquad \tau_c = 250 \times 10^3 / 23\ 013 = 10.86 \text{ N/mm}^2 = 10.86 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than 200 MPa, therefore the design for hub is safe.

2. Design for key

From Table 13.1, we find that the proportions of key for a 25 mm diameter shaft are :

Width of key, w = 10 mm **Ans.**

and thickness of key, t = 8 mm **Ans.**

The length of key (l) is taken equal to the length of hub,

$\therefore$ $l = L = 37.5$ mm **Ans.**

Let us now check the induced shear and crushing stresses in the key. Considering the key in shearing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 37.5 \times 10 \times \tau_k \times \frac{25}{2} = 4688\ \tau_k$$

$$\therefore \qquad \tau_k = 250 \times 10^3 / 4688 = 53.3 \text{ N/mm}^2 = 53.3 \text{ MPa}$$

Considering the key in crushing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 37.5 \times \frac{8}{2} \times \sigma_{ck} \times \frac{25}{2} = 1875\ \sigma_{ck}$$

$$\therefore \qquad \sigma_{ck} = 250 \times 10^3 / 1875 = 133.3 \text{ N/mm}^2 = 133.3 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the given stresses, therefore the design of key is safe.

3. Design for flange

The thickness of the flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\ d = 0.5 \times 25 = 12.5$ mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (50)^2}{2} \times \tau_c \times 12.5 = 49\ 094\ \tau_c$$

$$\therefore \qquad \tau_c = 250 \times 10^3 / 49\ 094 = 5.1 \text{ N/mm}^2 = 5.1 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 200 MPa, therefore design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

We know that the pitch circle diameter of bolts,

$\therefore$ $D_1 = 3\ d = 3 \times 25 = 75$ mm **Ans.**

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 100 \times 4 \times \frac{75}{2} = 11\ 780\ (d_1)^2$$

Contents

$\therefore \quad (d_1)^2 = 250 \times 10^3 / 11\,780 = 21.22 \quad$ or $\quad d_1 = 4.6$ mm

Assuming coarse threads, the nearest standard size of the bolt is M 6. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 25 = 100 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 25 = 6.25 \text{ mm} \quad \textbf{Ans.}$$

Example 13.10. *Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 125 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 r.p.m. For the safe stresses mentioned below, calculate 1. diameter of bolts ; 2. thickness of flanges ; 3. key dimensions ; 4. hub length; and 5. power transmitted.*

Safe shear stress for shaft material	= *63 MPa*
Safe stress for bolt material	= *56 MPa*
Safe stress for cast iron coupling	= *10 MPa*
Safe stress for key material	= *46 MPa*

Solution. Given : $d = 35$ mm ; $n = 6$; $D_1 = 125$ mm ; $T = 800$ N-m $= 800 \times 10^3$ N-mm ; $N = 350$ r.p.m.; $\tau_s = 63$ MPa $= 63$ N/mm^2 ; $\tau_b = 56$ MPa $= 56$ N/mm^2 ; $\tau_c = 10$ MPa $= 10$ N/mm^2 ; $\tau_k = 46$ MPa $= 46$ N/mm^2

1. *Diameter of bolts*

Let $\quad d_1$ = Nominal or outside diameter of bolt.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi}{4}(d_1)^2\,\tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\,56 \times 6 \times \frac{125}{2} = 16\,495\,(d_1)^2$$

$\therefore \quad (d_1)^2 = 800 \times 10^3 / 16\,495 = 48.5$ or $d_1 = 6.96$ say 8 mm **Ans.**

2. *Thickness of flanges*

Let $\quad t_f$ = Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi\,D^2}{2} \times \tau_c \times t_f = \frac{\pi\,(2 \times 35)^2}{2} \times 10 \times t_f = 76\,980\,t_f \quad \text{... } (\because D = 2d)$$

$\therefore \quad t_f = 800 \times 10^3 / 76\,980 = 10.4$ say 12 mm **Ans.**

3. *Key dimensions*

From Table 13.1, we find that the proportions of key for a 35 mm diameter shaft are :

Width of key, $\quad w$ = 12 mm **Ans.**

and thickness of key, $\quad t$ = 8 mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

$\therefore \quad l = L = 1.5\,d = 1.5 \times 35 = 52.5$ mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\,\tau_k$$

$\therefore \quad \tau_k = 800 \times 10^3 / 11\,025 = 72.5$ N/mm^2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length of key by substituting the value of $\tau_k = 46$ MPa in the above equation, *i.e.*

Top

Contents

$$800 \times 10^3 = l \times 12 \times 46 \times \frac{35}{2} = 9660\, l$$

$\therefore$ $l = 800 \times 10^3 / 9660 = 82.8$ say 85 mm **Ans.**

4. *Hub length*

Since the length of key is taken equal to the length of hub, therefore we shall take hub length,

$L = l = 85$ mm **Ans.**

5. *Power transmitted*

We know that the power transmitted,

$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60} = 29\,325 \text{ W} = 29.325 \text{ kW}$$ **Ans.**

Example 13.11. *The shaft and the flange of a marine engine are to be designed for flange coupling, in which the flange is forged on the end of the shaft. The following particulars are to be considered in the design :*

Power of the engine	= *3 MW*
Speed of the engine	= *100 r.p.m.*
Permissible shear stress in bolts and shaft	= *60 MPa*
Number of bolts used	= *8*
Pitch circle diameter of bolts	= *1.6 × Diameter of shaft*

Find : 1. diameter of shaft ; 2. diameter of bolts ; 3. thickness of flange ; and 4. diameter of flange.

Solution. Given : $P = 3$ MW $= 3 \times 10^6$ W ; $N = 100$ r.p.m. ; $\tau_b = \tau_s = 60$ MPa $= 60$ N/mm^2 ; $n = 8$; $D_1 = 1.6\, d$

1. *Diameter of shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$\therefore$ $d^3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$

or $d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. *Diameter of bolts*

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4} (d_1)^2\, \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$$= 90\,490\, (d_1)^2 \qquad \text{... } (\because D_1 = 1.6\, d)$$

$\therefore$ $(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160$ or $d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

Top

3. *Thickness of flange*

The thickness of flange (t_f) is taken as d / 3.

$\therefore \quad t_f = d / 3 = 300/3 = 100$ mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6 \tau_s$$

$\therefore \quad \tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2$ N/mm^2 = 20.2 MPa

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (t_f = 100 mm) is safe.

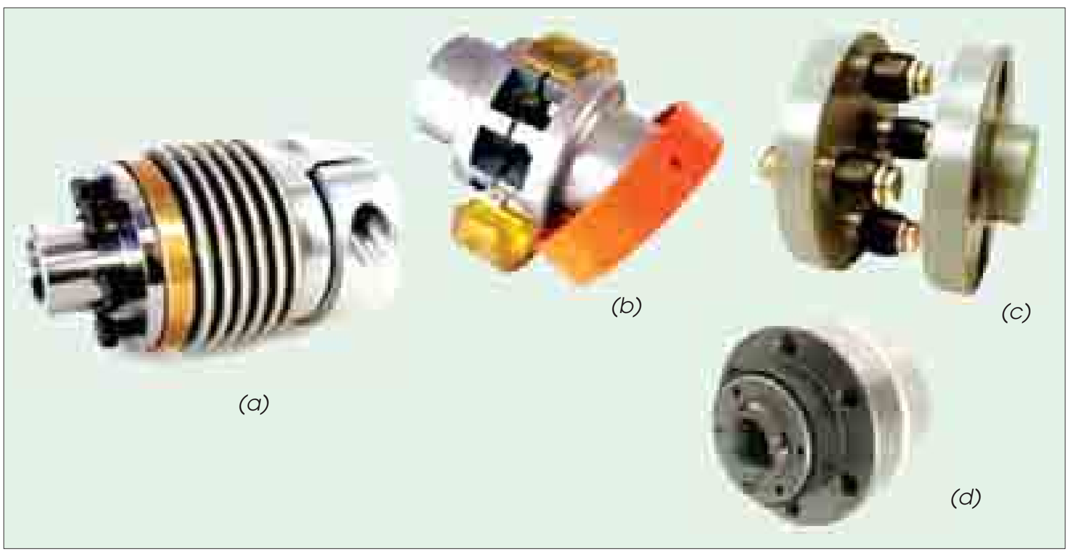
4. *Diameter of flange*

The diameter of flange (D_2) is taken as 2.2 d.

$\therefore \quad D_2 = 2.2\, d = 2.2 \times 300 = 660$ mm **Ans.**

13.18 Flexible Coupling

We have already discussed that a flexible coupling is used to join the abutting ends of shafts



(a) Bellows coupling , (b) Elastomeric coupling, (c) Flanged coupling , (d) Flexible coupling

Note : This picture is given as additional information and is not a direct example of the current chapter.

when they are not in exact alignment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a comparatively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalignemnt of the shaft without undue absorption of the power which the shaft are transmitting. Following are the different types of flexible couplings :

1. Bushed pin flexible coupling, **2.** Oldham's coupling, and **3.** Universal coupling.

We shall now discuss these types of couplings, in detail, in the following articles.

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* The flange material in case of marine flange coupling is same as that of shaft.

Contents

13.19 Bushed-pin Flexible Coupling

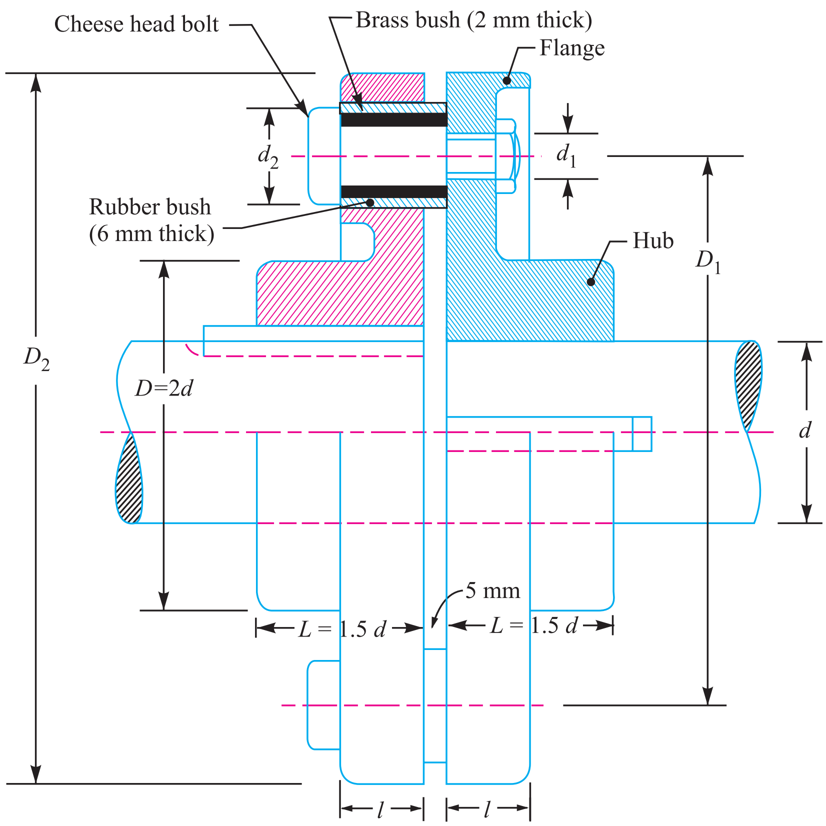

Fig. 13.15. Bushed-pin flexible coupling.

A bushed-pin flexible coupling, as shown in Fig. 13.15, is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm^2. In order to keep the low bearing pressure, the pitch circle diameter and the pin size is increased.

Let l = Length of bush in the flange,

d_2 = Diameter of bush,

p_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$$W = p_b \times d_2 \times l$$

∴ Total bearing load on the bush or pins

$$= W \times n = p_b \times d_2 \times l \times n$$

Top

Contents

and the torque transmitted by the coupling,

$$T = W \times n\left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n\left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit in the coupling hole to avoid bending stresses.

The threaded length of the pin should be as small as possible so that the direct shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$

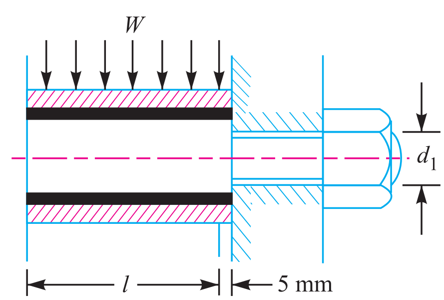

Fig. 13.16

Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. 13.16. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations :

(a) Taper bush (b) Locking-assembly (shaft or bush connectors)
(c) Friction joint bushing (d) Safety overload coupling.

Top

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

Example 13.12. *Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows :*

(*a*) *The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.*

(*b*) *The allowable shear stress for cast iron is 15 MPa.*

(*c*) *The allowable bearing pressure for rubber bush is 0.8 N/mm^2.*

(*d*) *The material of the pin is same as that of shaft and key.*

Draw neat sketch of the coupling.

Solution. Given : $P = 32$ kW $= 32 \times 10^3$ W ; $N = 960$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau_s = \tau_k = 40$ MPa $= 40$ N/mm^2 ; $\sigma_{cs} = \sigma_{ck} = 80$ MPa $= 80$ N/mm^2 ; $\tau_c = 15$ MPa $= 15$ N/mm^2 ; $p_b = 0.8$ N/mm^2

The bushed-pin flexible coupling is designed as discussed below :

1. *Design for pins and rubber bush*

First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$$\therefore \quad d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \quad \text{or} \quad d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$$\therefore \quad \text{Diameter of pins, } d_1 = \frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. **Ans.**

The length of the pin of least diameter *i.e.* $d_1 = 20$ mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

∴ Overall diameter of rubber bush,

$$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40 \text{ mm}$$ **Ans.**

and diameter of the pitch circle of the pins,

$$D_1 = 2\,d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132 \text{ mm}$$ **Ans.**

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\,l \text{ N}$$

and the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

$$\therefore \quad l = 382 \times 10^3/12\,672 = 30.1 \text{ say } 32 \text{ mm}$$

and $W = 32\,l = 32 \times 32 = 1024$ N

∴ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

and section modulus, $$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 40 = 80 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 40 = 60$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\,\tau_c$$

Contents

$\therefore \quad \tau_c = 382 \times 10^3 / 94.26 \times 10^3 = 4.05 \text{ N/mm}^2 = 4.05 \text{ MPa}$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\,\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, $w = 14$ mm **Ans.**

and thickness of key, $t = w = 14$ mm **Ans.**

The length of key (L) is taken equal to the length of hub, *i.e.*

$$L = 1.5\,d = 1.5 \times 40 = 60 \text{ mm}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\ \tau_k$$

$$\therefore \quad \tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 382 \times 10^3/8400 = 45.48 \text{ N/mm}^2 = 45.48 \text{ MPa}$$

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\,d = 0.5 \times 40 = 20 \text{ mm}$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi\,D^2}{2} \times \tau_c \times t_f = \frac{\pi(80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3\ \tau_c$$

$$\therefore \quad \tau_c = 382 \times 10^3 / 201 \times 10^3 = 1.9 \text{ N/mm}^2 = 1.9 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

13.20 Oldham Coupling

It is used to join two shafts which have lateral mis-alignment. It consists of two flanges A and B with slots and a central floating part E with two tongues T_1 and T_2 at right angles as shown in Fig. 13.17. The central floating part is held by means of a pin passing through the flanges and the floating part. The tongue T_1 fits into the slot of flange A and allows for 'to and fro' relative motion of the shafts, while the tongue T_2 fits into the slot of the flange B and allows for vertical relative motion of the parts. The resultant of these two components of motion will accommodate lateral misalignment of the shaft as they rotate.

Top

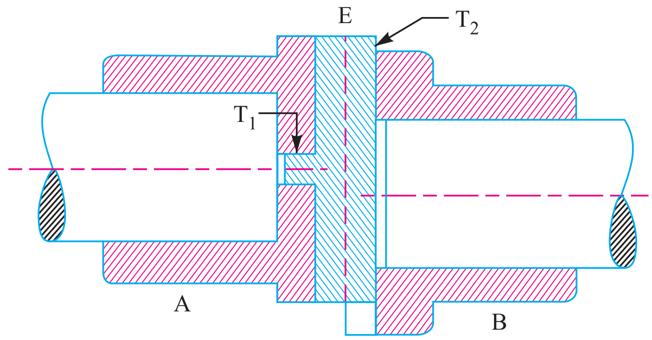

Fig. 13.17. Oldham coupling.

13.21 Universal (or Hooke's) Coupling

A universal or Hooke's coupling is used to connect two shafts whose axes intersect at a small angle. The inclination of the two shafts may be constant, but in actual practice, it varies when the motion is transmitted from one shaft to another. The main application of the universal or Hooke's coupling is found in the transmission from the gear box to the differential or back axle of the automobiles. In such a case, we use two Hooke's coupling, one at each end of the propeller shaft, connecting the gear box at one end and the differential on the other end. A Hooke's coupling is also used for transmission of power to different spindles of multiple drilling machine. It is used as a knee joint in milling machines.

In designing a universal coupling, the shaft diameter and the pin diameter is obtained as discussed below. The other dimensions of the coupling are fixed by proportions as shown in Fig. 13.18.

Let d = Diameter of shaft,

d_p = Diameter of pin, and

τ and τ_1 = Allowable shear stress for the material of the shaft and pin respectively.

We know that torque transmitted by the shafts,

$$T = \frac{\pi}{16} \times \tau \times d^3$$

A type of universal joint.

From this relation, the diameter of shafts may be determined.

Since the pin is in double shear, therefore the torque transmitted,

$$T = 2 \times \frac{\pi}{4}(d_p)^2 \tau_1 \times d$$

From this relation, the diameter of pin may be determined.

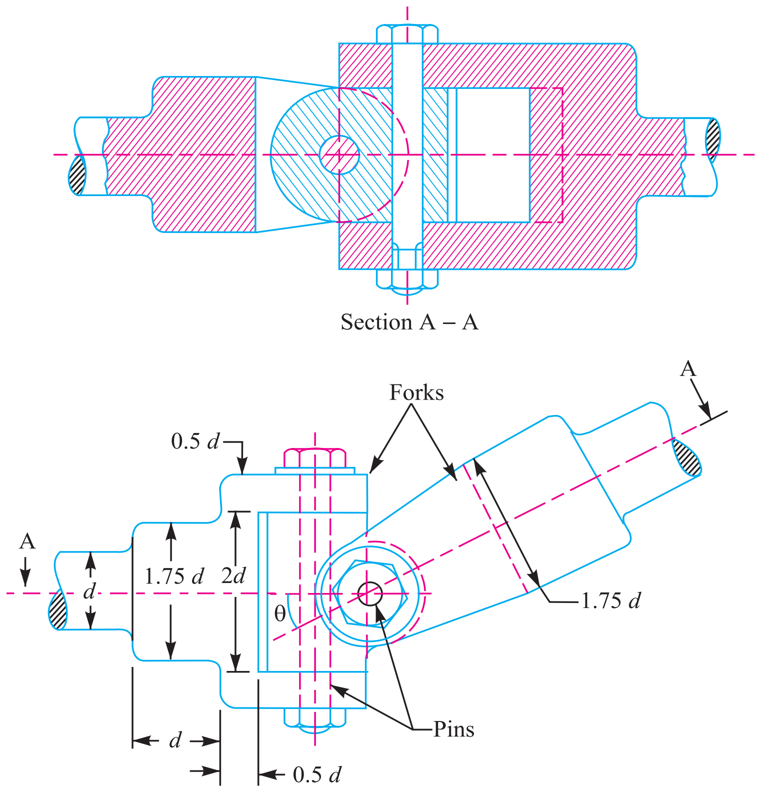

Fig. 13.18. Universal (or Hooke's) coupling.

Note: When a single Hooke's coupling is used, the ratio of the driving and driven shaft speeds is given by

$$\frac{N}{N_1} = \frac{1 - \cos^2\theta \times \sin^2\alpha}{\cos\alpha}$$

$$\therefore \quad N_1 = \frac{N \times \cos\alpha}{1 - \cos^2\theta \times \sin^2\alpha}$$

where N = Speed of the driving shaft in r.p.m.,

N_1 = Speed of the driven shaft in r.p.m.,

α = Angle of inclination of the shafts, and

θ = Angle of the driving shaft from the position where the pins of the driving shaft fork are in the plane of the two shafts.

We know that maximum speed of the driven shaft,

$$^*N_{1\,(max)} = \frac{N}{\cos\alpha}$$

and minimum speed of the driven shaft,

$$^*N_{1\,(min)} = N\cos\alpha$$

* For further details, please refer to authors' popular book on **'Theory of Machines'.**

From above we see that for a single Hooke's coupling, the speed of the driven shaft is not constant but varies from maximum to minimum. In order to have constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's coupling at each end (known as double Hooke's coupling) is used.

Example 13.13. *An universal coupling is used to connect two mild steel shafts transmitting a torque of 5000 N-m. Assuming that the shafts are subjected to torsion only, find the diameter of the shafts and pins. The allowable shear stresses for the shaft and pin may be taken as 60 MPa and 28 MPa respectively.*

Solution. Given : $T = 5000$ N-m $= 5 \times 10^6$ N-mm ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\tau_1 = 28$ MPa $= 28$ N/mm^2

Diameter of the shafts

Let d = Diameter of the shafts.

We know that the torque transmitted (T),

$$5 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 11.8 = 0.424 \times 10^6 \quad \text{or} \quad d = 75 \text{ mm}$$ **Ans.**

Diameter of the pins

Let d_p = Diameter of the pins.

We know that the torque transmitted (T),

$$5 \times 10^6 = 2 \times \frac{\pi}{4} (d_p)^2 \times \tau_1 \times d = 2 \times \frac{\pi}{4} (d_p)^2 \times 28 \times 75 = 3300\, (d_p)^2$$

$$\therefore \quad (d_p)^2 = 5 \times 10^6 / 3300 = 1515 \quad \text{or} \quad d_p = 39 \text{ say } 40 \text{ mm}$$ **Ans.**

EXERCISES

1. A shaft 80 mm diameter transmits power at maximum shear stress of 63 MPa. Find the length of a 20 mm wide key required to mount a pulley on the shaft so that the stress in the key does not exceed 42 MPa. **[Ans. 152 mm]**
2. A shaft 30 mm diameter is transmitting power at a maximum shear stress of 80 MPa. If a pulley is connected to the shaft by means of a key, find the dimensions of the key so that the stress in the key is not to exceed 50 MPa and length of the key is 4 times the width. **[Ans. l = 126 mm]**
3. A steel shaft has a diameter of 25 mm. The shaft rotates at a speed of 600 r.p.m. and transmits 30 kW through a gear. The tensile and yield strength of the material of shaft are 650 MPa and 353 MPa respectively. Taking a factor of safety 3, select a suitable key for the gear. Assume that the key and shaft are made of the same material. **[Ans. l = 102 mm]**
4. Design a muff coupling to connect two shafts transmitting 40 kW at 120 r.p.m. The permissible shear and crushing stress for the shaft and key material (mild steel) are 30 MPa and 80 MPa respectively. The material of muff is cast iron with permissible shear stress of 15 MPa. Assume that the maximum torque transmitted is 25 per cent greater than the mean torque.
   [Ans. d = 90 mm ; w = 28 mm, t = 16 mm, l = 157.5 mm ; D = 195 mm, L = 315 mm]
5. Design a compression coupling for a shaft to transmit 1300 N-m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are 4. The permissible tensile stress for the bolts material is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3. **[Ans. d = 55 mm ; D = 125 mm ; L = 192.5 mm ; d_b = 24 mm]**
6. Design a cast iron protective flange coupling to connect two shafts in order to transmit 7.5 kW at 720 r.p.m. The following permissible stresses may be used :
   Permissible shear stress for shaft, bolt and key material = 33 MPa
   Permissible crushing stress for bolt and key material = 60 MPa
   Permissible shear stress for the cast iron = 15 MPa **[Ans. d = 25 mm; D = 50 mm]**

7. Two shafts made of plain carbon steel are connected by a rigid protective type flange coupling. The shafts are running at 500 r.p.m. and transmit 25 kW power. Design the coupling completely for over-load capacity 25 per cent in excess of mean transmitted torque capacity.

Assume the following permissible stresses for the coupling components :

Shaft — Permissible tensile stress = 60 MPa; Permissible shear stress = 35 MPa

Keys — Rectangular formed end sunk key having permissible compressive strength = 60 MPa

Bolts — Six numbers made of steel having permissible shear stress = 28 MPa

Flanges — Cast iron having permissible shear stress = 12 MPa

Draw two views of the coupling you have designed. **[Ans. d = 45 mm ; D = 90 mm]**

8. Design a shaft and flange for a Diesel engine in which protected type of flange coupling is to be adopted for power transmission. The following data is available for design :

Power of engine = 75 kW; speed of engine = 200 r.p.m.; maximum permissible stress in shaft = 40 MPa; maximum permissible twist in shaft = 1° in length of shaft equal to 30 times the diameter of shaft; maximum torque = 1.25 × mean torque; pitch circle diameter of bolts = 3 × diameter of shaft; maximum permissible stress in bolts = 20 MPa.

Find out : 1. Diameter of shaft, 2. number of bolts, and 3. diameter of bolts.

[Ans. 100 mm ; 4 ; 22 mm]

9. A flanged protective type coupling is required to transmit 50 kW at 2000 r.p.m.. Find :

(*a*) Shaft diameters if the driving shaft is hollow with d_i / d_0 = 0.6 and driven shaft is a solid shaft. Take τ = 100 MPa.

(*b*) Diameter of bolts, if the coupling uses four bolts. Take $\sigma_c = \sigma_t$ = 70 MPa and τ = 25 MPa. Assume pitch circle diameter as about 3 times the outside diameter of the hollow shaft.

(*c*) Thickness of the flange and diameter of the hub. Assume σ_c = 100 MPa and τ = 125 MPa.

(*d*) Make a neat free hand sketch of the assembled coupling showing a longitudinal sectional elevation with the main dimensions. The other dimensions may be assumed suitably.

10. A marine type flange coupling is used to transmit 3.75 MW at 150 r.p.m. The allowable shear stress in the shaft and bolts may be taken as 50 MPa. Determine the shaft diameter and the diameter of the bolts. **[Ans. 300 mm ; 56 mm]**

11. Design a bushed-pin type flexible coupling for connecting a motor shaft to a pump shaft for the following service conditions :

Power to be transmitted = 40 kW ; speed of the motor shaft = 1000 r.p.m. ; diameter of the motor shaft = 50 mm ; diameter of the pump shaft = 45 mm.

The bearing pressure in the rubber bush and allowable stress in the pins are to be limited to 0.45 N/mm^2 and 25 MPa respectively. **[Ans. d_1 = 20 mm; n = 6; d_2 = 40 mm ; l = 152 mm]**

12. An universal coupling is used to connect two mild steel shafts transmitting a torque of 6000 N-m. Assuming that the shafts are subjected to torsion only, find the diameter of the shaft and the pin. The allowable shear stresses for the shaft and pin may be taken as 55 MPa and 30 MPa respectively.

[Ans. d = 85 mm ; d_p = 40 mm]

QUESTIONS

1. What is a key ? State its function.
2. How are the keys classified? Draw neat sketches of different types of keys and state their applications.
3. What are the considerations in the design of dimensions of formed and parallel key having rectangular cross-section ?
4. Write short note on the splined shaft covering the points of application, different types and method of manufacture.
5. What is the effect of keyway cut into the shaft ?
6. Discuss the function of a coupling. Give at least three practical applications.
7. Describe, with the help of neat sketches, the types of various shaft couplings mentioning the uses of each type.
8. How does the working of a clamp coupling differ from that of a muff coupling ? Explain.

Contents

9. Sketch a protective type flange coupling and indicate there on its leading dimensions for shaft size of '*d*'.
10. What are flexible couplings and what are their applications ? Illustrate your answer with suitable examples and sketches.
11. Write short note on universal coupling.
12. Why are two universal joints often used when there is angular misalignment between two shafts ?

OBJECTIVE TYPE QUESTIONS

1. The taper on a rectangular sunk key is
   (*a*) 1 in 16 (*b*) 1 in 32
   (*c*) 1 in 48 (*d*) 1 in 100
2. The usual proportion for the width of key is
   (*a*) d/8 (*b*) d/6
   (*c*) d/4 (*d*) d/2
   where *d* = Diameter of shaft.
3. When a pulley or other mating piece is required to slide along the shaft, a .... sunk key is used.
   (*a*) rectangular (*b*) square (*c*) parallel
4. A key made from a cylindrical disc having segmental cross-section, is known as
   (*a*) feather key (*b*) gib head key
   (*c*) woodruff key (*d*) flat saddle key
5. A feather key is generally
   (*a*) loose in shaft and tight in hub (*b*) tight in shaft and loose in hub
   (*c*) tight in both shaft and hub (*d*) loose in both shaft and hub.
6. The type of stresses developed in the key is/are
   (*a*) shear stress alone (*b*) bearing stress alone
   (*c*) both shear and bearing stresses (*d*) shearing, bearing and bending stresses
7. For a square key made of mild steel, the shear and crushing strengths are related as
   (*a*) shear strength = crushing strength (*b*) shear strength > crushing strength
   (*c*) shear strength < crushing strength (*d*) none of the above
8. A keyway lowers
   (*a*) the strength of the shaft (*b*) the rigidity of the shaft
   (*c*) both the strength and rigidity of the shaft (*d*) the ductility of the material of the shaft
9. The sleeve or muff coupling is designed as a
   (*a*) thin cylinder (*b*) thick cylinder
   (*c*) solid shaft (*d*) hollow shaft
10. Oldham coupling is used to connect two shafts
    (*a*) which are perfectly aligned (*b*) which are not in exact alignment
    (*c*) which have lateral misalignment (*d*) whose axes intersect at a small angle

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)
6. (*c*)	**7.** (*a*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)

Top