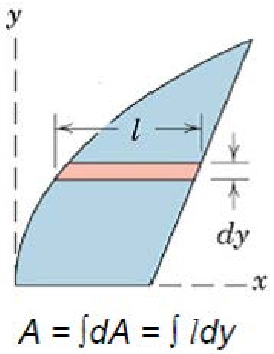
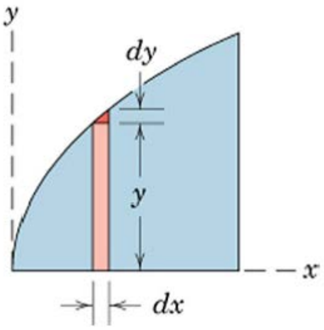
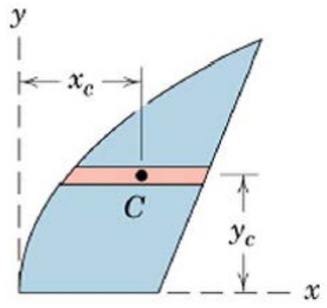
Lecturer: Luay Hashem Abbud
E-mail: LuayHashemAbbud@mustaqbal-college.edu.iq

Centroids





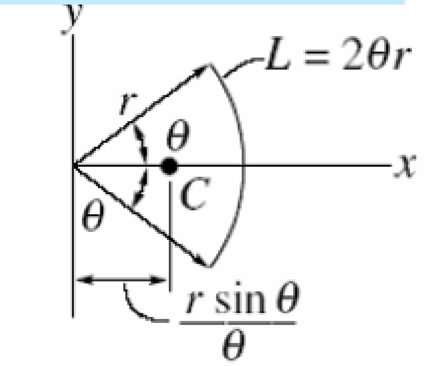
Vertical strip of area under the curve $dA = ydx$



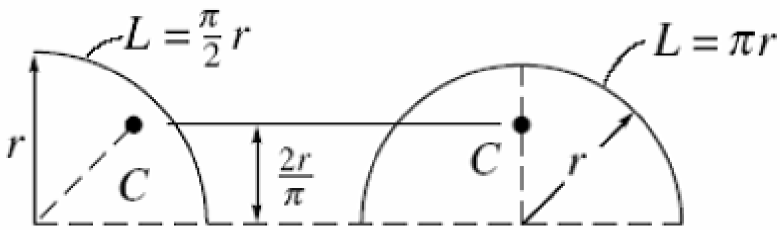
$$\bar{x} = \frac{\int x_c dA}{A} \qquad \bar{y} = \frac{\int y_c dA}{A}$$

Geometric Properties of Line and Area Elements

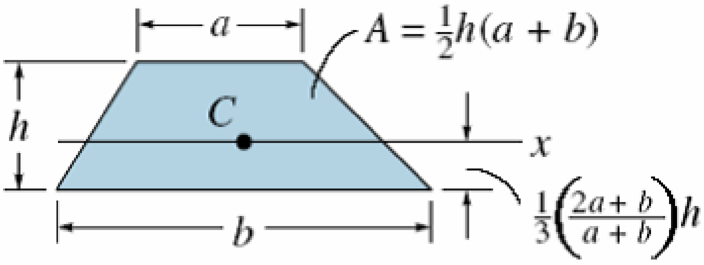
Centroid Location



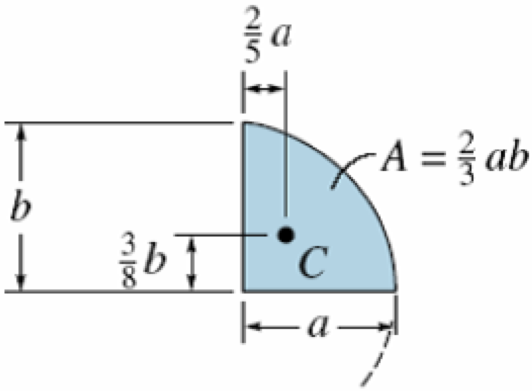
Circular arc segment



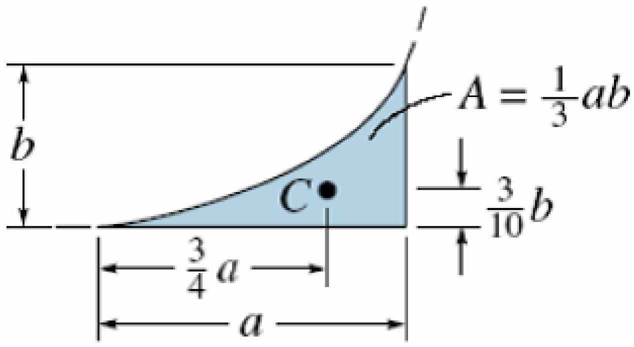
Quarter and semicircle arcs



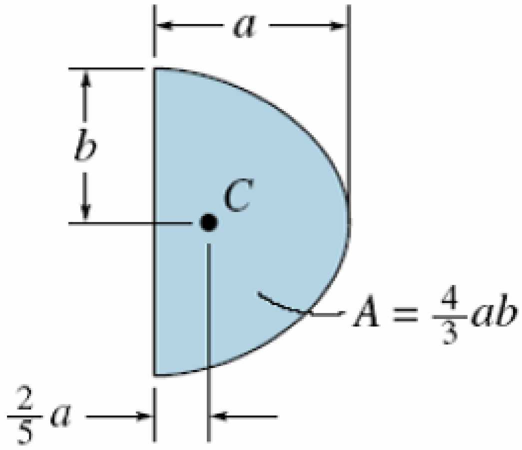
Trapezoidal area



Semiparabolic area

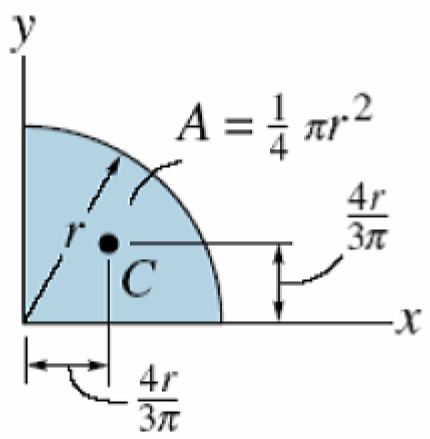


Exparabolic area



Parabolic area

Centroid Location | **Area Moment of Inertia**

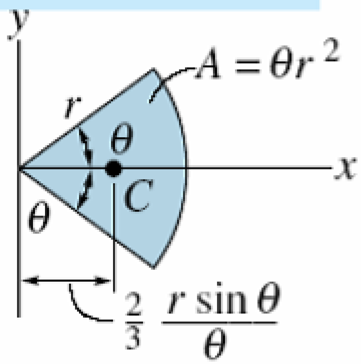


$$I_x = \tfrac{1}{16}\pi r^4$$

$$I_y = \tfrac{1}{16}\pi r^4$$

Quarter circle area

Centroid Location | **Area Moment of Inertia**

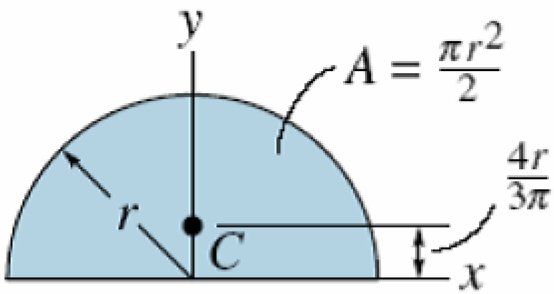


$$I_x = \tfrac{1}{4} r^4 (\theta - \tfrac{1}{2}\sin 2\theta)$$

$$I_y = \tfrac{1}{4} r^4 (\theta + \tfrac{1}{2}\sin 2\theta)$$

Circular sector area

Centroid Location | **Area Moment of Inertia**

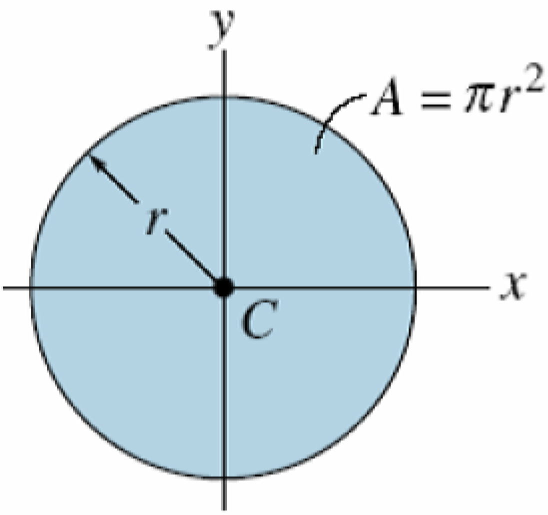


$$I_x = \tfrac{1}{8}\pi r^4$$

$$I_y = \tfrac{1}{8}\pi r^4$$

Semicircular area

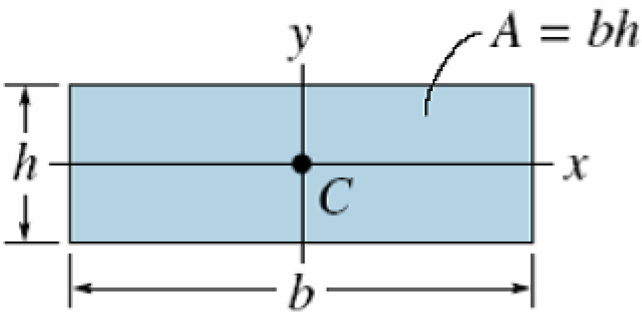
Centroid Location | Area Moment of Inertia



Circular area

$$I_x = \tfrac{1}{4}\pi r^4$$

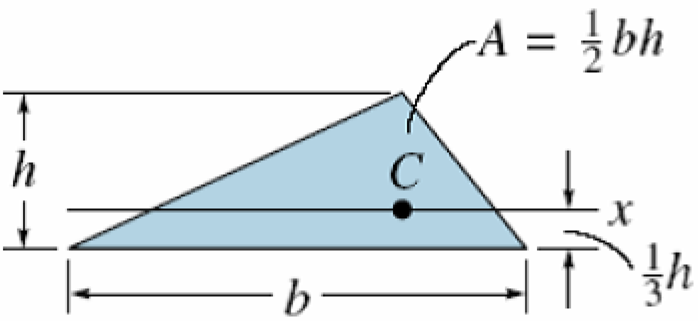
$$I_y = \tfrac{1}{4}\pi r^4$$



Rectangular area

$$I_x = \tfrac{1}{12}bh^3$$

$$I_y = \tfrac{1}{12}hb^3$$

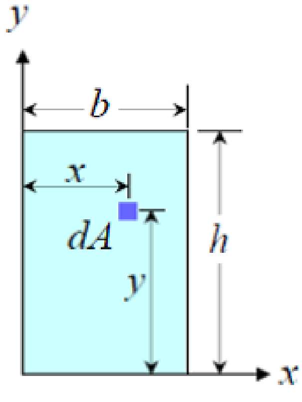


Triangular area

$$I_x = \tfrac{1}{36}bh^3$$

Problem 1

Find the Centroid of A Rectangular Area by integration



Solution

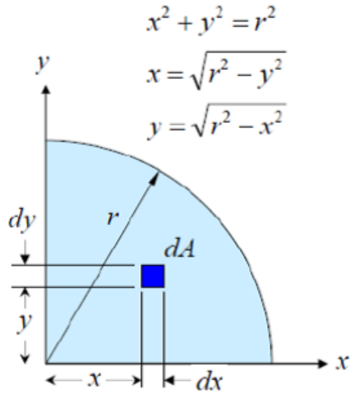
$$A = \int_A dA = \int_0^h \left(\int_0^b dx \right) dy = bh$$

$$x_c = \frac{M_y}{A} = \frac{1}{A}\int_A x dA = \frac{1}{bh}\int_0^h \left(\int_0^b x dx \right) dy = \frac{1}{bh}\int_0^h \frac{b^2}{2} dy = \frac{b^2 h}{2bh} = \frac{b}{2}$$

$$y_c = \frac{M_x}{A} = \frac{1}{A}\int_A y dA = \frac{1}{bh}\int_0^h \left(\int_0^b y dx \right) dy = \frac{1}{bh}\int_0^h by dy = \frac{bh^2}{2bh} = \frac{h}{2}$$

Problem 2

Find the centroid of a quarter circle by double integration in rectangular coordinates



Solution

$$M_x = \int_A y dA = \int_0^r \left(\int_0^{\sqrt{r^2-x^2}} y dy \right) dx$$

$$= \int_0^r \left(\left[\frac{y^2}{2} \right]_0^{\sqrt{r^2-x^2}} \right) dx = \int_0^r \frac{r^2 - x^2}{2} dx = \left[\frac{r^2 x}{2} - \frac{x^3}{6} \right]_0^r = \frac{r^3}{3}$$

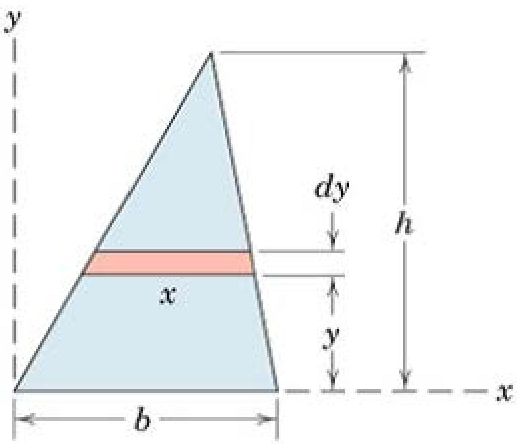
$$A = \int_A dA = \frac{\pi r^2}{4}$$

$$y_c = \frac{M_x}{A} = \frac{\int_A y dA}{A} = \frac{r^3/3}{\pi r^2/4} = \frac{4r}{3\pi}$$

$$x_c = \frac{M_y}{A} = \frac{\int_A x dA}{A} = \frac{4r}{3\pi}$$

Problem 3

Locate the centroid of the triangle along h from the base



Solution

$$\frac{x}{b} = \frac{h-y}{h}$$

$$\therefore x = (h-y)\frac{b}{h}$$

$$dA = x\,dy = (h-y)\frac{b}{h}dy$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA}$$

$$= \frac{\int_0^h y(h-y)\frac{b}{h}dy}{\int_0^h (h-y)\frac{b}{h}dy}$$

$$= \frac{\frac{b}{h}\int_0^h (yh-y^2)dy}{\frac{b}{h}\int_0^h (h-y)dy} = \frac{\left[h\frac{y^2}{2}-\frac{y^3}{3}\right]_0^h}{\left[hy-\frac{y^2}{2}\right]_0^h} = \frac{h}{3}$$