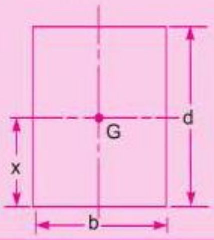
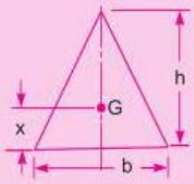
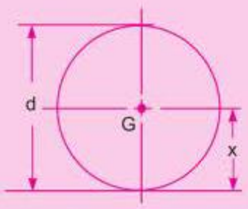
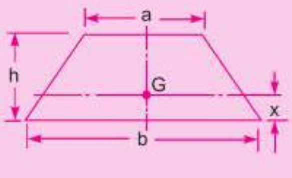
P = ρ g h **(where h is depth of the surface)**

Table (3.1) The moments of inertia and other geometric properties of some important plane surfaces .

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Contd...

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

3.3 / Inclined Plane surface submerged in liquid :

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle ϴ with the free surface of the liquid as shown in Fig.(3.2).

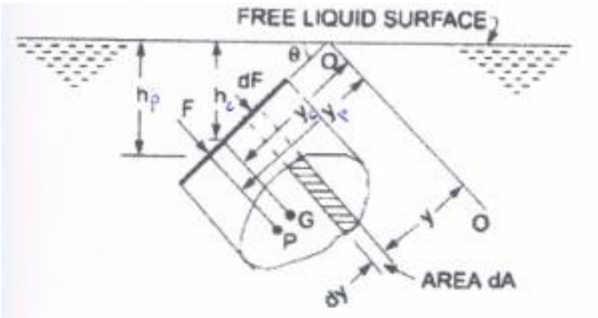


Fig.(3.2) Inclined immersed surface

Let , A total area of inclined surface , h_c depth of C.G of inclined area from free surface , h_p distance of center of pressure from free surface of liquid , ϴ angle made by the plane of the surface with free surface , y_c distance of the C.G of the inclined surface from O – O , y_p distance of the center of pressure from the O – O .

Consider a small strip of area dA at a depth (h) from free surface and at a distance y from the axis O – O as shown in Fig.(3.2).

Force dF on the strip = p × Area of strip = ρ g h ×dA

Total Force on the whole area , F = ∫ dF = ∫ ρ g h dA

But from Fig.(3.2) , $\sin\theta = \frac{h}{y} = \frac{h_c}{y_c} = \frac{h_p}{y_p}$

Therefore , $h = y \sin\theta$

$$F = \int \rho\, g \times y \sin\theta \times dA = \rho\, g \sin\theta \int y\, dA$$

But , $\int y\, dA = A\, y_c$

Therefore , $F = \rho\, g \sin\theta \times A \times y_c$

$$F = \rho\, g\, A\, h_c \qquad (3.6)$$

Force on the strip , dF = ρ g h dA

$\sin\theta = \frac{h}{y}$, $h = y \sin\theta$

$dF = \rho\, g\, y \sin\Theta\, dA$

Moment of force (dF) about axis O – O ,

$dF \times y = \rho\, g\, y \sin\Theta\, dA \times y = \rho\, g \sin\Theta\, y^2\, dA$

Sum of moments of all such forces about O - O ,

$M = \int \rho\, g \sin\Theta\, y^2\, dA = \rho\, g \sin\Theta \int y^2\, dA$

But $\int y^2\, dA = I_o$

Therefore , $M = \rho\, g \sin\Theta\, I_o$ **(3.7)**

Moment of the total force F , about O – O is given by : $F \times y_p$ **(3.8)**

Equating the two values given by equations (3.7) & (3.8)

$F \times y_p = \rho\, g \sin\Theta\, I_o$

$y_p = \frac{\rho\, g \sin\theta\, I_o}{F}$ **(3.9)**

But , $\sin\Theta = \frac{h_p}{y_p}$, $y_p = \frac{h_p}{\sin\theta}$ **, and** $F = \rho\, g\, A\, h_c$

And $I_o = I_G + A\, y_c^2$, **Substituting these values in eq.(3.9) , we get :**

$\frac{h_p}{\sin\theta} = \frac{\rho\, g \sin\theta}{\rho\, g\, A h_c} (I_G + A\, y_c^2)$ **(× sin Θ)**

But , $\sin\Theta = \frac{h_c}{y_c}$, $y_c = \frac{h_c}{\sin\theta}$

$h_p = \frac{\sin^2\theta}{A h_c} \left(I_G + A \frac{h_c^2}{\sin^2\theta} \right)$

$h_p = \frac{I_G \sin^2\theta}{A\, h_c} + h_c$ **(3.10)**

If the $\Theta = 90^0$**, equation (3.10) becomes same as equation (3.5) (vertical plane submerged) .**

3.4 / Curved Surface Submerged in Liquid :

Consider a curved surface (AB) , submerged in a static liquid as shown in Fig.(3.3) . Let dA is the area of a small strip at a depth of (h) from water surface.

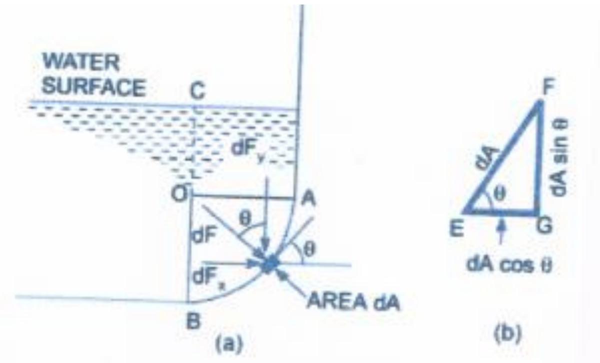


Fig.(3.3)

Then pressure (p) = ρ g h

Force (dF) = p × area = ρ g h × dA (3.11) This force dF acts normal to the surface , hence , total force on the curved surface should be:

$$F = \int \rho g h \, dA \quad (3.12)$$

By resolving the force dF in two components dF , and dF_x and dF_y in the x and y directions respectively . The total force in the x and y directions , i .e , F_x and F_y are obtained by integrating dF_x and dF_y , Then total force on the curved surface is :

$$F = \sqrt{F_x^2 + F_y^2} \qquad (3.13)$$

And inclination of resultant with horizontal is ,

$$\tan \Theta = \frac{F_y}{F_x} \qquad (3.14)$$

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin\Theta = \rho g h \, dA \sin\Theta$$

$$dF_y = dF \cos\Theta = \rho g h \, dA \cos\Theta$$

Total forces in the x and y directions are :

$$F_x = \int dF_x = \rho g \int h \, dA \sin\Theta \qquad (3.15)$$

$$F_y = \int dF_y = \rho g \int h \, dA \cos\Theta \qquad (3.16)$$

Fig.(3.3) b , shows the enlarged area dA , from this figure , i.e. , Δ EFG :

$$EF = dA \quad , \quad FG = dA \sin\Theta \quad , \quad EG = dA \cos\Theta$$

Thus , in Eq.(3.15) , $dA \sin\Theta = FG$ = Vertical projection of the area dA .

Therefore , F_x force on the projected area on the vertical plane .

Thus , in Eq.(3.16) , $dA \cos\Theta = EG$ = Horizontal projection of the area dA.

Therefore , $\int h \, dA \cos\Theta$ is the total volume contained between the curved surface , extended up to free surface .

Hence , $\rho g \int h \, dA \cos\Theta$ is the total weight supported by the curved surface , thus , $F_y = \rho g \int h dA \cos \Theta$ = Weight of liquid supported by the curved surface up to free surface of liquid.