$$0.1F + 0.3D = 0.2P$$
$$0.9F + 0.7D = 0.8P$$
$$F + D = P$$

3. How many values of the concentrations and flow rates in the process shown in Figure SAT7.2P3 are unknown? List them. The streams contain two components, 1 and 2.

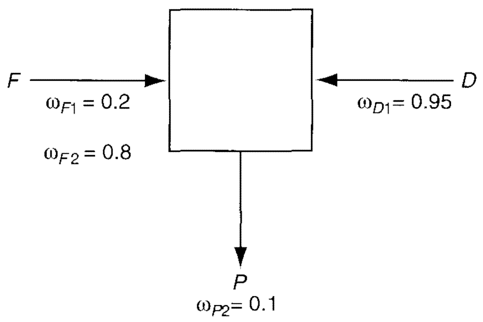


Figure SAT7.2P3

4. How many material balances are needed to solve problem 3? Is the number the same as the number of unknown variables? Explain.

Answers:

1. (a) Two; (b) two of these three: acetic acid, water, total; (c) two; (d) feed of the 10% solution (say F) and mass fraction ω of the acetic acid in P; (e) 14% acetic acid and 86% water
2. Not for a unique solution because only two of the equations are independent.
3. F, D, P, ω_{D2}, ω_{P1}
4. Three unknowns exist. Because only two independent material balances can be written for the problem, one value of F, D, or P must be specified to obtain a solution. Note that specifying values of ω_{D2} or ω_{P1} will not help.

2.3 Solving Material Balance Problems for Single Units without Reaction

The use of material balances in a process allows you **(a)** to calculate the values of the total flows and flows of species in the streams that enter and leave the plant equipment, and **(b)** to calculate the change of conditions inside the equipment.

(أ) لحساب قيم التدفقات الإجمالية وتدفق الأنواع في الجداول التي تدخل وتخرج من معدات المصنع.
(ب) حساب تغير الظروف داخل المعدات.

Example 9

Determine the mass fraction of Streptomycin in the exit organic solvent assuming that no water exits with the solvent and no solvent exits with the aqueous solution. Assume that the density of the aqueous solution is 1 g/cm^3 and the density of the organic solvent is 0.6 g/cm^3. Figure E8. 1 shows the overall process.

Solution

This is an **open** (flow), **steady-state** process without reaction. Assume because of the low concentration of Strep. in the aqueous and organic fluids that the **flow rates** of the **entering** fluids **equal** the flow rates of the **exit** fluids.

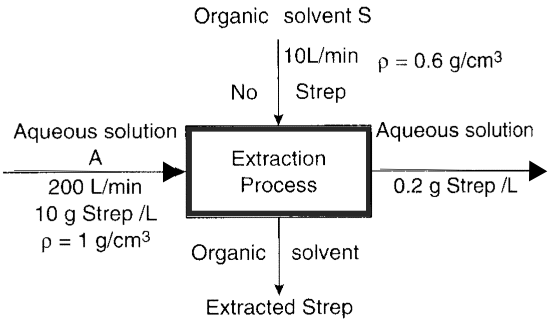


Figure E8.1

Basis: 1 min

Basis: Feed = 200 L (flow of aqueous entering aqueous solution)

- Flow of exiting aqueous solution (same as existing flow)
- Flow of exiting organic solution (same as existing flow)

The material balances are **in = out** in **grams**. Let **x** be the **g** of Strep per **L** of solvent **S**

Strep. balance:

$$\frac{200 \text{ L of A}}{}\left|\frac{10 \text{ g Strep}}{1 \text{ L of A}}\right. + \frac{10 \text{ L of S}}{}\left|\frac{0 \text{ g Strep}}{1 \text{ L of S}}\right. = \frac{200 \text{ L of A}}{}\left|\frac{0.2 \text{ g Strep}}{1 \text{ L of A}}\right. + \frac{10 \text{ L of S}}{}\left|\frac{x \text{ g Strep}}{1 \text{ L of S}}\right.$$

x = 196 g Strep/L of solvent

To get the g Strep/g solvent, use the density of the solvent:

$$\frac{196 \text{ g Strep}}{1 \text{ L of S}}\left|\frac{1 \text{ L of S}}{1000 \text{ cm}^3 \text{ of S}}\right|\frac{1 \text{ cm}^3 \text{ of S}}{0.6 \text{ g of S}} = 0.3267 \text{ g Strep/g of S} \rightarrow \frac{0.3267 \text{ g Strep}}{1 \text{ g of solvent}}$$

$$\text{The mass fraction Strep} = \frac{0.3267}{1 + 0.3267} = 0.246$$

Example 10

Membranes represent a relatively new technology for the separation of gases. One use that has attracted attention is the separation of nitrogen and oxygen from air. Figure E8.2a illustrates a nanoporous membrane that is made by coating a very thin layer of polymer on a porous graphite supporting layer. What is the composition of the waste stream if the waste stream amounts to 80% of the input stream?

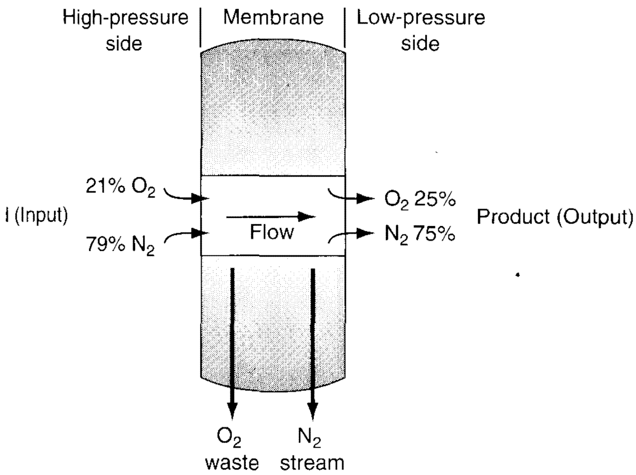
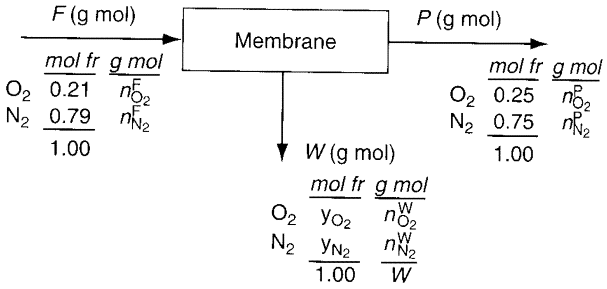


Figure E8.2a

Solution

This is an **open**, **steady-state process** without chemical reaction.



Basis: 100 g mol = F

Basis: F = 100

Specifications: $n_{O_2}^F = 0.21(100) = 21$

$n_{N_2}^F = 0.79(100) = 79$

$y_{O_2}^P = n_{O_2}^P/P = 0.25 \qquad n_{O_2}^P = 0.25P$

$y_{N_2}^P = n_{N_2}^P/P = 0.75 \qquad n_{N_2}^P = 0.75P$

$W = 0.80(100) = 80$

Material balances: O_2 and N_2

Implicit equations: $\Sigma n_i^W = W$ or $\Sigma y_i^W = 1$

	In	*Out*		*In*	*Out*
O_2:	0.21 (100)	$= 0.25P + y_{O_2}^W(80)$	or	0.21 (100)	$= 0.25P + n_{O_2}^W$
N_2:	0.79 (100)	$= 0.75P + y_{N_2}^W(80)$	or	0.79 (100)	$= 0.75P + n_{N_2}^W$
	1.00	$= y_{O_2}^W + y_{N_2}^W$	or	80	$= n_{O_2}^W + n_{N_2}^W$

The solution of these equations is

$n_{O_2}^W = 16$ and $n_{N_2}^W = 64$, or $y_{O_2}^W = 0.20$ and $y_{N_2}^W = 0.80$, and $P = 20$ g mol.

Check: total balance 100 = 20 + 80 OK

❖ <u>Another method for solution</u>

The overall balance is easy to solve because

F = P + W or 100=P+ 80

Gives P = 20 straight off. Then, the oxygen balance would be

$0.21(100) = 0.25(20) + n_{O_2}^W$

$n_{O_2}^W = 16$ g mol, and $n_{O_2}^W = 80 - 16 = 64$ g mol.

<u>Note (Example 10)</u>

$n_{O_2}^F + n_{N_2}^F = F$ is a redundant equation because it repeats some of the specifications.

Also, $n^P_{O_2} + n^P_{N_2} = P$ is redundant. Divide the equation by P to get $y^P_{O_2} + y^P_{N_2} = 1$, a relation that is equivalent to the sum of two of the specifications.

Example 11

A novice manufacturer of ethyl alcohol (denoted as EtOH) for gasohol is having a bit of difficulty with a distillation column. The process is shown in Figure E8.3. It appears that too much alcohol is lost in the bottoms (waste). Calculate the composition of the bottoms and the mass of the alcohol lost in the bottoms based on the data shown in Figure E8.3 that was collected during 1 hour of operation.

Solution

The process is an **open system**, and we assume it is in the **steady state**. No **reaction** occurs.

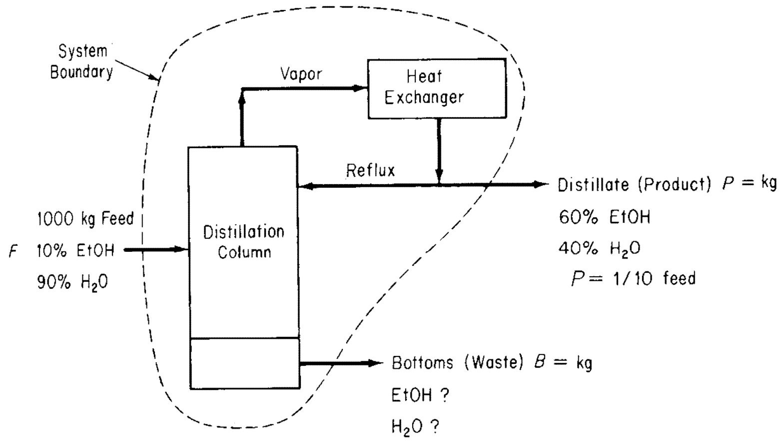


Figure E8.3

Basis: 1 hour so that **F = 1000 kg** of feed

We are given that P is (1/10) of F, so that P = 0.1(1000) = 100 kg

Basis: F= 1000 kg

Specifications: $m^F_{EtOH} = 1000(0.10) = 100$

$$m^F_{H_2O} = 1000(0.90) = 900$$

$$m^P_{EtOH} = 0.60P$$

$$m^P_{H_2O} = 0.40P$$

P = (0.1) (F) = 100 kg

Material balances: EtOH and H_2O

Implicit equations: $\Sigma m_i^B = B$ or $\Sigma \omega_i^B = 1$

The total mass balance: F = P + B

B = 1000 – 100 = 900 kg

The solution for the composition of the **bottoms** can then be computed directly from the material balances:

	kg feed in		*kg distillate out*		*kg bottoms out*	*Mass fraction*
EtOH balance:	0.10(1000)	−	0.60(100)	=	40	0.044
H_2O balance:	0.90(1000)	−	0.40(100)	=	860	0.956
					900	1.000

As a **check** let's use the redundant equation

$$m^B_{EtOH} + m^B_{H_2O} = B \quad \text{or} \quad \omega^B_{EtOH} + \omega^B_{H_2O} = 1$$

40 + 860 = 900 = B

Example 12

You are asked to prepare a batch of 18.63% battery acid as follows. A tank of old weak battery acid (H_2SO_4) solution contains 12.43% H_2SO_4 (the remainder is pure water). If 200 kg of 77.7% H_2SO_4 is added to the tank, and the final solution is to be 18.63% H_2SO_4, how many kilograms of battery acid have been made? See Figure E8.4.

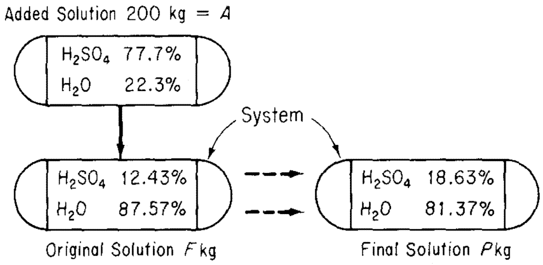


Figure E8.4

Solution

1. An unsteady-state process (the tank initially contains sulfuric acid solution).

Accumulation = In – Out

2. Steady-state process (the tank as initially being empty)

In = Out (Because no **accumulation** occurs in the tank)

1) Solve the problem with the mixing treated as an **unsteady-state process**.

Basis = 200 kg of A

Material balances: H_2SO_4 and H_2O

The balances will be in **kilograms**.

Type of Balance	*Accumulation in Tank*				*In*		*Out*
	Final		Initial				
H_2SO_4	$P(0.1863)$	−	$F(0.1243)$	=	200(0.777)	−	0
H_2O	$P(0.8137)$	−	$F(0.8757)$	=	200(0.223)	−	0
Total	P	−	F	=	200	−	0

Note that any **pair** of the three equations is **independent**. P = 2110 kg acid & F = 1910 kg acid.

2) The problem could also be solved by considering the mixing to be a **steady- state process**.

	A in		*F in*		*P out*
H_2SO_4	200(0.777)	+	F(0.1243)	=	P(0.1863)
H_2O	200(0.223)	+	F(0.8757)	=	P(0.8137)
Total	*A*	+	*F*	=	*P*

Note: You can see by inspection that these equations are no different than the first set of mass balances except for the arrangement and labels.

Example 13

In a given batch of fish cake that contains 80% water (the remainder is dry cake), 100 kg of water is removed, and it is found that the fish cake is then 40% water. Calculate the weight of the fish cake originally put into the dryer. Figure E8.5 is a diagram of the process.

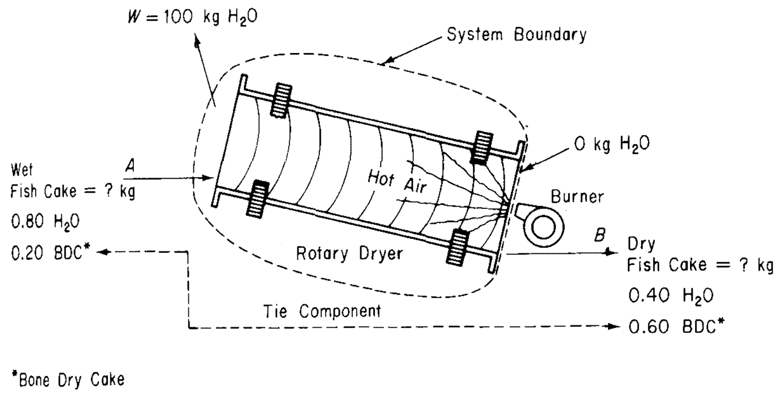


Figure E8.5

Solution

This is a steady-state process without reaction.

Basis: 100 kg of water evaporated = W

	In	*Out*	
Total balance:	A	$= B + W = B + 100$	mass balances
BDC balance:	$0.20A$	$= 0.60B$	

A = 150 kg initial cake and B = (150)(0.20/0.60) = 50kg

Check via the water balance: 0.80 A = 0.40 B + 100

$$0.80(150) \approx 0.40(50) + 100$$

$$120 = 120$$

Note

In Example 8.5 the BDC in the wet and dry fish cake is known as a **tie component** because the BDC goes from a single stream in the process to another single stream **without loss, addition, or splitting**.

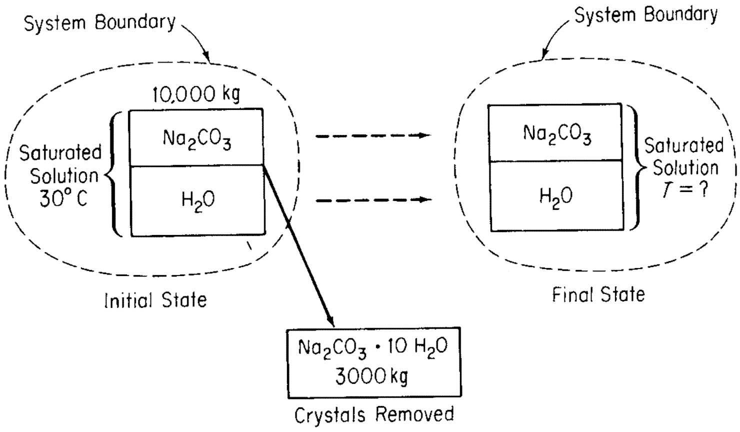
Example 14

A tank holds 10,000 kg of a saturated solution of Na_2CO_3 at 30°C. You want to crystallize from this solution 3000 kg of $Na_2CO_3.10\ H_2O$ without any accompanying water. To what temperature must the solution be cooled? You definitely need solubility data for Na_2CO_3 as a function of the temperature:

Temp.(°C)	Solubility (g Na_2CO_3/100 g H_2O)
0	7
10	12.5
20	21.5
30	38.8

Solution

No **reaction** occurs. Although the problem could be set up as a **steady-state problem** with flows in and out of the system (the tank), it is equally justified to treat the process as an -**unsteady-state process**.



Because the initial solution is saturated at 30°C, you can calculate the composition of the initial solution:

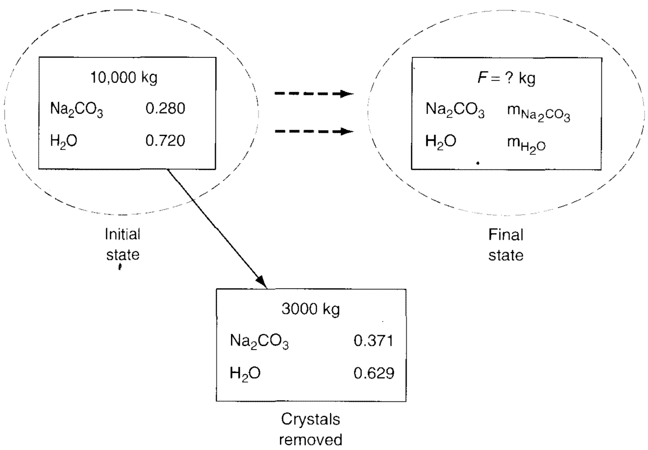
$$\frac{38.8 \text{ g } Na_2CO_3}{38.8 \text{ g } Na_2CO_3 + 100 \text{ g } H_2O} = 0.280 \text{ mass fraction } Na_2CO_3$$

Next, you should calculate the composition of the crystals.

Basis: 1 g mol Na_2CO_3.10 H_2O

Comp.	*Mol*	*Mol wt.*	*Mass*	*Mass fr*
Na_2CO_3	1	106	106	0.371
H_2O	10	18	180	0.629
Total			286	1.00

Basis: 10,000 kg of saturated solution at 30°C



An **unsteady-state** problem, the mass balance reduces to (the flow in = 0)

Accumulation = In – Out

Basis: I = 10,000 kg

Material balances: Na_2CO_3, H_2O

Note that $\omega_i^I I = m_i^I$, $\omega_i^F F = m_i^F$, and $\omega_i^C C = m_i^C$ are redundant equations. C = Crystals

Also redundant are equations such as $\Sigma\omega_i = 1$ and $\Sigma m_i = m_{total}$.

M.B.:

	Accumulation in Tank: *Final*		Accumulation in Tank: *Initial*		*Transport out*
Na_2CO_3	$m^F_{Na_2CO_3}$	–	10,000(0.280)	=	–3000(0.371)
H_2O	$m^F_{H_2O}$	–	10,000(0.720)	=	–3000(0.629)
Total	F	–	10,000	=	–3000

The solution for the composition and amount of the final solution is

Component	*kg*
$m^F_{Na_2CO_3}$	1687
$m^F_{H_2O}$	5313
F (total)	7000

Check using the total balance: 7,000 + 3,000 = 10,000

To find the temperature of the final solution, $\dfrac{1{,}687 \text{ kg } Na_2CO_3}{5{,}313 \text{ kg } H_2O} = \dfrac{31.8 \text{ g } Na_2CO_3}{100 \text{ g } H_2O}$

Thus, the temperature to which the solution must be cooled lies between **20°C** and **30°C**. By **linear interpolation**

$$30°C - \frac{38.8-31.8}{38.8-21.5}(10.0°C) = 26°C$$

Example 14

This example focuses on the plasma components of the streams: water, uric acid (UR), creatinine (CR), urea (U), P, K, and Na. You can ignore the initial filling of the dialyzer because the treatment lasts for an interval of two or three hours. Given the measurements obtained from one treatment shown in Figure E8.7b, calculate the grams per liter of each component of the plasma in the outlet solution.

Solution

This is an open steady-state system. **Basis: 1 minute**

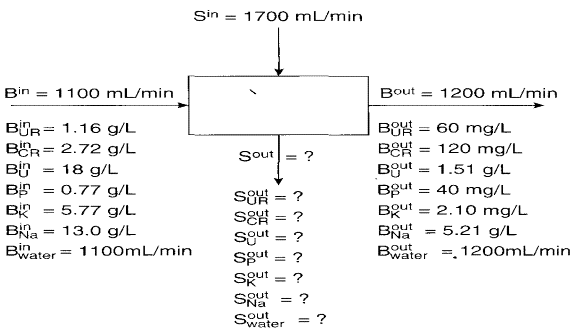


Figure E8.7b

- The **entering solution** is assumed to be essentially **water**.

The water balance in grams, assuming that I mL is equivalent to 1 gram, is:

$$1100 + 1700 = 1200 + S_{water}^{out} \quad \text{hence:} \quad S_{water}^{out} = 1600 \text{ mL}$$

The component balances in grams are:

		g/L
UR:	$1.1(1.16) + 0 = 1.2(0.060) + 1.6\, S_{UR}^{out}$	$S_{UR}^{out} = 0.75$
CR:	$1.1(2.72) + 0 = 1.2(0.120) + 1.6\, S_{CR}^{out}$	$S_{CR}^{out} = 1.78$
U:	$1.1(18) + 0 = 1.2(1.51) + 1.6\, S_{U}^{out}$	$S_{U}^{out} = 11.2$
P:	$1.1(0.77) + 0 = 1.2(0.040) + 1.6\, S_{P}^{out}$	$S_{P}^{out} = 0.50$
K:	$1.1(5.77) + 0 = 1.2(0.120) + 16\, S_{K}^{out}$	$S_{K}^{out} = 3.8$
Na:	$1.1(13.0) + 0 = 1.2(3.21) + 1.6\, S_{Na}^{out}$	$S_{Na}^{out} = 6.53$

Questions

1. Answer the following questions true or false:
   a. The most difficult part of solving material balance problems is the collection and formulation of the data specifying the compositions of the streams into and out of the system, and of the material inside the system.
   b. All open processes involving two components with three streams involve zero degrees of freedom.
   c. An unsteady-state process problem can be analyzed and solved as a steady-state process problem.
   d. If a flow rate is given in kg/min, you should convert it to kg mol/min.
2. Under what circumstances do equations or specifications become redundant?

Answers:

1. (a) T; (b) F; (c) T; (d) F
2. When they are not independent.

Problems

1. A cellulose solution contains 5.2% cellulose by weight in water. How many kilograms of 1.2% solution are required to dilute 100 kg of the 5.2% solution to 4.2%?
2. A cereal product containing 55% water is made at the rate of 500 kg/hr. You need to dry the product so that it contains only 30% water. How much water has to be evaporated per hour?
3. If 100 g of Na_2SO_4 is dissolved in 200 g of H_2O and the solution is cooled until 100 g of $Na_2SO_4.10H_2O$ crystallizes out; find (a) the composition of the remaining solution (the mother liquor) and (b) the grams of crystals recovered per 100 g of initial solution.
4. Salt in crude oil must be removed before the oil undergoes processing in a refinery. The crude oil is fed to a washing unit where freshwater fed to the unit mixes with the oil and dissolves a portion of the salt contained in the oil. The oil (containing some salt but no water), being less dense than the water, can be removed at the top of the washer. If the "spent" wash water contains 15% salt and the crude oil contains 5% salt, determine the concentration of salt in the "washed" oil product if the ratio of crud oil (with salt) to water used is 4:1.

Answers:

1. 33.3 kg
2. 178 kg/hr
3. (a) 28% Na_2SO_4 ; (b) 33.3
4. Salt: 0.00617; Oil: 0.99393.

2.4 The Chemical Reaction Equation and Stoichiometry

Stoichiometry

- The stoichiometric coefficients in the chemical reaction equation
  $C_7H_{16}(\ell) + 11\ O_2(g) \rightarrow 7\ CO_2(g) + 8\ H_2O(g)$ C_7H_{16}, 11 for O_2 and so on).
- Another way to use the chemical reaction equation is to indicate that **1 mole of CO_2** is formed from each **(1/7) mole of C_7H_{16}**, and **1 mole of H_2O** is formed with each **(7/8) mole of CO_2**. The latter ratios indicate the use of **stoichiometric ratios** in determining the relative proportions of products and reactants.

For example how many kg of CO_2 will be produced as the product if 10 kg of C_7H_{16} react completely with the **stoichiometric quantity** of O_2? On the basis of 10 kg of C_7H_6

$$\frac{10\ \text{kg}\ C_7H_{16}}{}\left|\frac{1\ \text{kg mol}\ C_7H_{16}}{100.1\ \text{kg}\ C_7H_{16}}\right|\frac{7\ \text{kg mol}\ CO_2}{1\ \text{kg mol}\ C_7H_{16}}\left|\frac{44.0\ \text{kg}\ CO_2}{1\ \text{kg mol}\ CO_2}\right. = 30.8\ \text{kg}\ CO_2$$

Example 15

The primary energy source for cells is the aerobic catabolism (oxidation) of glucose ($C_6H_{12}O_6$, a sugar). The overall oxidation of glucose produces CO_2 and H_2O by the following reaction

$$C_6H_{12}O_6 + aO_2 \rightarrow b\ CO_2 + c\ H_2O$$

Determine the values of a, b, and c that balance this chemical reaction equation.

Solution

Basis: The given reaction

By inspection, the carbon balance gives b = 6, the hydrogen balance gives c = 6, and an oxygen balance

$$6 + 2a = 6 * 2 + 6$$

Gives a = 6. Therefore, the balanced equation is $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$

Example 16

In the combustion of heptane, CO_2 is produced. Assume that you want to produce 500 kg of dry ice per hour, and that 50% of the CO_2 can be converted into dry ice, as shown in Figure E9.2. How many kilograms of heptane must be burned per hour? (MW: CO_2 = 44 and C_7H_{16} = 100.1)

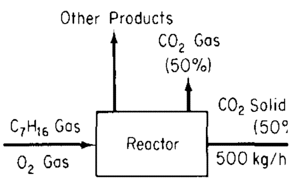


Figure E9.2