CH.4 Unsteady state conduction.

- Unsteady heat transfer.
- Transient heat " .

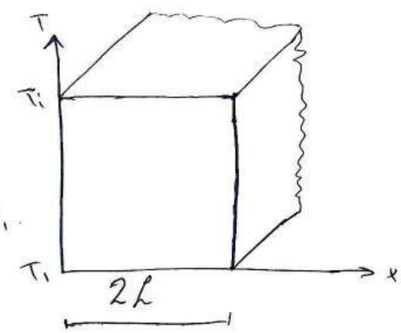
* For plane wall.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

In This case temp. is not only a function of x, it is also a function of time. $T(x, \tau)$

* Assumption:

1) Infinite plate with thickness $2L$
2) T_i is the initial temp @ $\tau = 0.0$
3) sudden change while cooling to T_1
4) constant thermal conductivity.
5) Unsteady. (درجة الحرارة تتغير مع الزمن)



$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{0.0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0.0} + \cancel{\frac{\dot{q}}{k}}^{0.0} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}} \quad \text{--- (1)} \qquad \boxed{\theta = T - T_1}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial \tau}$$

Boundary and initial condition.

$\theta = \theta_i = T_i - T_1$ @ $\tau = 0$ and $0 \le x < 2L$

$\theta = 0$ @ $\tau > 0$ $x = 0$ ---- (*)

$\theta = 0$ @ $\tau > 0$ $x = 2L$ ---- (**)

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$\theta(x, \tau)$

$\theta = X , H(\tau)$

$$\frac{\partial \theta}{\partial \tau} = x_{(x)} \frac{\partial H}{\partial \tau} \Rightarrow \frac{\partial \theta}{\partial x} = H(\tau) \frac{\partial x_{(x)}}{\partial x} \Rightarrow \frac{\partial^2 \theta}{\partial x^2} =$$

$$H(\tau) \frac{\partial^2 X_m}{\partial x^2}$$

$$H(\tau) \frac{\partial^2 X(m)}{\partial x^2} = \frac{1}{\alpha} X_{(x)} \frac{\partial H}{\partial \tau}$$

$$\frac{1}{x_{(x)}} \frac{\partial^2 X(x)}{\partial x^2} = \frac{1}{\alpha} \frac{1}{H(\tau)} \frac{\partial H}{\partial \tau} = -\lambda^2$$

λ^2: separation parameter

$$\frac{\partial^2 x_{(x)}}{\partial x^2} + \lambda^2 x_{(x)} = 0$$

$$\frac{\partial H}{\partial \tau} + \alpha \lambda^2 H(\tau) = 0$$

$$\theta = [C_1 \cos \lambda x + C_2 \sin \lambda x] e^{-\alpha \lambda^2 \tau}$$

using B-C $\Rightarrow 0 = (C_1 \cos 0 + C_2 \sin(0)) e^{-\alpha \lambda^2 \tau}$

$C_1 = 0$

$$\theta = (C_2 \sin \lambda x) e^{-\alpha \lambda^2 \tau} \quad \text{.....} (2)$$

using B-C $\Rightarrow 0 = (C_2 \sin 2L\lambda) e^{-\alpha \lambda^2 \tau}$

$\sin 2\lambda L = 0 \qquad 0, \lambda, 2\pi, 3\pi$

$2\lambda L = n\lambda$

$$\lambda = \frac{n\pi}{2L}$$

$$\boxed{\lambda = \frac{n\pi}{2L}}$$

$$\theta = \sum_{n=1}^{\infty}\left(C_n \sin\frac{n\pi x}{2L}\right) e^{-\alpha\left(\frac{n\pi}{2L}\right)^2 \tau}$$

substitute in equ.②.

Infinite plate having thickness... = 2L

$$\frac{T-T_1}{T_i-T_1} = \frac{\theta}{\theta_i} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\, e^{-\alpha\left(\frac{n\pi}{2L}\right)^2 \tau} \sin\left(\frac{n\pi x}{2L}\right) \qquad n = 1, 3, 5, 7, 9, \ldots$$

هنا الزوايا بالراديان.

$$\alpha = \frac{k}{\rho c}$$

4-2 An infinite plate having a thickness of 2.5 cm is initially at a temperature of 150◦C, and the surface temperature is suddenly lowered to 30◦C. The thermal diffusivity of the material is 1.8×10−6 m2/s. Calculate the center-plate temperature after 1 min by summing the first four nonzero terms of Equation (4-3). Check the answer using the Heisler charts.

thickness = $2L = 2.5$ cm , $T_i = 150°C$, $T_1 = 30°C$

thermal diffusivity $(\alpha) = 1.8 \times 10^{-6}$ m²/s , find $T\big|_{\tau = 60s,\ x = L}$

— calculate the center plate ($x = L$) temp. after 1 min. using the first 4 nonzero terms of equation.

solu:- $n = 1, 3, 5, 7$

$$\frac{T-30}{150-30} = \frac{4}{\pi}\Bigg[\frac{1}{1} e^{-1.8\times10^{-6}\times60\left[\frac{\pi}{0.025}\right]^2} \sin\frac{1\pi L}{2L} + \frac{1}{3} e^{-1.8\times10^{-6}\times60\left[\frac{3\pi}{0.025}\right]^2} \sin\frac{3\pi L}{2L}$$

$$+ \frac{1}{5} e^{-1.8\times10^{-6}\times60\left[\frac{5\pi}{0.025}\right]^2} \sin\frac{5\pi}{2} + \frac{1}{7} e^{-1.8\times10^{-6}\times60\left[\frac{7\pi}{0.025}\right]^2} \sin\frac{7\pi}{2}\Bigg]$$

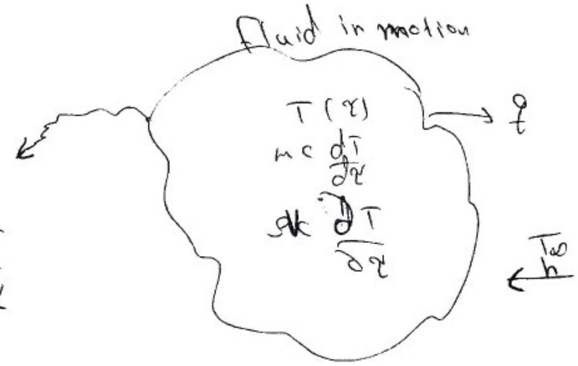
$$\frac{T-30}{120} = \frac{4}{\pi}\left[0.1818 - 7.1912\times10^{-8} + 6.0834\times10^{-20} - 7.27\times10^{-38}\right]$$

$$\frac{T-30}{120} = 0.231 \Rightarrow \boxed{T = 57.777°C} \quad \#$$

* lumped - Heat-capacity system.

- The system is assumed to be uniform in Temp. This means that temp defference has been decayed. (فتذبذب متلاشي)
- The internal thermal conductive resistance is smaller than the external convective resistance.

جسم غمر في مائع .



* Energy Balance.

$$q = hA(T - T_\infty) = -\rho c V \frac{dT}{d\tau}$$

$V \Rightarrow$ volume
$A \Rightarrow$ surface area
$c \Rightarrow$ spesific heat transfer (kJ/kg) - c
$T_\infty \Rightarrow$ Temp fluid

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\theta}{\theta_o} = e^{\frac{-hA\tau}{\rho c V}}$$

$A = m^2$, T_o = Temp, intial
τ = scond T = center temp , T_∞ = fluid Temp (air, water)
$c = J/kg\text{-}c$ $V = m^3$ T_o = initial temp

4-6 A piece of aluminum weighing 6 kg and initially at a temperature of 300◦C is suddenly immersed in a fluid at 20◦C. The convection heat-transfer coefficient is 58W/m2◦C.Taking the aluminum as a sphere having the same weight as that given, estimate the time required to cool the aluminum to 90◦C, using the lumped-capacity method of analysis.

$m = 6\,kg$, $T_o = 300°C$, $T_\infty = 20°C$, $h = 58\ W/m^2.°C$.
Aluminum as a sphere. , $T = 90°C$, find τ (time) using lumped capacity method.

تذكر:
sphere $\Rightarrow$ $A = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

sol:- $$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}}$$

from Table A-2 :- Aluminum. $\rho = 2707\ kg/m^3$
$c = 0.896\ kJ/kg.°C$
$\boxed{c = 896\ J/kg.°C}$

To find volume and Area. we must to find (r)

$\boxed{\rho = \frac{m}{V}}$

$$m = \rho V$$

$$6 = 2707 * \frac{4}{3}\pi r^3 \Rightarrow \boxed{r = 0.0807\ m}$$

$$A = 4\pi r^2 = 4 * \pi * (0.0807)^2 = 0.0822\ m^2$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}} = e^{-\frac{hA\tau}{cm}}$$

$$Ln\ \frac{T - T_\infty}{T_o - T_\infty} = Ln\ e^{-\frac{hA\tau}{cm}}.$$

$$Ln\ \frac{90 - 20}{300 - 20} = -\frac{58 * 0.0822 * \tau}{896 * 6}$$

$$\boxed{\tau = 1563\ sec}$$

4-10 A stainless-steel rod (18% Cr, 8% Ni) 6.4 mm in diameter is initially at a uniform temperature of 25◦C and is suddenly immersed in a liquid at 150◦C with *h*= 120 W/m2 ◦C. Using the lumped-capacity method of analysis, calculate the time necessary for the rod temperature to reach 120◦C.

stainless - steel rod (cylinder) (18% Cr, 8% Ni) $d = 6.4$ mm.

$T_o = 25$, $T_\infty = 150°C$, $h = 120$, using lumped - capacity method.

find $\tau|_{T=120°C}$.

sol:

from table A-2. (page 651) $\rho = 7817$ Kg/m³
$c = 460$ J/Kg.c

cylinder.

$A = \pi d L$
$V = \frac{\pi}{4} d^2 L$

Note: L is unknown. so...?

$$\frac{A}{V} = \frac{\pi d L}{\frac{\pi}{4} d^2 L} = \boxed{\frac{4}{d}}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}}$$

$$\frac{120 - 150}{25 - 150} = e^{\frac{-120 \times 4 \times \tau}{7817 \times 460 \times 6.4 \times 10^{-3}}}$$

Ln للطرفين

$$\boxed{\tau = 1316 \text{ sec}}$$

Time constant $\tau = \frac{\rho c V}{h A}$.

special case: IF time is equal to time constant. in lumped capacity method.

$$\frac{\theta}{\theta_o} = e^{\frac{-hA\tau}{\rho c V}} = e^{-\frac{hA}{\rho c V}\left(\frac{\rho c V}{hA}\right)} = e^{-1}$$

$$\frac{\theta}{\theta_o} = 0.368. \qquad \therefore \boxed{T - T_\infty = 36.8\% \ [T_o - T_\infty]}$$

في جميع الاسئلة السابقه كان السؤال محدد طريقه الحل و هي (lumped) . لكن في حاله عدم تحديد طريقه الحل نستخدم biot number لتحديد طريقه الحل.

$Bi < 0.1 \Rightarrow$ lumped can be used
$Bi > 0.1 \Rightarrow$ from figures (Heisler charts)

$$Bi = \frac{h\,(V/A)}{K} = \frac{hs}{K}$$

$s \rightarrow$ characteristic length. $\frac{V}{A}$

$$s\,(\text{plate}) = \frac{V}{A} = \frac{L^3}{L^2} = L$$

$$s\,(\text{cylinder}) = \frac{V}{A} = \frac{\frac{\pi}{4} d^2 L}{\pi d L} = \frac{d}{4} = \frac{R}{2}$$

$$s\,(\text{sphere}) = \frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{d}{6}$$

4-16 A 12-mm-diameter aluminum sphere is heated to a uniform temperature of 400◦C and then suddenly subjected to room air at 20◦C with a convection heat-transfer coefficient of 10 W/m2 ・ ◦C. Calculate the time for the center temperature of the sphere to reach 200◦C.

$d = 12\,mm$. (Aluminum sphere). , $T_o = 400°C$. $T_\infty = 20°C$
$h = 10\ w/m^2.c°$, $\tau|_{T=200°C}$. ?

Sol: Table: $K = 204$, $\rho = 2707$, $c = 896$.

check. sphere $\left(\frac{V}{A}\right) = \frac{d}{6}$ $\frac{A}{V} = \frac{6}{d}$

$$Bi = \frac{h}{K}\frac{V}{A} = \frac{10}{204} * \frac{12 * 10^{-3}}{6} = 9.8 * 10^{-5} < 0.1 \therefore \text{lumped can use.}$$

$$\therefore \frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}} \Rightarrow \frac{200-20}{400-20} = e^{\frac{-10 * 6 * \tau}{2707 * 896 * 12 * 10^{-3}}}$$

$$\boxed{\tau = 362\,sec}$$