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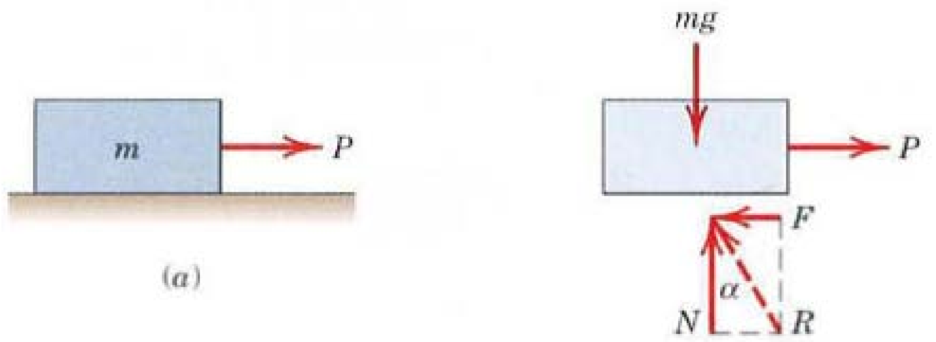
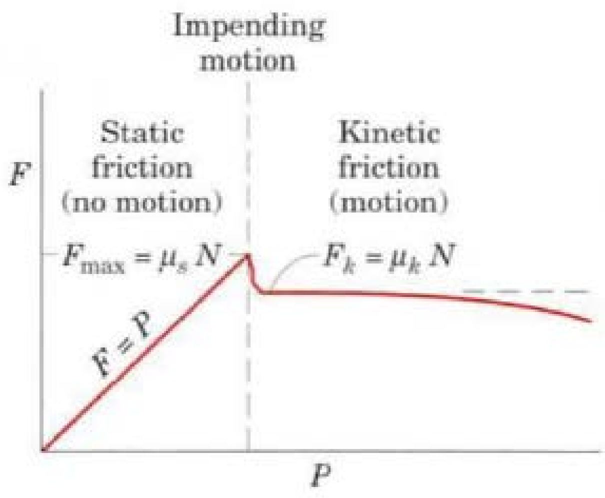
FRICTION

Consider a solid block of mass m resting on a horizontal surface, as shown in Figures we assume that the contacting surfaces have some roughness.



(a) (b)



(d)

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Static Friction

The region in Figure d up to the point of slippage or impending motion is called the range of static friction, and in this range the value of the friction force is determined by the equations of equilibrium.

$$F_{max} = \mu_s N$$

Where μ_s, is the proportionality constant, called the coefficient of static friction.

$$F < \mu_s N$$

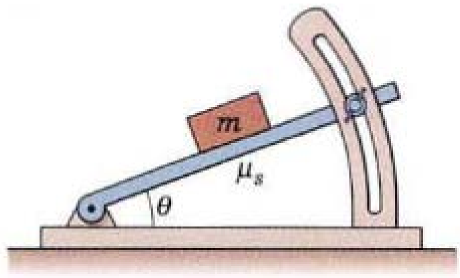
Kinetic Friction

After slippage occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k is also proportional to the normal force. Thus,

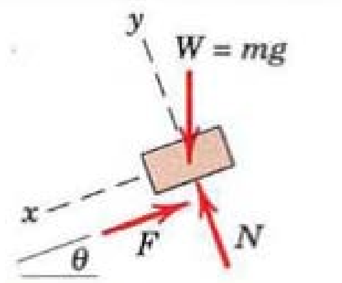
$$F_k = \mu_k N$$

Problem 1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s .



Solution



Equilibrium in the x- and y-directions requires

$\Sigma F_x=0$ $mg \sin\theta - F = 0$ $F = mg \sin\theta$

$\Sigma F_y=0$ $-mg\cos\theta + N = 0$ $N = mg\cos\theta$

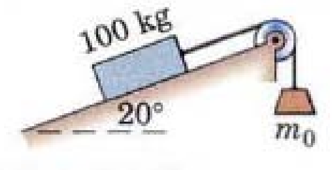
$F/N = \tan\theta$

$F = F_{max} = \mu_s N$

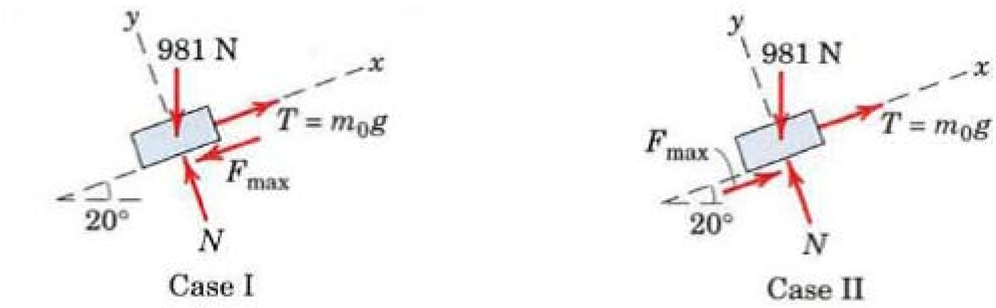
$\mu_s = \tan\theta_{max}$ or $\theta_{max} = \tan^{-1}\mu_s$,

Problem 2

Determine the range of values which the mass m_o may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30



Solution



mg = 100(9.81) = 981 N , $\Sigma F_y=0$ N - 981 cos 20° = 0 N = 922 N

$F_{max} = \mu_s N$ F_{max} = 0.30(922) = 277 N

The block for Case I in the figure

$\Sigma F_x=0$ $m_o(9.81) - 277 - 981 \sin 20^\circ = 0$ m_o = 62.4 kg

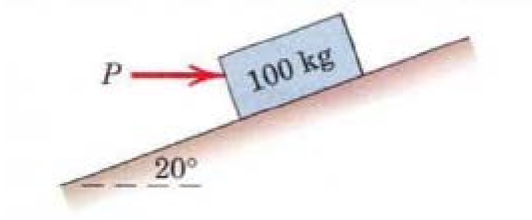
The block for Case II in the figure

$\Sigma F_x=0$ $m_o(9.81) + 277 - 981 \sin 20^\circ = 0$ m_o = 6.01 kg

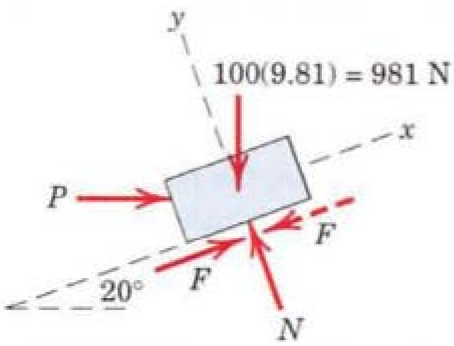
Thus, m_o may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

Problem 3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, P = 500 N and, second, P = 100 N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution



$\Sigma F_x=0$ $\quad$ $P\cos 20^\circ + F - 981\sin 20^\circ = 0$

$\Sigma F_y=0$ $\quad$ $N - P\sin 20^\circ - 981\cos 20^\circ = 0$

Case I. P = 500 N

F = - 134.3 N , N = 1093N

$F_{max} = \mu_s N$ $\quad$ $F_{max} = 0.20(1093) = 219$ N

Case II. P = 100 N

F = 242 N , N = 956 N

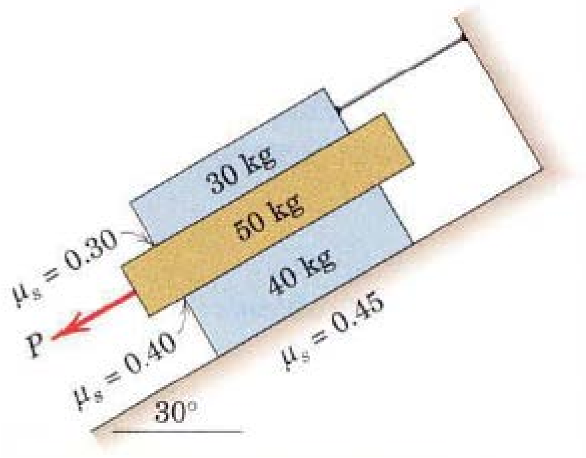
$F_{max} = \mu_s N$ $\quad$ $F_{max} = 0.20(956) = 191.2$ N

It follows that 242 N of friction cannot be supported. Therefore, equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the mot ion down the plane.

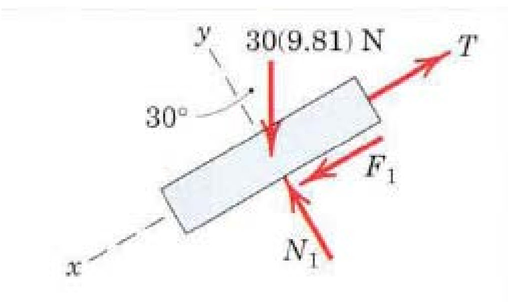
$F_k = \mu_k N$ $\quad$ $F_k = 0.17(956) = 162.5$ N

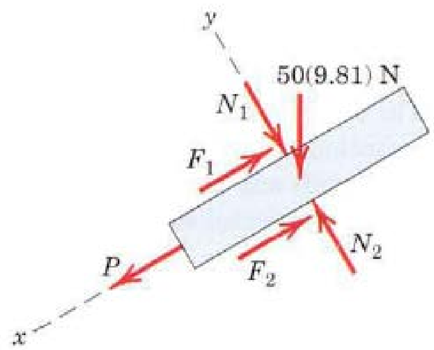
Problem 4

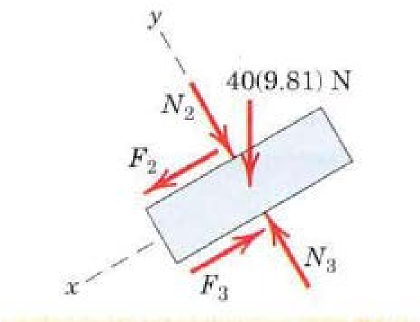
The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.



Solution







$\Sigma F_y=0$ (30 kg) $N_1 - 30(9.81)\cos 30^\circ = 0$ $N_1 = 255$ N

(50 kg) $N_2 - 50(9.81)\cos 30^\circ - 255 = 0$ $N_2 = 680$ N

(40 kg) $N_3 - 40(9.81)\cos 30^\circ - 680 = 0$ $N_3 = 1091$ N

$F_{max} = \mu_s N$ $F_1 = 0.30(255) = 76.5$ N $F_2 = 0.40(680) = 272$ N

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$\Sigma F_x=0$ $P - 76.5 - 272 + 50(9.81)\sin 30^\circ = 0$ $P = 103.1$ N

We now check on the validity of our initial assumption. For the 40·kg block with F2 == 272 N the friction force F3 would be given by

$\Sigma F_x=0$ $272 + 40(9.81)\sin 30 – F_3 = 0$ $F_3 = 468$ N

$F_3 = \mu_s N_3 = 0.45(1019) = 459$ N.

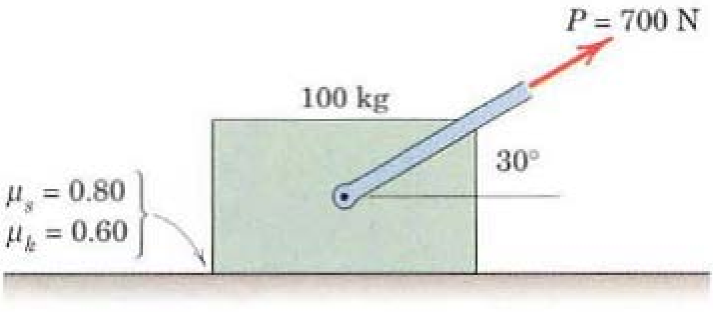
$\Sigma F_x=0$ $F_2 + 40(9.81)\sin 30 - 459 = 0$ $F_2 = 263$ N

Equilibrium of the 50.kg block gives, finally,

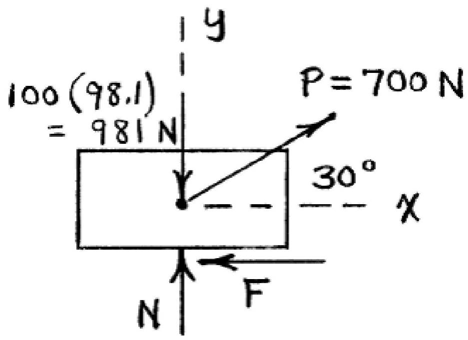
$\Sigma F_x=0$ $P + 50(9.81)\sin 30 - 263 - 76.5 = 0$ $P = 93.8$ N

Problem 5

The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force *F* exerted by the horizontal surface on the block.



Solution



$$\Sigma F_x = 0: \quad 700 \cos 30° - F = 0, \quad F = 606 \text{ N}$$

$$\Sigma F_y = 0: \quad N - 981 + 700 \sin 30° = 0, \quad N = 631 \text{ N}$$

$$F_{max} = \mu_s N = 0.8(631) = 505 \text{ N} < F = 606 \text{ N}$$

Assumption invalid, motion occurs.

$$F = \mu_k N = 0.6(631) = \underline{379 \text{ N}}$$