# *Chapter four*

## *Shear and bending moment diagrams*

In this chapter we use the term beam or bar. The beam is defined as a bar subjected to forces or couples that lie in a plane containing the longitudinal axis of the bar and acting perpendicular to this axis. During the context of this chapter different types of beams will be used depending on the types of support or loading(i.e. simply supported, cantilever, overhanging beam----etc). The beams also divided as:-

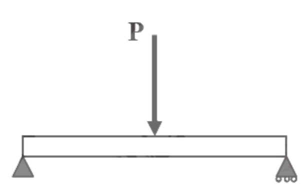
- Statically determinate beams: all the beams are ones in which the reactions of the supports may be determined by use of the equations of static equilibrium. The values of reactions are independent of the deformation of the beam.
- Statically indeterminate beams: if the number of the reactions upon the beam exceeds the number of equations of static equilibrium, then the statics equations must be supplemented by equations based upon the deformations of the beam.

It's very important to know how to draw axial force, shear force and bending moment diagrams for any part of any structure

1- To determine the location of max and min values of these function then we can determine the min and max stresses of axial, shear and bending moment.
2- To find deflection of beams by different methods, such as area- moment method.
3- To solve and analysis of statically indeterminate structures.

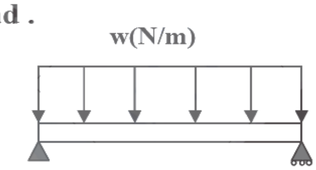
**Loads are classified as:-**

**1- Concentrated load .**



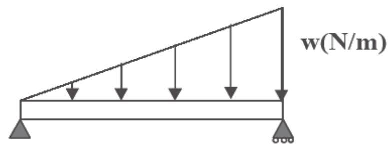

1- Concentrated Load

**2- Uniformly distributed load .**




2- Uniformly Distributed Load

**3- Varyingly distributed loads .**

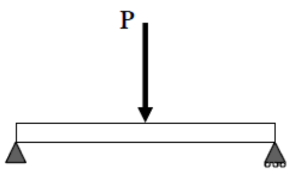



3- Varying Distributed Load

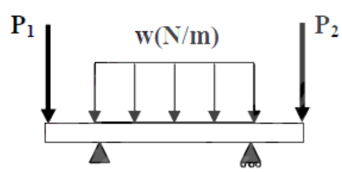
**There are different types of beams supporting such as:**

**A-** Statically determinate beams:

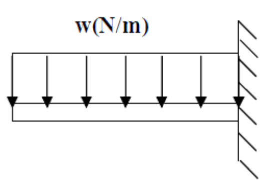
**1- Simply supporting beam :**




**2- Overhanging beam :**

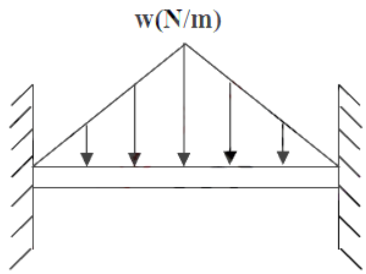



**3- Cantilever beam :**

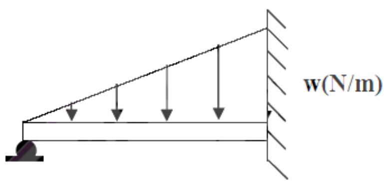


**B-** Statically indeterminate beams:

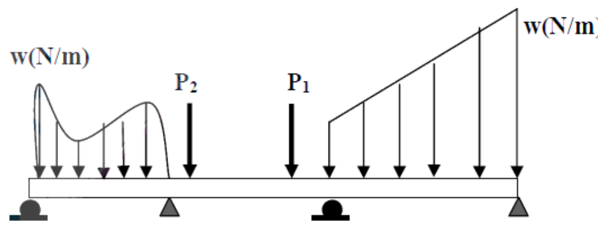
**4- Fixed-end beam :**




**5- Propped beam :**



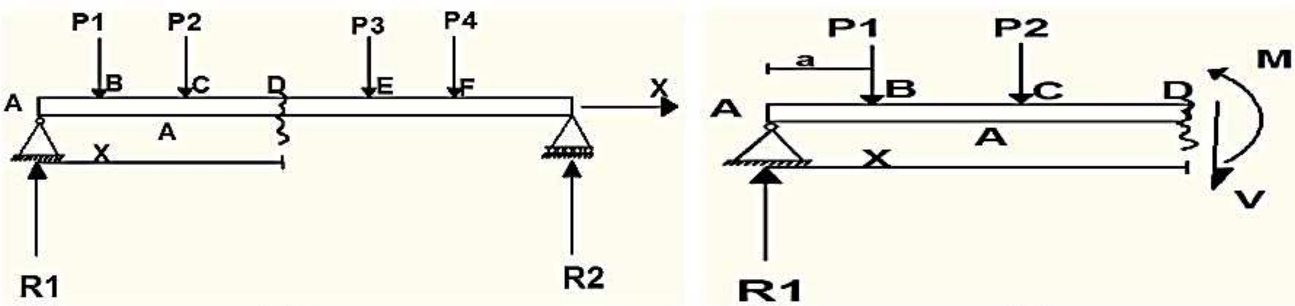

**6- Continues beam :**




Generally, two kind of stresses act over the transverse section of beams:
1) Shearing stresses: which varies directly with the shear force (V).
2) Bending stress: which varies directly with the bending moment (M).

## Shear and Moment :

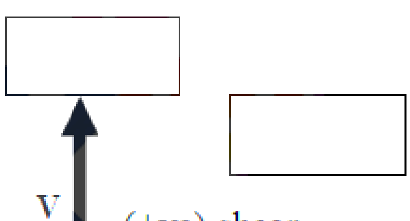
When the beam loaded by forces and couples, internal stresses arise in the bar. In general, both normal and shearing stresses will occur. The resultant of forces and moment acting are necessary to be found and equations of static equilibrium are applied to find these stresses. Fig. 3-1 shows a simple supported beam with several concentrated loads.



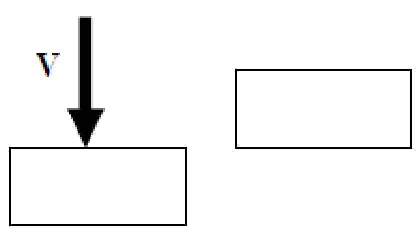

(a) (b)

To study the internal stresses at section D let us consider the beam to be cut at D and the portion of the beam to the right of D removed. The portion removed must then be replaced by the effect it exerted upon the portion to the left of D and this effect will consist of a vertical shearing force together with a couple, as represented by vectors V and M respectively in the free body diagram Fig. 3-1b.

The force V and the couple M hold the left portion of the bar in equilibrium under the action of the forces $R_1$ ,$P_1$, $P_2$.




(+ve) shear




(-ve) shear

**Bending moment is positive if it produces bending of the beam concave upward and vise-versa as shown in figure .**

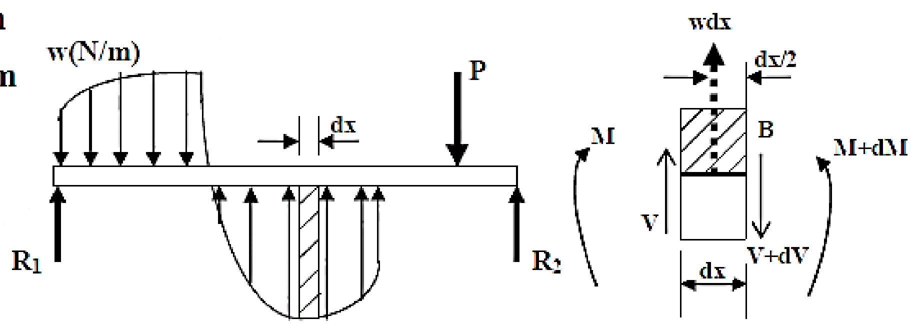

**Upward acting forces generally causes (+ve) bending regardless to weather they act to the left or to right of exploratory section .**

## Relation between load , shear and moment :-

**We will discuss the relations existing between the loads, shears and bending moments in any beam . Theses relations provide a method of constructing shear and moment diagrams without writing shear and moment equations .**

**The beam shown in figure (1) is assumed to carry any general loading .**

**The free body diagram of segment of this beam of length (dx) is shown in figure (2) .**




(1) (2)

$$\sum F_y = 0 \Rightarrow V + wdx - (V + dV) = 0$$

$w$---------Intensity of load (N/m)

$$\therefore dV = wdx$$

$$\sum M_B = 0 \Rightarrow M + Vdx + (wdx)\frac{dx}{2} - (M + dM) = 0$$

where : $w\frac{(dx)^2}{2} \approx zero$

$$\therefore dM = Vdx$$

for ( $dV = wdx$ ) we can integrating

$$\therefore \int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w\,dx$$

but $\int_{x_1}^{x_2} w\,dx \Rightarrow$ is the summation of area between ( $x_1$ and $x_2$ )

$$\therefore V_2 - V_1 = \Delta V = (area)_{load}$$

similarly

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\,dx$$

$$M_2 - M_1 = \Delta M = (area)_{shear}$$

$$\therefore w = \frac{dV}{dx} \qquad \text{(slope of shear diagram)}$$

$$\therefore V = \frac{dM}{dx} \qquad \text{(slope of moment diagram)}$$