## CHEMICAL ENERGY

Determining the energy released by a given fuel in a particular combustion situation is a critical aspect of combustion engineering. This section introduces the tools needed by the combustion engineer to make this determination. This includes the concepts of heat of reaction at constant pressure and constant volume, higher and lower heating value, heat of formation, and absolute enthalpy.

## Heat of Reaction

Heat of reaction is the chemical energy released when a fuel reacts with air to form products. The starting point for determining the heat of reaction is to specify the chemical species and their states in the reactants and products. This is done by writing a balanced reaction with the phase of each species noted. The heat of vaporization of liquid fuels and the heat of pyrolysis of solid fuels are typically small compared to the chemical energy released by combustion. However, the effect of water condensation can be important. For example, there is a substantial increase in furnace efficiency when water is condensed in the heat exchanger of a home gas-fired furnace.
For very lean hydrocarbon-air mixtures where the temperature is low, the products may be assumed to be complete (usually CO2, H2O, O2, and N2). However, with high product temperatures and for rich mixtures, it is generally necessary to include other species and assume chemical equilibrium to determine the species mole fractions. If the products are not in equilibrium, then a chemical kinetic analysis or direct measurement is required to determine the end state. For example, the amount of solid carbon produced by very rich mixture combustion is typically not predicted correctly by equilibrium. The gaseous composition of rich mixture products is also somewhat uncertain, even if equilibrium prevails, because the identity of the unburned hydrocarbon species is not easily determined. A solution to this problem is to complete the oxidation in a separate reactor and measure the additional energy release.
To understand the heat of reaction, consider the reaction of a fuel and air mixture of mass $m$ (the total mass of the system). For constant volume combustion with heat transfer there is no work and the first law of thermodynamics (Equation 3.4) is

$$
Q_{v}=m\left[\left(u_{2}-u_{1}\right)_{s}+\left(u_{2}-u_{1}\right)_{\text {chem }}\right]
$$

where the subscripts $s$ and chem refer to sensible and chemical energy. Note that in Equation 3.46, the values of $u$ at State 1 and State 2 are obtained by summing over all I species, $\mathrm{i}=1,2, \ldots$, I. Thus

$$
\left[u_{1}\right]_{s}=\left[u_{T_{1}}\right]_{s}=\sum_{i=1}^{I} y_{i}\left[u_{i, T_{1}}\right]_{s}=\sum_{i=1}^{I} y_{i} \int_{T_{0}}^{T_{1}}\left(c_{v, i}\right)_{\text {react }} d T=\int_{T_{0}}^{T_{1}}\left(c_{v}\right)_{\text {react }} d T
$$

where $T_{0}$ is the reference temperature. The quantity $\left(c_{\nu}\right)_{\text {react }}$ is the specific heat of the reactant mixture. Similarly

$$
\left[u_{2}\right]_{s}=\int_{T_{0}}^{T_{2}}\left(c_{v}\right)_{\operatorname{prod}} d T
$$

where $T_{0}$ is the reference temperature, and the quantity $\left(c_{\nu}\right)_{\text {prod }}$ is the specific heat of the product mixture.

If the heat transfer is just large enough to bring the product's temperature back to the reactant temperature, and if this temperature is taken as the reference temperature, $T_{0}$, for the sensible energy, then $\left(u_{2}-u_{1}\right)_{s}=0$ by definition, and $Q_{v}$ is the chemical energy released by the reaction. For constant volume combustion if the water in the products is not condensed, then the lower heating value can be expressed as

$$
\begin{equation*}
\mathrm{LHV}=\frac{-Q_{v}}{m}\left(\frac{1+f}{f}\right) \tag{3.47}
\end{equation*}
$$

If the water in the products is condensed, then the higher heating value,

$$
\begin{equation*}
\mathrm{HHV}=\frac{-Q_{v}}{m}\left(\frac{1+f}{f}\right) \tag{3.48}
\end{equation*}
$$

is obtained for constant volume combustion.
If the reaction takes place at constant pressure and the total heat transfer is $Q_{p}$, then the energy equation (Equation 3.12) becomes

$$
\begin{equation*}
m\left[\left(h_{2}-h_{1}\right)_{s}+\left(h_{2}-h_{1}\right)_{c}\right]=Q_{p} \tag{3.49}
\end{equation*}
$$

Again, if $T_{1}=T_{2}=T_{0}$, then $Q_{p}$ is the chemical energy released. For the constant pressure case if the number of moles of gaseous products, $N_{\text {prod }}$, is larger than the number of moles of gaseous reactants, $N_{\text {react }}$, then some of the chemical energy is expended as work as the expanding gases push aside the ambient pressure. Thus for reactants and products which are ideal gases,

$$
\begin{equation*}
Q_{p}-Q_{v}=\Delta(p V)=\left(N_{\text {prod }}-N_{\text {react }}\right) \hat{R} T_{0}=\Delta N \hat{R} T \tag{3.50}
\end{equation*}
$$

Consider the generalized combustion reaction

$$
\begin{align*}
& \mathrm{C}_{\alpha} \mathrm{H}_{\beta} \mathrm{O}_{\gamma}+\left(\alpha+\frac{\beta}{4}-\frac{\gamma}{2}\right)\left(\mathrm{O}_{2}+3.76 \mathrm{~N}_{2}\right)  \tag{3.51}\\
& \quad \rightarrow \alpha \mathrm{CO}_{2}+\frac{\beta}{2} \mathrm{H}_{2} \mathrm{O}+3.76\left(\alpha+\frac{\beta}{4}-\frac{\gamma}{2}\right) \mathrm{N}_{2}
\end{align*}
$$

Assuming that the fuel is in the gas phase and the water remains water vapor, then

$$
\Delta N=\frac{\beta}{4}+\frac{\gamma}{2}-1
$$

For methane (CH4) $\beta=4$ and $\gamma=0$. Therefore, the number of moles of reactants equals the number of moles of reactants $(\Delta N=0)$ for methane. Because both $Q p$ and $Q v$ are typically negative, $\Delta N>0$ implies $|Q v|>|Q p|$ as anticipated. For most cases of interest the difference, $Q p-Q v$, is only a few kilocalories and is thus often neglected.

The heat of reaction may be calculated for reactions taking place at temperatures other than $T 0$ and for cases where the initial and final temperature are not equal by use of the heat of reaction data taken at $T 0$. Consider the reaction at constant pressure with reactant temperature $T 1$ and product temperature $T 2$. Assume, for example, $T 2>T 1>T 0$. To use the $Q p(T 0)$ value, first imagine the reactants are cooled from $T 1$ to $T 0$, then the reaction takes place at $T 0$, and finally the products are heated from $T 0$ to $T 2$.

$$
Q_{p}=m \int_{T_{1}}^{T_{0}} c_{p, \text { reat }} d T+Q_{p, T_{0}}+m \int_{T_{0}}^{T_{2}} c_{p, \text { prod }} d T
$$

or

$$
\begin{equation*}
Q_{p}-Q_{p, T_{0}}=m\left(h_{s 2}-h_{s 1}\right) \tag{3.52}
\end{equation*}
$$

Remember that $Q_{p, T_{0}}$ is negative for an exothermic reaction.

## Example 1:

The higher heating value of gaseous methane and air at $25^{\circ} \mathrm{C}$ is $55.5 \mathrm{MJ} / \mathrm{kg}$.
Find the heat of reaction at constant pressure of a stoichiometric mixture of methane and air if the reactants and products are at 500 K .

## Solution

The reaction is

$$
\mathrm{CH}_{4}+2\left(\mathrm{O}_{2}+3.76 \mathrm{~N}_{2}\right) \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}+7.52 \mathrm{~N}_{2}
$$

Determining the molecular weight of the reactant mixture,

| Reactants | $N_{i}$ (moles) | $\boldsymbol{x}_{\boldsymbol{i}}$ | $\mathcal{M}_{\boldsymbol{i}}(\mathbf{k g} / \mathbf{k g m o l})$ | $\boldsymbol{M}_{i} \boldsymbol{x}_{\boldsymbol{i}}(\mathbf{k g} / \mathbf{k g m o l})$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{CH}_{4}$ | 1 | 0.095 | 16 | 1.5 |
| $\mathrm{O}_{2}$ | 2 | 0.190 | 32 | 6.1 |
| $\mathrm{~N}_{2}$ | 7.52 | 0.715 | 28 | 20.0 |
|  | 10.52 | 1.000 |  | 27.6 |

$$
\begin{gathered}
N_{\text {react }}=1+2(4.76)=10.52 \\
M_{\text {react }}=\sum_{i} x_{i} M_{i}=27.6 \mathrm{~kg} / \mathrm{kgmol}
\end{gathered}
$$

Analysis of the reactants using sensible enthalpies from Appendix $C$ yields

| Reactants | $\boldsymbol{x}_{\boldsymbol{i}}$ | $\hat{\boldsymbol{h}}_{\boldsymbol{i}}(\mathbf{M J} / \mathbf{k g m o l})$ | $\hat{\boldsymbol{h}}_{\boldsymbol{i}} \boldsymbol{x}_{\boldsymbol{i}}(\mathbf{k g} / \mathbf{k g m o l})$ |
| :--- | :---: | :---: | :---: |
| $\mathrm{CH}_{4}$ | 0.095 | 8.20 | 0.779 |
| $\mathrm{O}_{2}$ | 0.190 | 6.09 | 1.157 |
| $\mathrm{~N}_{2}$ | 0.715 | 5.91 | 4.226 |
|  | 1.000 |  | 6.162 |

$$
\begin{aligned}
& \hat{h}_{s, \text { react }}=\sum_{i} x_{i} h_{s i}=6.16 \mathrm{MJ} / \mathrm{kgmol} \\
& h_{s, \text { react }}=\frac{6.16 \mathrm{MJ}}{\mathrm{kgmol}} \cdot \frac{\mathrm{kgmol}}{27.6 \mathrm{~kg}}=\frac{223 \mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

The fuel-air mass ratio is

| Reactants | $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{M}_{\boldsymbol{i}}$ | $\boldsymbol{M}_{\boldsymbol{i}} / \boldsymbol{M}$ | $\boldsymbol{y}_{\boldsymbol{i}}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{CH}_{4}$ | 0.095 | 16 | 0.580 | 0.055 |
| $\mathrm{O}_{2}$ | 0.190 | 32 | 1.159 | 0.220 |
| $\mathrm{~N}_{2}$ | 0.715 | 28 | 1.014 | 0.725 |
|  | 1.000 |  |  | 1.000 |

$$
f=\frac{y_{\mathrm{f}}}{y_{\text {air }}}=\frac{0.055}{0.220+0.725}=0.0582
$$

In the same way analysis of the products yields

| Products | $\boldsymbol{x}_{\boldsymbol{j}}$ | $\hat{\boldsymbol{h}}_{\boldsymbol{s} \boldsymbol{j}}(\mathrm{MJ} / \mathrm{kgmol})$ | $\boldsymbol{M}_{\boldsymbol{j}}$ |
| :--- | :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.095 | 8.31 | 44 |
| $\mathrm{H}_{2} \mathrm{O}$ | 0.190 | 6.92 | 18 |
| $\mathrm{~N}_{2}$ | 0.715 | 5.91 | 28 |

$$
\begin{aligned}
& N_{\text {prod }}=10.52 \\
& \hat{h}_{s, \text { prod }}=6.33 \mathrm{MJ} / \mathrm{kgmol} \\
& M_{\text {prod }}=2.76 \mathrm{~kg} / \mathrm{kgmol} \\
& h_{s . \text { prod }}=229 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

Because the water vapor does not condense at 500 K , the lower heating value is used;

Because the water vapor does not condense at 500 K , the lower heating value is used;

$$
\begin{aligned}
\mathrm{LHV} & =\frac{55,000 \mathrm{~kJ}}{\mathrm{~kg}_{\mathrm{H}_{2} \mathrm{O}}}-\frac{2394 \mathrm{~kJ}}{\mathrm{~kg}_{\mathrm{H}_{2} \mathrm{O}}} \cdot \frac{2 \mathrm{kmol}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{kmol}_{\mathrm{CH}_{4}}} \cdot \frac{18 \mathrm{~kg}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{kgmol}_{\mathrm{H}_{2} \mathrm{O}}} \cdot \frac{\mathrm{kgmol}_{\mathrm{CH}_{4}}}{16 \mathrm{~kg}_{\mathrm{CH}_{4}}} \\
& =\frac{50,113 \mathrm{KJ}}{\mathrm{~kg}_{\mathrm{CH} 4}}
\end{aligned}
$$

Remembering that

$$
\frac{m_{\mathrm{f}}}{m_{\text {react }}}=\frac{m_{\mathrm{f}}}{m_{\text {air }}+m_{\mathrm{f}}}=\frac{f}{1+f}
$$

we can write

$$
\begin{aligned}
\frac{Q_{p, T_{0}}}{m}=\frac{f}{1+f}(\mathrm{LHV}) & =\frac{0.0582}{1+0.0582}(50,113) \mathrm{kJ} / \mathrm{kg}_{\text {reat }} \\
& =-2747 \mathrm{~kJ} / \mathrm{kg}_{\text {react }}
\end{aligned}
$$

From Equation 3.52 the heat of reaction per unit mass of reactants at 500 K is

$$
\mathrm{Q}_{p}=-2747+(229-223)=-2741 \mathrm{~kJ} / \mathrm{kg}_{\text {react }}
$$

The negative sign indicates that the heat flows out of the system.

The heat of reaction of fuels combusting in air (or oxygen) with the starting and ending points at $25^{\circ} \mathrm{C}$ and 1 atm gives the fuel heating value. These heating values are then tabulated for common fuels. Following this same practice for all possible reactions would, however, lead to an enormous amount of tabulated data. The
solution to this problem comes by noting that we may simply add selected reactions and their heats of reaction to obtain any given reaction and its heat of reaction. The most basic set of reactions are thus for the formation of compounds from their elements. Once the data for this basic set is tabulated, any reaction can be constructed from it.

## Example 2:

The higher heating value of gaseous methane and air at 25 C IS $55.5 \mathrm{MJ} / \mathrm{kg}$ .find the heat of reaction at
constant pressure of a stoichiometeric mixture of methane and air if the reactant and products are at 500 k .
since the water vapour dose not $\mathrm{Np}=10.52$,hsp=6.33 MJ/kgmole , $\mathrm{Mp}=27.6$ $\mathrm{kg} / \mathrm{kgmole}, \mathrm{hsp}=229 \mathrm{KJ} / \mathrm{kg}$ condense , the lower heating value is used $=-2747 \mathrm{KJ} / \mathrm{kg}$ fuel reactants

The reaction is :- $\mathrm{CH} 4+2(\mathrm{O} 2+3.76 \mathrm{~N} 2) \rightarrow \mathrm{CO} 2+2 \mathrm{H} 2 \mathrm{O}+7.52 \mathrm{~N} 2$
Analysis of reactants and products using sensible enthalpies from appendix yield

| Reactants | Xi | his(MJ/kgmole) | Mj | Yi(kg) |
| ---: | ---: | ---: | ---: | ---: |
| CH4 | 0.095 | 8.2 | 16 | $\mathbf{0 . 0 5 5}$ |
| O2 | 0.19 | 6.09 | 32 | 0.22 |
| N2 | 0.715 | 5.91 | 28 | $\mathbf{0 . 7 2 5}$ |

$\mathrm{Nr}=1+2(4.76)=10.52$
$\mathrm{Mr}=\sum \mathrm{XiMI}=27.6 \mathrm{~kg} \mathrm{~kg} \mathrm{~mole}$
hsr= XXihsi= 6.16 MJ/kgmole hsr=6.16/27.8 KJ/kg
the fuel to air mass ratio is $f=0.055 /(0.22+0.725)=0.0582$

| Products | Xi | His | Mj |
| :--- | :--- | :--- | :--- |
| CO2 | 0.095 | 8.31 | 44 |
| H2O | 0.19 | 6.92 | 18 |
| N2 | 0.715 | 5.91 | 28 |

$\mathrm{Qp}=-2747+(229-223)=-2741 \mathrm{KJ} / \mathrm{kg}$ reactants
The negative sign indicates that the heat flow out of the system.

