جامعة المستقبل

كلية الهندسة والتقنيات الهندسية

قسم هندسة تقنيات الاجهزة الطبية

Course: Digital Signal Processing

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Lecture 5 & 6

(Discrete Time Signals)

Third Year 2023-2024

1/ Overview

1 / A –Target population :-

For students of third class

Department of Medical Instrumentation Eng. Techniques

1 / B –Rationale :-

This unit introduces principles of the discrete time signals, types of signals and demonstrates some fundamental signals.

1 / C –Central Idea :-

The major topics discussed in this unit are included in the following outline.

- **Analog – to – digital and Digital – to Analog Conversion**.
- **Sampling of Analog Signals**
- **The sampling theorem**
- **Quantization of Continuous – Amplitude signal**

2/ Performance Objectives :-

After studying the 2nd modular unit , the student will be able to:-

Investigates the sampling process, sampling theory, and the signal reconstruction process. It also includes practical considerations for anti-aliasing and anti-image filters and signal quantization.

3/ Pre test :-

Circle the correct answer :-

1. **Most signals of practical interest, such as biological signals are:**
   a- Continuous in nature.
   b- Discrete in nature.
   c- Random in nature.
2. **ADC is the abbreviation of:**
   a- Digital to Analog Converter.
   b- Analog to Digital Converter.
   c- Analog to Analog Converter.

d- Digital to Digital Converter.

3. DAC is the abbreviation of

a- Frequency Sampling.

b- Time sampling.

c- Digital to Analog Converter.

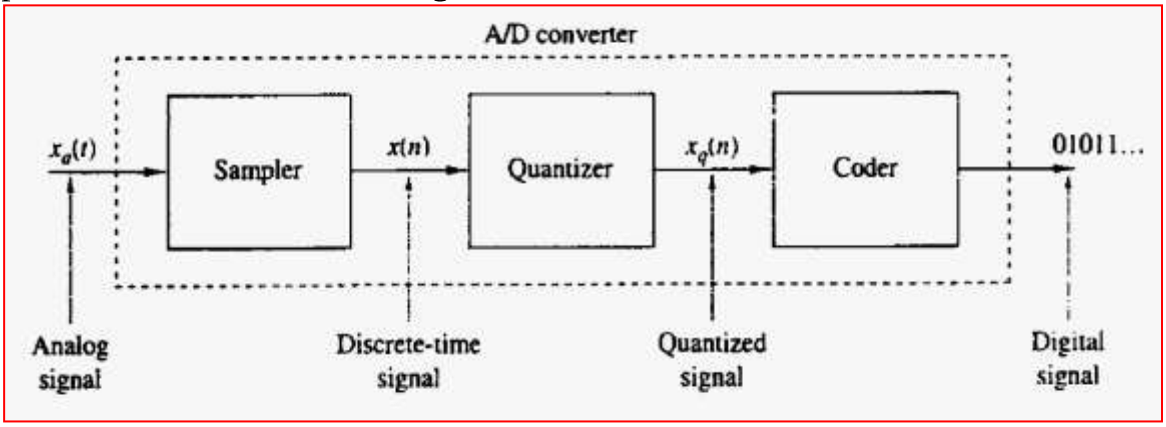
d- Impulse sampling.

4/ the text :-

Analog – to – digital and Digital – to Analog Conversion:

Most signals of practical interest, such as biological signals are analog. To process analog signals by digital means, it is first necessary to convert them in digital form. That is, to convert them to a sequence of numbers having finite precision. This procedure is called analog – to – digital (A/D) conversion.

Conceptually, we view A/D conversion as a three – step process. This process is illustrated in the figure below

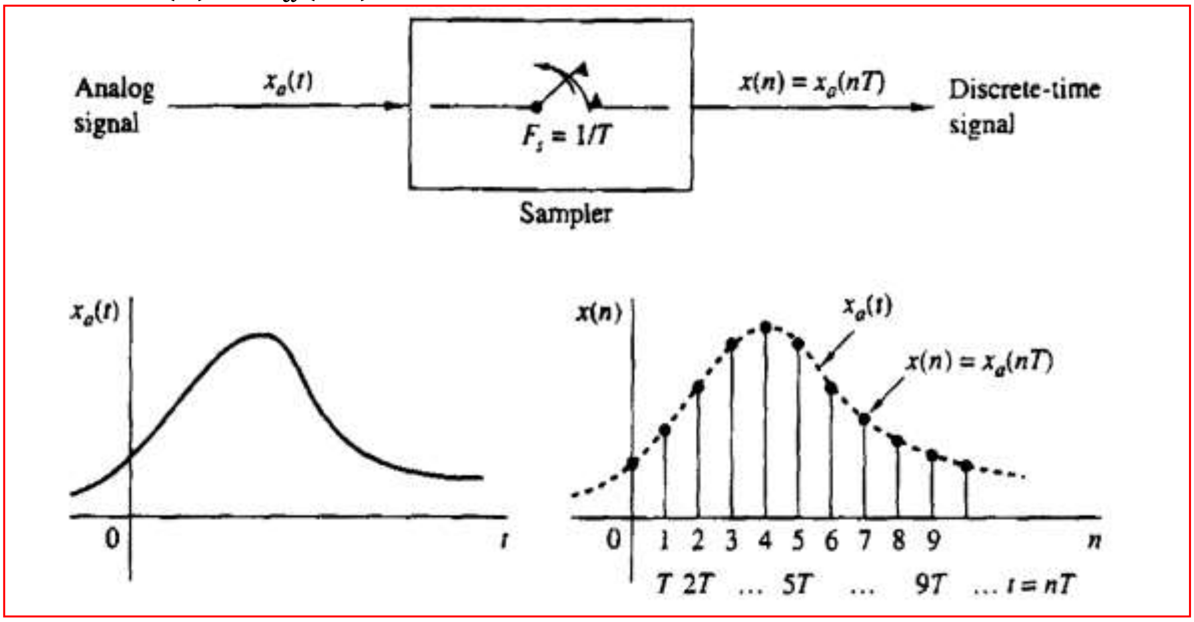


1. Sampling. This is the conversion of continuous – time signal into a discrete – time signal obtained by taking "samples" of the continuous – time signal at discrete – time instants. Thus, if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) \equiv x(n)$, where T is called the sampling interval.
2. Quantization. This is the conversion of a discrete – time continuous – valued signal into a discrete – time, discrete – valued (digital) signal. The value of each signal sample is represented by a value selected from a finite set of possible values. The difference between the unquantized sample x(n) and the quantized output $x_q(n)$ is called the quantization error.
3. Coding. In the coding process, each discrete value $x_q(n)$ is represented by a b – bit binary sequence.

Sampling of Analog Signals.

There are many ways to sample an analog signal. We limit our discussion to periodic or uniform sampling, which is the type of sampling used most often in practice. This is described by the relation:

$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$



The time interval T between successive samples is called the sampling period or sample interval and its reciprocal 1/T = F_s is called the sampling rate or sampling frequency.

The variables t and n are linearly related through the sampling period T or, equivalently, through the sampling rate F_s = 1/T, as

$$t = nT = \frac{n}{F_s}$$

As a consequence of above equation, there exists a relationship between the frequency variable F (or Ω) for analog signals and the frequency variable f (or ω) for discrete – time signals. To establish this relationship, consider an analog sinusoidal signal of the form:

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi FnT + \theta)$$

$$= A\cos\left(\frac{2\pi nF}{F_s} + \theta\right)$$

The frequency variables F and f are linearly related as:

$$f = \frac{F}{F_s}$$

Or, equivalently, as $\omega = \Omega T$

The frequency variable f is relative or normalized frequency. We can use f to determine the frequency F in hertz only if the sampling frequency F_s is known.

The relations are summarized in following table

Continuous – time signals		Discrete – time signals	
$\Omega = 2\pi F$		$\omega = 2\pi f$	
$\frac{radians}{sec}$	Hz	$\frac{radians}{sample}$	$\frac{cycles}{sample}$

$\omega = \Omega T,\ f = F/F_s$ →

← $\Omega = \omega/T,\ F = f.F_s$

$-\infty < \Omega < \infty$	$-\pi/T \le \omega \le \pi/T$
$-\infty < F < \infty$	$-F_s/2 \le f \le F_s/2$

From these relations we observe that

$$f_{max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\omega_{max} = \pi F_s = \frac{\pi}{T}$$

The sampling theorem

If the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$ and the signal is sampled at a rate $F_s > 2F_{max} \equiv 2B$, then $x_a(t)$ can be exactly recovered from its sample values using the interpolation function:

$$g(t) = \frac{sin2\pi Bt}{2\pi Bt}$$

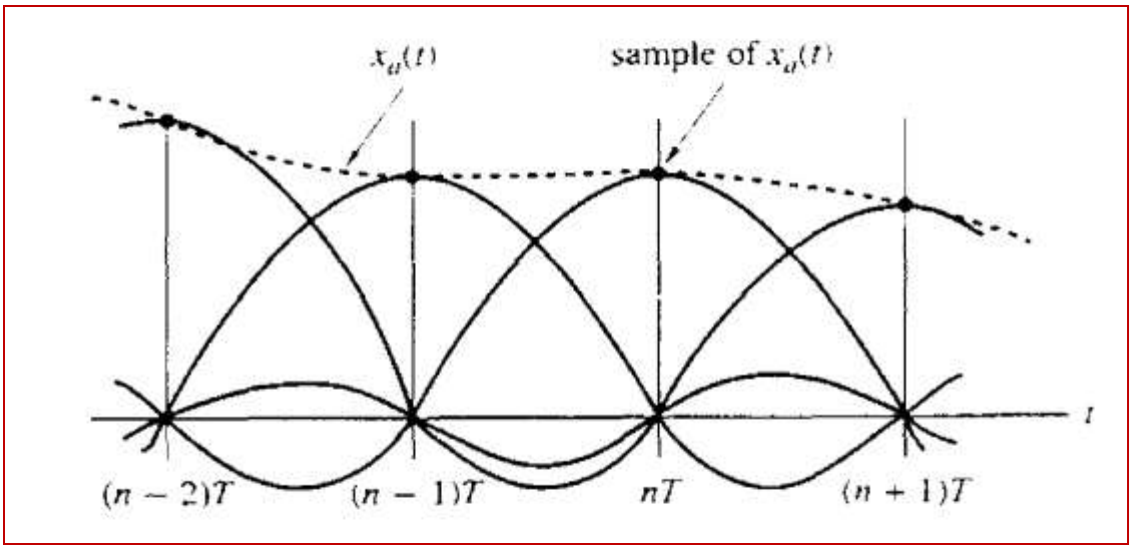
Thus $x_a(t)$ may be expressed as

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{n}{F_s}\right)$$

Where $x_a(n/F_s) = x_a(nT) \equiv x(n)$ are the samples of $x_a(t)$.
When the sampling of $x_a(t)$ is performed at the minimum sampling rate $F_s = 2B$, the reconstruction formula becomes

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{2B}\right) \frac{sin2\pi B(t - \frac{n}{2B})}{2\pi B(t - \frac{n}{2B})}$$

And the sampling rate $F_N = 2B$ is called Nyquist rate.
Figure below illustrate the ideal D/A conversion process using the interpolation function.



Example

Consider the analog signal

$$x_a(t) = 3cos50\pi t + 10sin300\pi t - cos100\pi t$$

What is the Nyquist rate for this signal?

Solution

The frequencies present in the signal above are

F_1 = 25 Hz $\quad$ F_2 = 150 Hz $\quad$ F_3 = 50 Hz

Thus F_{max} = 150 Hz and $F_s > 2F_{max}$ = 300 Hz = F_N

Example

Consider the analog signal

$$x_a = 3cos2000\pi t + 5sin6000\pi t + 10cos12000\pi t$$

a) What is the Nyquist rate for this signal?
b) Assume now that we sample this signal using a sampling rate F_s = 5000 samples/sec. what is the discrete – time signal obtained after sampling?
c) What is the analog signal y_a (t) we can reconstruct from the samples if we use ideal interpolation?

Solution

a) The frequencies existing in the analog signal are

   F_1 = 1KHz, $\quad$ F_2 = 3 KHz, $\quad$ F_3 = 6 KHz

   Thus F_{max} = 6 KHz, and according to the sampling theorem

   $F_s > 2\ F_{max}$ = 12 KHz and the Nyquist rate is F_N = 12 KHz

b) Since we have chosen F_s = 5 KHz, the folding frequency is

$$\frac{F_s}{2} = 2.5\ KHz$$

   And this is the maximum frequency that can be represented uniquely by the sampled signal. We obtain

$$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(\frac{3}{5}\right)n + 10cos2\pi\left(\frac{6}{5}\right)n$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(1-\frac{2}{5}\right)n$$
$$+ 10cos2\pi\left(1+\frac{1}{5}\right)n$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(-\frac{2}{5}\right)n + 10cos2\pi\left(\frac{1}{5}\right)n$$

Finally, we obtain

$$x(n) = 13cos2\pi\left(\frac{1}{5}\right)n - 5sin2\pi\left(\frac{2}{5}\right)n$$

c) Since only the frequency components at 1 KHz and 2 KHz are present in the sampled signal, the analog signal we can recover is

$$y_a(t) = 13cos2000\pi t - 5sin4000\pi t$$

Which is obviously different from the original signal $x_a(t)$. This distortion of the original analog signal was caused by the aliasing effect, due to the low sampling rate used.

Quantization of Continuous – Amplitude signal

The process of converting a discrete – time continuous – amplitude signal into a digital signal by expressing each sample value as a finite (instead of an infinite) number of digits, is called quantization. The error introduced in representing the continuous – valued signal by a finite set of discrete value levels is called quantization error or quantization noise.

We denote the quantizer operation on the samples x(n) as Q[x(n)] and let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer. Hence

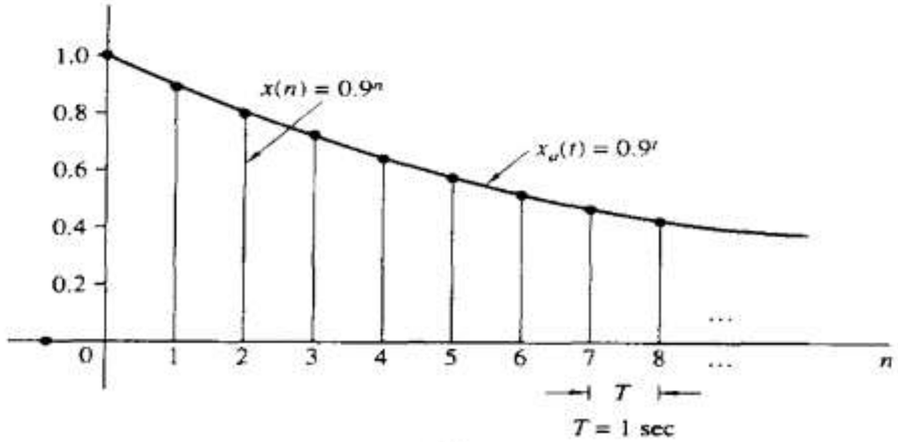
$$x_q(n) = Q[x(n)]$$

Then the quantization error is a sequence e_q (n) defined as the difference between the quantized value and the actual sample value. Thus

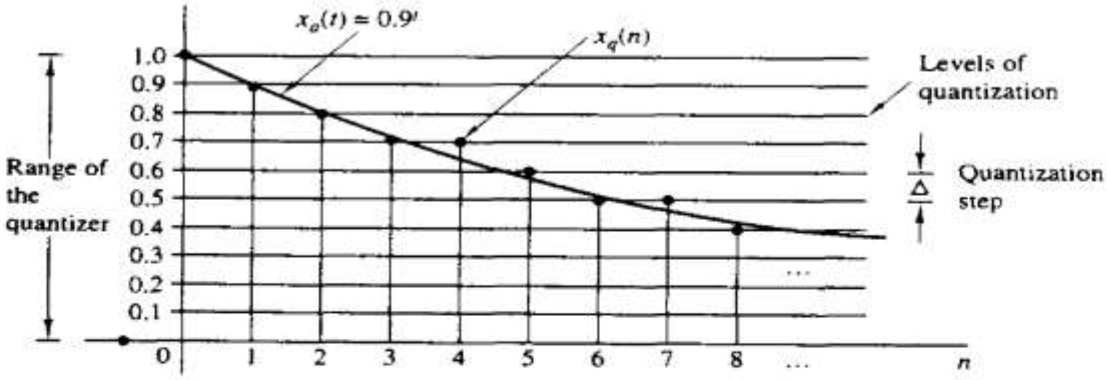
$$e_q(n) = x_q(n) - x(n)$$

Let us consider the discrete – time signal

$$x(n) = \begin{cases} 0.9^n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$



(a)



(b)

TABLE 1.2 NUMERICAL ILLUSTRATION OF QUANTIZATION WITH ONE SIGNIFICANT DIGIT USING TRUNCATION OR ROUNDING

n	$x(n)$ Discrete-time signal	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding)	$e_q(n) = x_q(n) - x(n)$ (Rounding)
0	1	1.0	1.0	0.0
1	0.9	0.9	0.9	0.0
2	0.81	0.8	0.8	−0.01
3	0.729	0.7	0.7	−0.029
4	0.6561	0.6	0.7	0.0439
5	0.59049	0.5	0.6	0.00951
6	0.531441	0.5	0.5	−0.031441
7	0.4782969	0.4	0.5	0.0217031
8	0.43046721	0.4	0.4	−0.03046721
9	0.387420489	0.3	0.4	0.012579511

5/ Post test :-

Circle the correct answer:-

1- Sampling Process is:

a- Is the process of converting continuous time signal to discrete time signal.

b- Is the process of converting continuous time signal to digital signal.

c- Is the process of converting discrete time signal to digital signal.

d- Is the process of converting digital signal to analog signal.

2- Quantization Process is:

a- Conversion of discrete time discrete valued signal to digital signal.

b- Conversion of discrete time continuous valued signal to discrete time discrete valued signal.

c- Conversion of discrete time continuous valued signal to continuous time discrete valued signal.

d- Conversion of digital signal to continuous time discrete valued signal.

3- Coding Process is:

a- Representing of continuous valued signal by binary numbers.

b- Representing of continuous valued signal by real numbers.

c- Representing of discrete valued signal by binary numbers.

6/ key answer :-

1- Pre test :-

1. a
2. b
3. c

2- Post test :-

1. a
2. b
3. c

7/References :-

1. Schaum's Outline of Theory and Problems of Digital Signal processing.
2. Digital signal processing, principles, algorithms, and applications by John G. Proakis and Dimitris G. Manolakis.
3. Signal and systems, Alan Oppenheim.