Convection Heat Transfer

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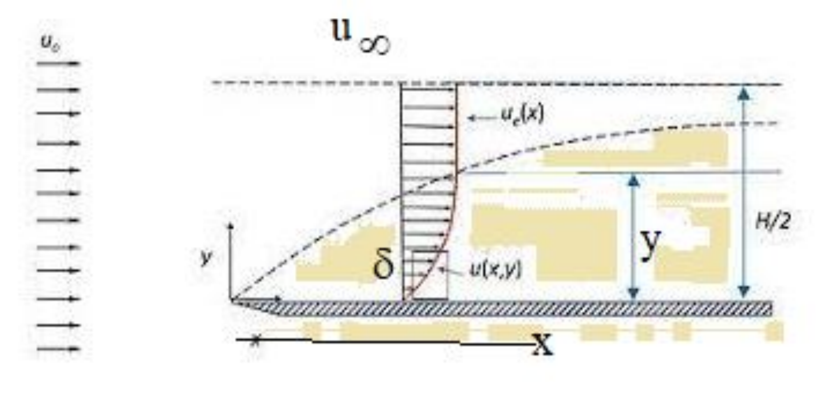
- **Viscous flows** occur when the effects of fluid viscosity are balanced by those arising from fluid inertia, body forces, and/or pressure gradients
- In a **viscous flow**, the forces associated with viscous shear stresses are large enough to effect the dynamic motion of the particles that comprise the flow. Indeed, both laminar and turbulent flows are types of viscous flows.
- In an **inviscid flow**, the forces associated with viscous shear stresses are small enough that they do not affect the dynamic motion of the particles that comprise the fl ow. Thus, in an inviscid flow, the viscous stresses can be neglected in the equations for motion.

LAMINAR VELOCITY IN BOUNDARY LAYER ON A FLAT PLATE

- The Flow of fluid over a flat plate is laminar if Re< $5x10^5$
- $Re = \frac{\rho u x}{\mu}$
- The velocity distribution through boundary layer is
- $\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$
- The boundary layer thicknes
  is
- $\frac{\delta}{x} = \frac{4.64}{Re_x^{1/2}}$
- And also from exact solutio
- $\frac{\delta}{x} = \frac{5.0}{Re_x^{1/2}}$

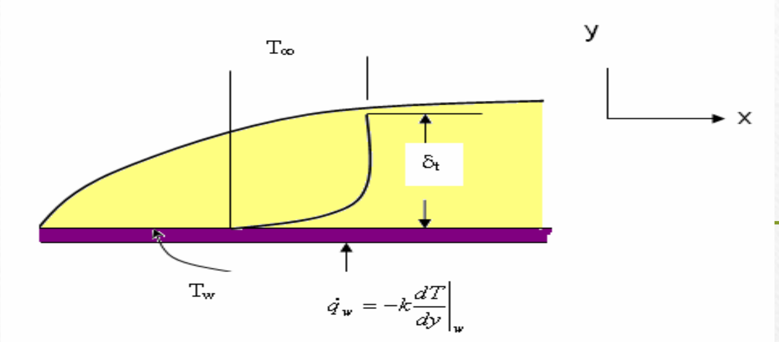


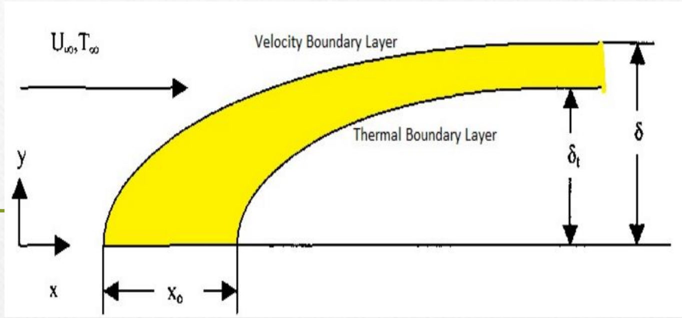
- **Example 1. Air at (27^{o}C) and (1.5atm) flows over a flat surface with a velocity (2m/s). Calculate the boundary layer thickness at a distance of (10cm) and (20cm) from the leading edge of the surface.**
- <u>**Solution**</u>: It is to calculate the thickness of boundary layer of air flowing on a flat surface with T=27^{o}C and P=1.5atm=1.5x1.0132x10^{5}Pa=1.52x10^{5}Pa and velocity, u=2m/s at the distances 10cm and 20cm.

- **Assumption**: Steady state condition. The air is an ideal gas
- **Properties**: Constant properties μ=1.85x10^{-5}kg/m.S, R=287J/kg.K
- **Analysis**: to calculate the thickness of the boundary layer, we need to calculate the Reynolds number. Then
- $Re_x = \frac{\rho u_\infty x}{\mu}$
- At the pressure not the standard at that at the table we can calculate the density for this value
- $\rho = \frac{P}{RT} = \frac{1.52\times10^5}{278\times(27+273)} = 1765 kg/m^3$

- At the distance x=10cm $Re = \frac{1.76 \times 2 \times 0.1}{1.85 \times 10^{-5}} = 0.191 \times 10^5$
- $\delta = \frac{4.64x}{Re_x^{0.5}} = \frac{4.64 \times 0.1}{(0.191 \times 10^5)^{0.5}} = 0.00336m = 3.36mm$
- At the distance x=20cm $Re = \frac{1.76 \times 2 \times 0.2}{1.85 \times 10^{-5}} = 0.382 \times 10^5$
- $\delta = \frac{4.64x}{Re_x^{0.5}} = \frac{4.64 \times 0.2}{(0.382 \times 10^5)^{0.5}} = 0.00476m = 4.76mm$
- **he Drag Coefficient (friction Coefficient)**
- $C_{Dx} = \frac{0.646}{\sqrt{Re_x}}$ the local Drag Coefficient
- $\bar{C}_{DL} = \frac{1.292}{\sqrt{Re_L}}$ The average Drag Coefficient
- The Drag Force $F_D = \bar{C}_{Dl} A \frac{\rho u_\infty^2}{2} = \bar{C}_{Dl} (LW) \frac{\rho u_\infty^2}{2}$

- <u>**Thermal Boundary Layer**</u>





- $\zeta = \frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-\frac{1}{3}}$ **and** Pr: Prandtl number= $\frac{\mu Cp}{k}$
- Heat transfer Coefficient for Laminar flow over flat plate.
- $Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$
- Nu: Nusselt Number $Nu = \frac{hL}{k}$
- The mean value of Nu is $\overline{Nu} = 2Nu_x$
- Then $\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3}$

- It is recommended that the properties to be evaluated at the **Film Temperature** (T_f). Film temperature is defined as the arithmetic mean of the wall and free-stream temperature $T_F = \frac{T_W + T_\infty}{2}$.
- **<u>Example</u>** **Atmospheric air at a temperature of (27°C) flows over a flat plate at a temperature of (77°C) by a velocity of (10m/S). Find the heat transfer per unit width from the plate to the air at the laminar region.**
- **<u>Solution</u>**: A flat plate is of temperature (T_w=77°C). Air at temperature (T_∞=27°C) is flow over the plat with velocity (u_∞=10m/S). Heat transfer between the plat and the air is to be determine at the laminar region per unit width.
- **<u>Assumption:</u>** steady state laminar one dimensional flow over a flat plate.

- <u>Property:</u> The properties of air is evaluated at the film temperature T_f where

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- **<u>Analysis:</u>** At the beginning, the critical length is to be determined
- $Re_c = 5 \times 10^5 = \frac{\rho u L_c}{\mu} = \frac{1.0877 \times 10 \times L_c}{1.961 \times 10^{-5}} \rightarrow L_c = 0.9m$
- Now we can calculate the average Nusselt number
- $\overline{Nu} = 0.664 Re_L^{0.5} Re^{1/3} = 0.664(5 \times 10^5)^{0.5}(0.7025)^{1/3}$
- $\overline{Nu} = 417.38$

- We can determine average coefficient of heat transfer.
- $\bar{h} = \frac{\overline{Nu}k}{L} = \frac{417.38 \times 0.02814}{0.9} = 13.05 W/m^{2o}C$
- The heat transfer is
- $\dot{Q} = \bar{h}A(T_w - T_\infty) = 13.05(0.9 \times 1)(77 - 27)$
  $= 587.25W$
- <u>Heat Transfer Coefficient and Drag Coefficient Relation For Laminar Flow Over Flat Plate</u>
- We can express a relation between the heat transfer coefficient and the drag coefficient for the flow over a flat plate in the laminar region as follows.
- $$\frac{Nu_x}{Re_x Pr} = \frac{\frac{h_x x}{k}}{Re_x Pr} = \frac{0.332 Re_x^{0.5} Pr^{1/3}}{Re_x Pr}$$

- $\frac{Nu_x}{Re_x Pr} = \frac{h_x}{\rho C p u_\infty} = \frac{0.332 Re_x^{0.5} Pr^{1/3}}{Re_x Pr}$ $\quad St_x = \frac{h_x}{\rho C p u_\infty}$
- $St_x = 0.332 Re_x^{-1/2} Pr^{-2/3}$ $\quad \frac{C_{Dx}}{2} = 0.332 Re_x^{-1/2}$
- $St_x Pr^{2/3} = 0.332 Re_x^{-1/2} = \frac{C_{Dx}}{2}$
- $\frac{h_x}{\rho C p u_\infty} Pr^{2/3} = \frac{C_{Dx}}{2}$
- This expression gives the relation between the coefficient of heat transfer and the drag coefficient.
- **Example** **Atmospheric air at temperature of (27°C) flows over a flat plate at temperature of (127°C). The plate is of width (2m) and (2.5m) length in the direction of flow. The average drag coefficient of flow of air over the plate is measured to be (0.0027). Find the drag force on the plate and the amount of heat transfer from the plat to the air.**

- **<u>Solution</u>**: Air at atmospheric pressure and temperature $T_\infty = 27^o C$. The plate at $T_w = 127^o C$. For plate W=2m and L=2.5m. The average drag coefficient is $\bar{C}_D = 0.0027$.
- The drag force and the heat transfer is to be calculated.

- **<u>Assumption:</u>** The flow is laminar steady state and one dimensional on a flat plate.
- **Property:** The properties of air must be evaluated at the Film temperature T_f
- $T_F = \frac{T_w + T_\infty}{2} = \frac{127+27}{2} = 77^o C = 350K$
- $\rho = 0.998 kg/m^3 \;\; Cp = 1009 J/kgK \;\; \mu = 20.72 \times \frac{10^{-6} kg}{m.s}$
- $k = 0.03003 W/mk$ Pr=0.697

- **Analysis:** To calculate the Drag Force we need to Determine the velocity (u_∞).

- $\bar{C}_{Dl} = \frac{1.323}{\sqrt{Re_L}} \rightarrow Re_L = \left(\frac{1.323}{\bar{C}_{DL}}\right)^2 = \left(\frac{1.323}{0.0027}\right)^2 = 2.401 \times 10^5$

- $Re_L = \frac{\rho u_\infty L}{\mu} \rightarrow 2.401 \times 10^5 = \frac{0.998 \times 2.5 u_\infty}{20.72 \times 10^{-6}} \rightarrow$
- $u_\infty = 2m/s$
- $F_D = \bar{C}_{DL}(WL)\frac{\rho u_\infty^2}{2} = 0.0027 \times (2.0 \times 2.5)\frac{0.998(2.0)^2}{2}$
- $F_D = 0.027N$
- $St_x Pr^{2/3} = \frac{C_{Dx}}{2} = \frac{\bar{C}_{DL}}{4} \rightarrow St_x = \frac{\bar{C}_{DL}}{4}(Pr)^{-2/3}$
$= \frac{00027}{4}(0.697)^{-2/3} = 0.00086$

- $St_X = \frac{h_x}{\rho Cpu_\infty} \rightarrow h_x = St_x \rho Cpu_\infty = 0.00086 \times 0.998(1009)$2=1.73W/m^2oC

$$\bar{h} = 2h_x = 2 \times 1.73 = \frac{3.46W}{m^{2o}C}$$

$$\dot{Q} = Ah(T_w - T_\infty) = 3.46(2 \times 2.5)(127 - 27) = 1730W$$

For flow across the tubes

$$\overline{Nu}_{cy} = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62Re^{1/2}Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

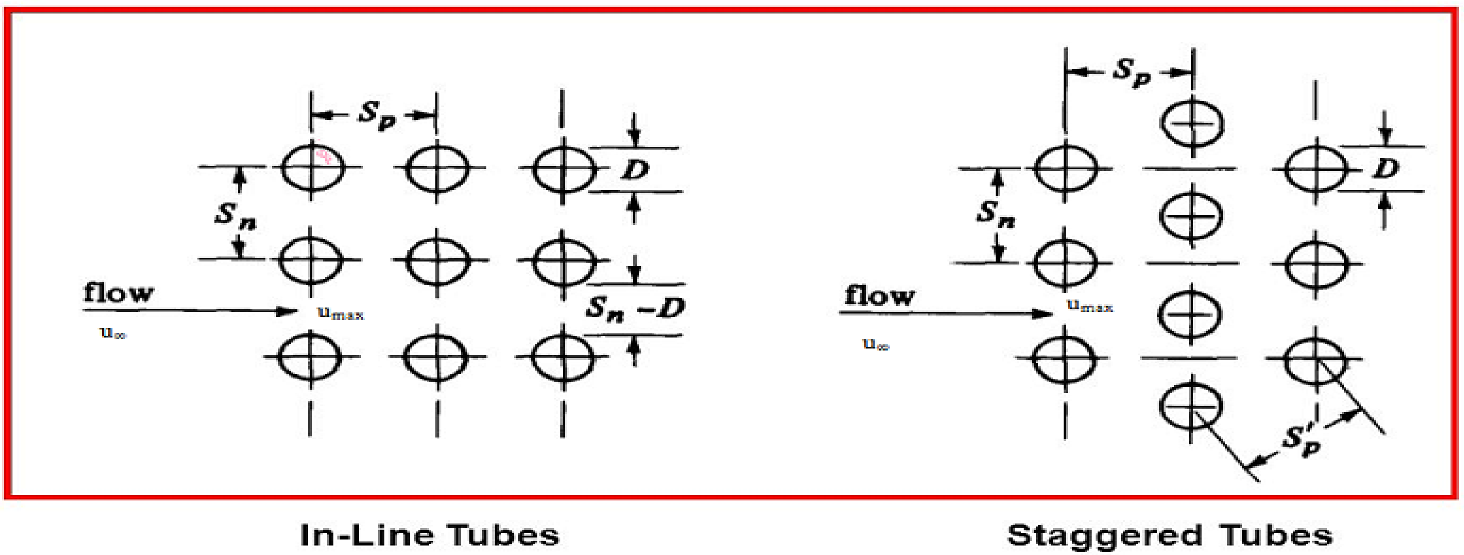
Where $Re = \frac{\rho uD}{\mu}$ **and** This equation can be used for $10^2 < Re < 10^7$ and $RePr > 0.2$. The properties is at T_F

- Or $\overline{Nu}_{cyc} = \frac{\bar{h}D}{k} = CRe_D^m Pr^{1/3}$
- The table below gives the value of C and m for different value of Re. Properties at T_F

Re_D	C	m
0.4-4.0	0.989	0.330
4-40.0	0.910	0.385
40-4000	0.683	0.466
4000-40000	0.193	0.618
40000-400000	0.0266	0.805

- **For flow in tubes or pipes.**
- For laminar flow where Re< 2300 where $Re = \frac{\rho u d}{\mu}$
- For the turbulent flow in tube
- $\overline{Nu} = 0.023Re^{0.8}Pr^{1/3}$ it is employed
- for 0.7$\leq Pr \leq 160$ and Re>10000

- **FLOW ACROSS BANKS OF TUBES**
- $\overline{Nu} = CRe^{m}Pr^{1/3}$ this relation is used to find Nu
- The tube may be arranged in in-line or staggered as in fig



S_p = Tube spacing parallel to flow.

S_n = Tube spacing normal to flow.

S'_p = Diagonal tube spacing for staggered rows.

- **Modified Correlation of Grimson for heat transfer in tube Banks of 10 Rows or more.**

	S_n/d							
Sp/d	1.25		1.5		2.0		3.0	
	C	M	C	M	C	m	C	M
In- Line								
1.25	0.386	0.592	0.305	0.608	0.111	0.704	0.0703	0.752
1.50	0.407	0.586	0.278	0.620	0.112	0.702	0.0753	0.744
2.00	0.464	0.570	0.332	0.602	0.254	0.632	0.220	0.480
3.00	0.322	0.601	0.396	0.584	0.415	0.581	0.317	0.608
Staggered								
0.6	-	-	-	-	-	-	0.236	0.636
0.9	-	-	-	-	0.495	0.571	0.445	0.581
1.0	-	-	0.552	0.558	-	-	-	-
1.125	-	-	-	-	0.531	0.565	0.575	0.560
1.25	0.575	0.556	0.561	0.554	0.576	0.556	0.579	0.562
1.5	0.501	0.568	0.511	0.562	0.502	0.568	0.542	0.568
2.0	0.448	0.572	0.462	0.568	0.535	0.556	0.498	0.570
3.0	0.344	0.592	0.395	0.580	0.488	0.562	0.467	0.574

- **Example: A water flow across a pipe with (20cm) diameter with a free stream temperature and velocity of (20°C) and (0.5m/S) respectively. The temperature of outer surface of the pipe is (60°C). Determine the heat transfer coefficient by using the different correlations that available.**

- **Solution**: The water at T_∞=20°C, and u_∞=0.5m/S, flows across a pipe with D=20cm=0.2m and temperature T_S=60°C.
- **Assumption**: The flow is across a pipe. The pipe is at constant temperature and the flow is of constant temperature. The heat transfer occurs by convection only
- **Properties**: The use of different correlations need to prepare all the data at free stream, surface and film temperature. And T_F=(20+60)/2=40°C. The Properties are
- ρ=994kg/m^3, $\mu = 0.654 \times \frac{10^{-3} kg}{m.s}$, k=0.628W/m.K, Pr=4.34

- **Analysis:** $Re_D = \frac{\rho u_\infty D}{\mu} = \frac{994 \times 0.5 \times 0.2}{0.654 \times 10^{-3}} = 151988$

- $\overline{Nu}_{cy} = 0.3 + \frac{0.62(151988)^{1/2}(4.34)^{1/3}}{\left[1+(0.4/4.34)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{151988}{282000}\right)^{5/8}\right]^{4/5}$
  $= 632.423$

$h = \frac{k \times \overline{Nu}}{D} = \frac{0.628 \times 632.423}{0.2} = 1986 \mathrm{W}/m^{2o}C$

and also we can find $\overline{Nu}_{cy}$ from $\overline{Nu}_{cyc} = CRe_D^m Pr^{1/3}$

Where C=0.0266 and m=0.805

$\overline{Nu}_{cyc} = CRe_D^m Pr^{1/3} = 0.0266(151988)^{0.805}(4.34)^{1/3} = 683$

$h = \frac{k \times \overline{Nu}}{D} = \frac{0.628 \times 683.0}{0.2} = 2146 \mathrm{W}/m^{2o}C$

- **<u>Example:</u> In a straight tube of (60mm) diameter, water is flowing at a velocity of (12m/S). The tube surface temperature is maintained at (70oC) and the flowing water is heated from the inlet temperature (15oC) to an outlet temperature of (45oC). Taking the physical properties of ~~water at its mean bulk temperature, determine the following:~~ (i) The coefficient of heat transfer (ii) The heat transfer rate to the water, (iii) The length of the tube.**
- **Solution:** A straight tube of diameter (D=0.06m) with velocity (u=12m/S). Surface Temperature is $T_s = 70^oC$. The temperatures at inlet and outlet are (T_1=15^oC) and (T_2=45^oC). It is to determine
- **The heat transfer coefficient, h**
- **Amount of heat transfer per time $\dot{Q}$**
- **The length of the tube. L**

- **Assumption:** The flow is fully developed and the flow is turbulent
- **Property:** The properties are evaluated at the mean bulk temperature. The properties of water at
- $T_b = \frac{T_1+T_2}{2} = \frac{15+45}{2} = 30^o C.$
- The properties of water at $30^o C$ are ρ=997.56kg/m^3, Cp=4180J/kg.K, μ=0.830 $\times 10^{-3} kg/msec$, k=0.6125W/$m.^o\ C$, Pr=5.68
- **Analysis:** the cross-sectional area is $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.06)^2 = 0.00283 m^2$.
- $\boldsymbol{\dot{m} = \rho A u = 997.56 \times 0.00283 \times 12 = 33.877 kg/sec}$

- $Re = \frac{\rho u D}{\mu} = \frac{997.56 \times 12 \times 0.06}{0.830 \times 10^{-3}} = 8.65 \times 10^{5}$
- $\overline{Nu} = 0.023 Re^{0.8} Pr^{1/3} = 0.023(8.65 \times 10^{5})^{0.8}(5.68)^{1/3}$
  $= 2306.4$
- $\bar{h} = \frac{Nu.k}{D} = \frac{2306.4 \times 0.6125}{0.06} = 23544 W/m^{2o}C$
- $\dot{Q} = \dot{m} Cp(T_2 - T_1) = 33.877 \times 4180(45 - 15)$
  $= 4.248 MW$
- $\dot{Q} = \pi D L h(T_s - T_m) = \pi(0.06) L(23544)(70 - 30)$
  $= 4.248 \times 10^{6} \rightarrow L = 24m$