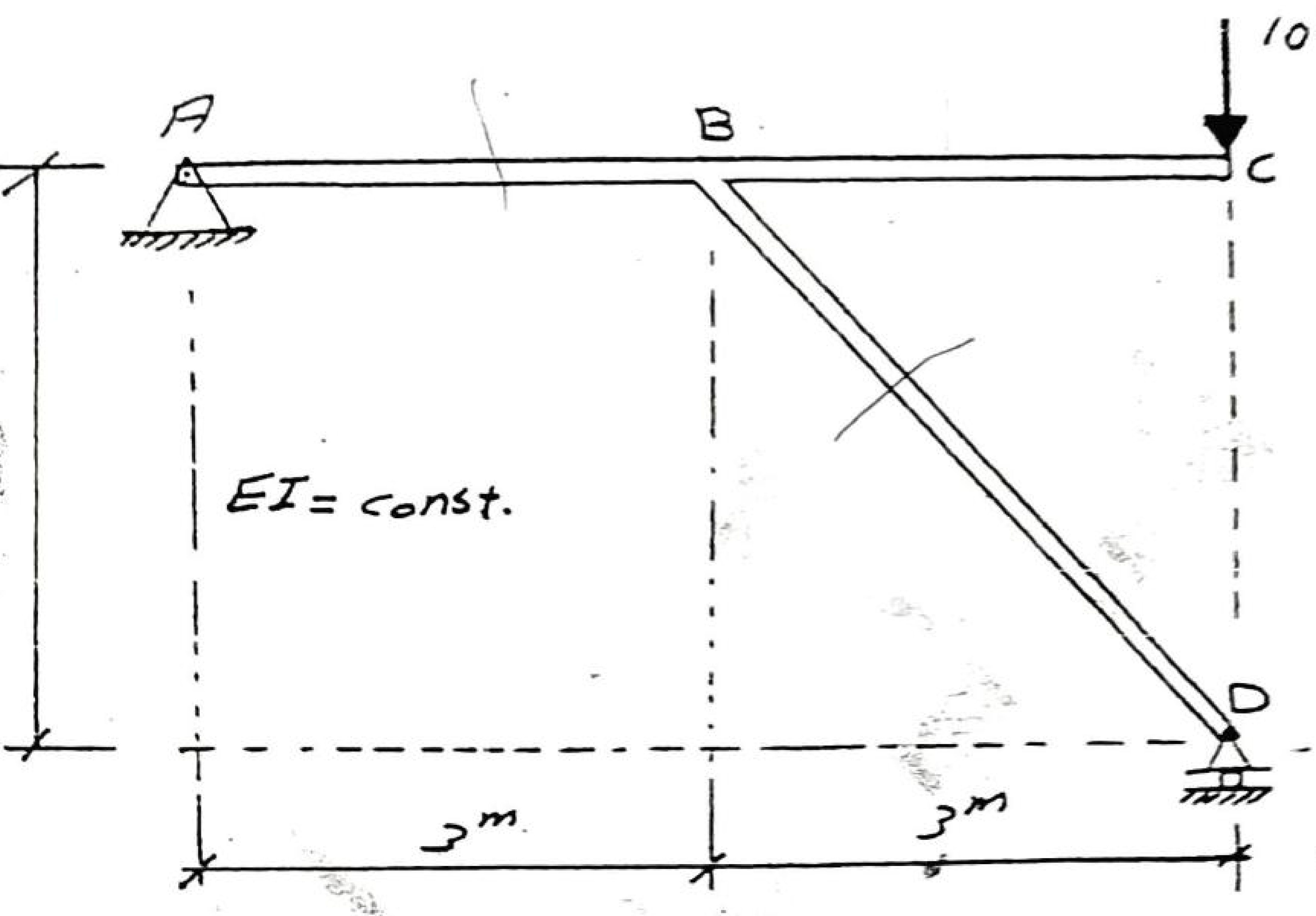
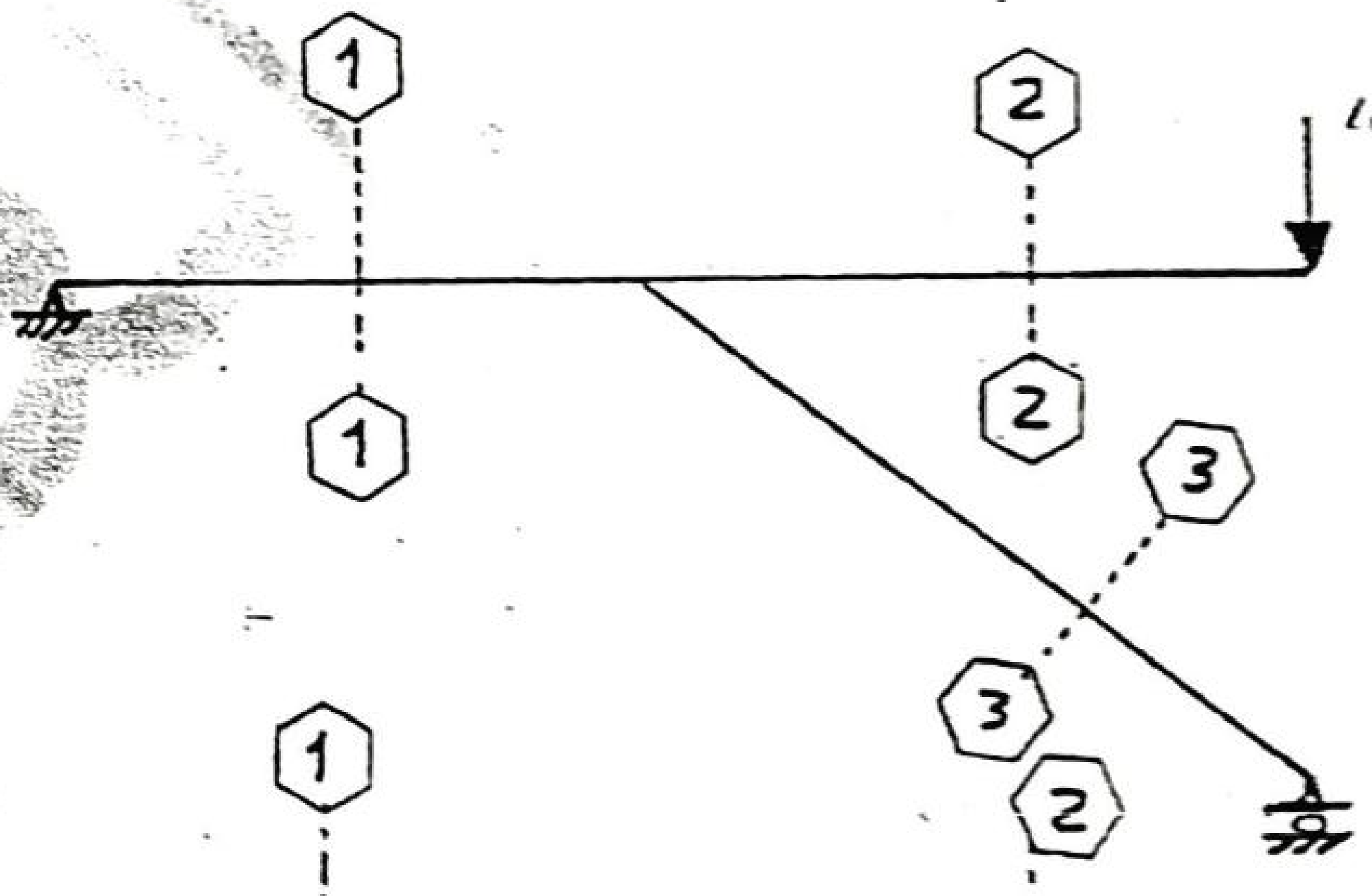
Ex:- For the frame shown in fig, compute the horizantal movement of point (D) use the method of virtual work.



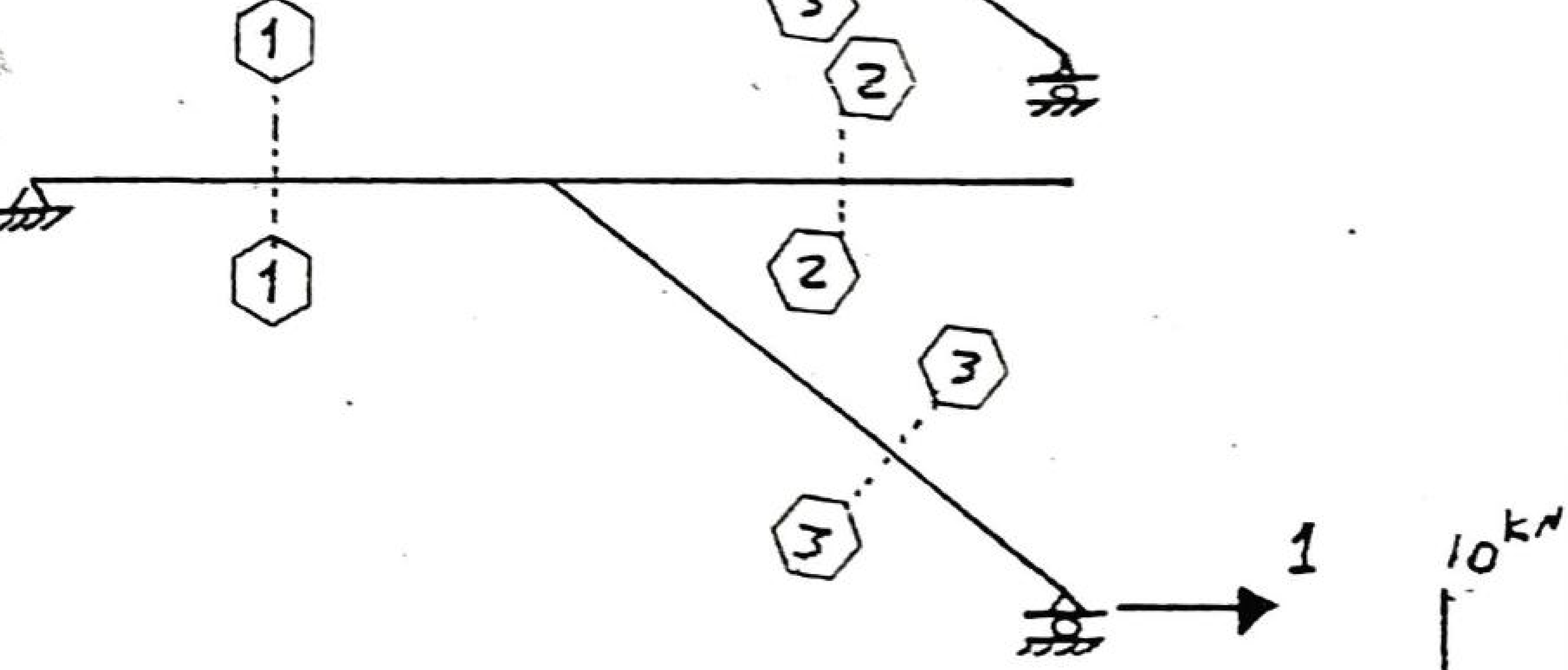
Sol.

* لغرض التأكد من عدد المقطوعات :-

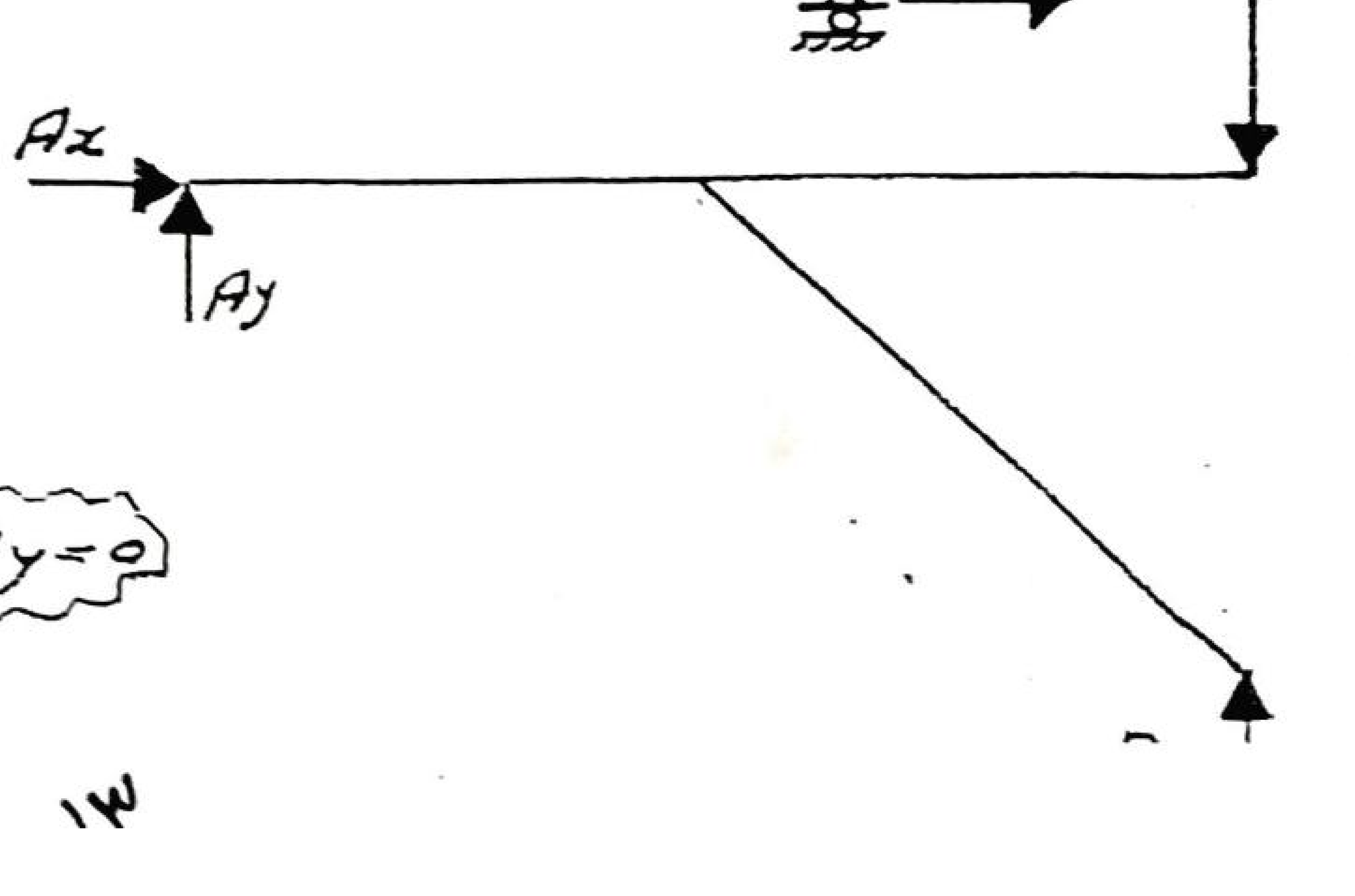
Find M ⇐



Find m ⇐



* To find M :-



$\sum M_A = 0$

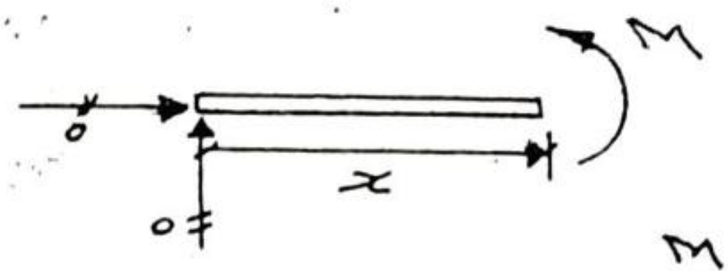
$10 * 6 - D_y * 6 = 0 \Rightarrow D_y = 10 \uparrow$

$\sum F_y = 0 \Rightarrow A_y + 10 - 10 = 0 \Rightarrow A_y = 0$

$\sum F_x = 0 \Rightarrow A_x = 0$

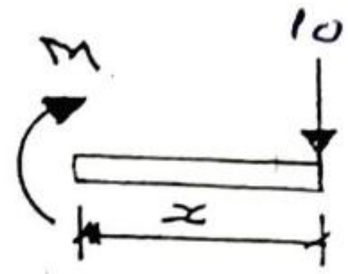
① For part AB :-

$M = 0$



② For part Bc :-

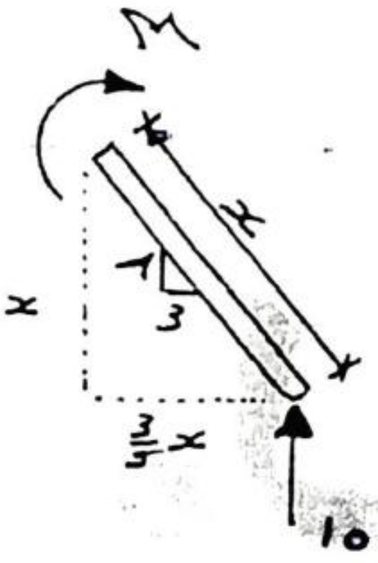
$M = -10x$



③ For part BD :-

$M = 10 * \frac{3}{5} x$

$M = 6x$



تشابه مثلثات

$\frac{4}{5} = \frac{y}{x}$ طول المقابل / طول المقابل

$y = \frac{4}{5}x$

$\frac{4}{5}x$

* To find m :-

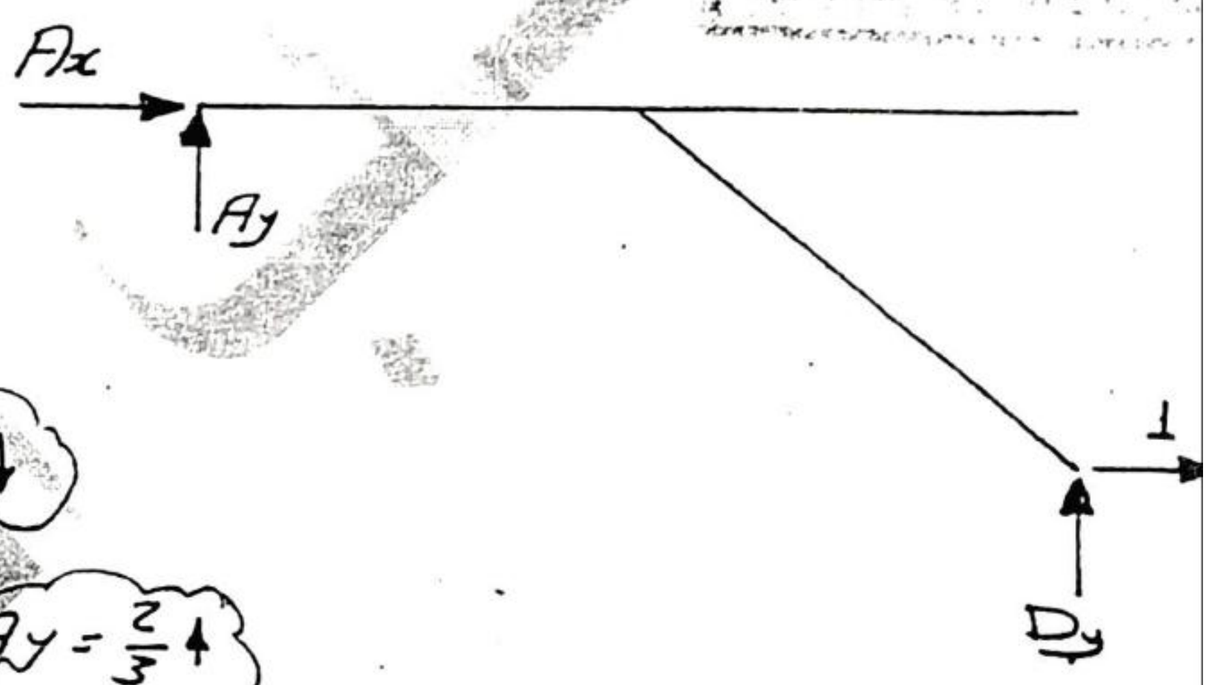
To find reaction :-

$\Sigma M_A = 0$

$D_y * 6 + 1 * 4 = 0 \Rightarrow D_y = \frac{2}{3} \downarrow$

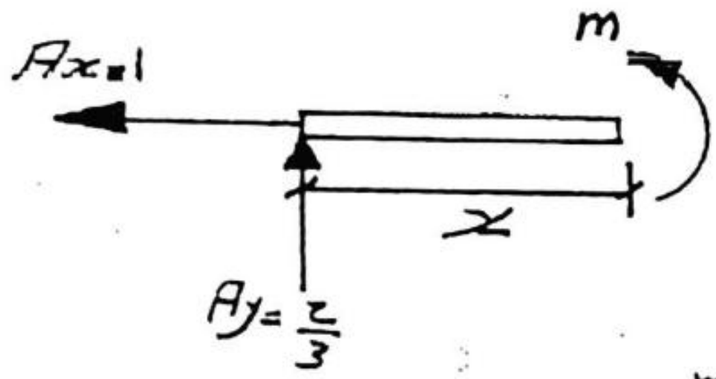
$\Sigma F_y = 0 \Rightarrow A_y - \frac{2}{3} = 0 \Rightarrow A_y = \frac{2}{3} \uparrow$

$\Sigma F_x = 0 \Rightarrow A_x + 1 = 0 \Rightarrow A_x = 1 \leftarrow$



① For part AB :-

$m = \frac{2}{3} x$



② For part Bc :-

$m = 0$

③ For part BD :-

$m = 1 * \frac{4}{5} x - \frac{2}{3} * (\frac{3}{5} x)$

$m = 0.4x$

member	origin	Limit	EI	M	m
AB	A	$0 \to 3$	1	0	$\frac{x}{3}x$
Bc	C	$0 \to 3$	1	$-1.0x$	0
BD	D	$0 \to 5$	1	$6x$	$0.4x$

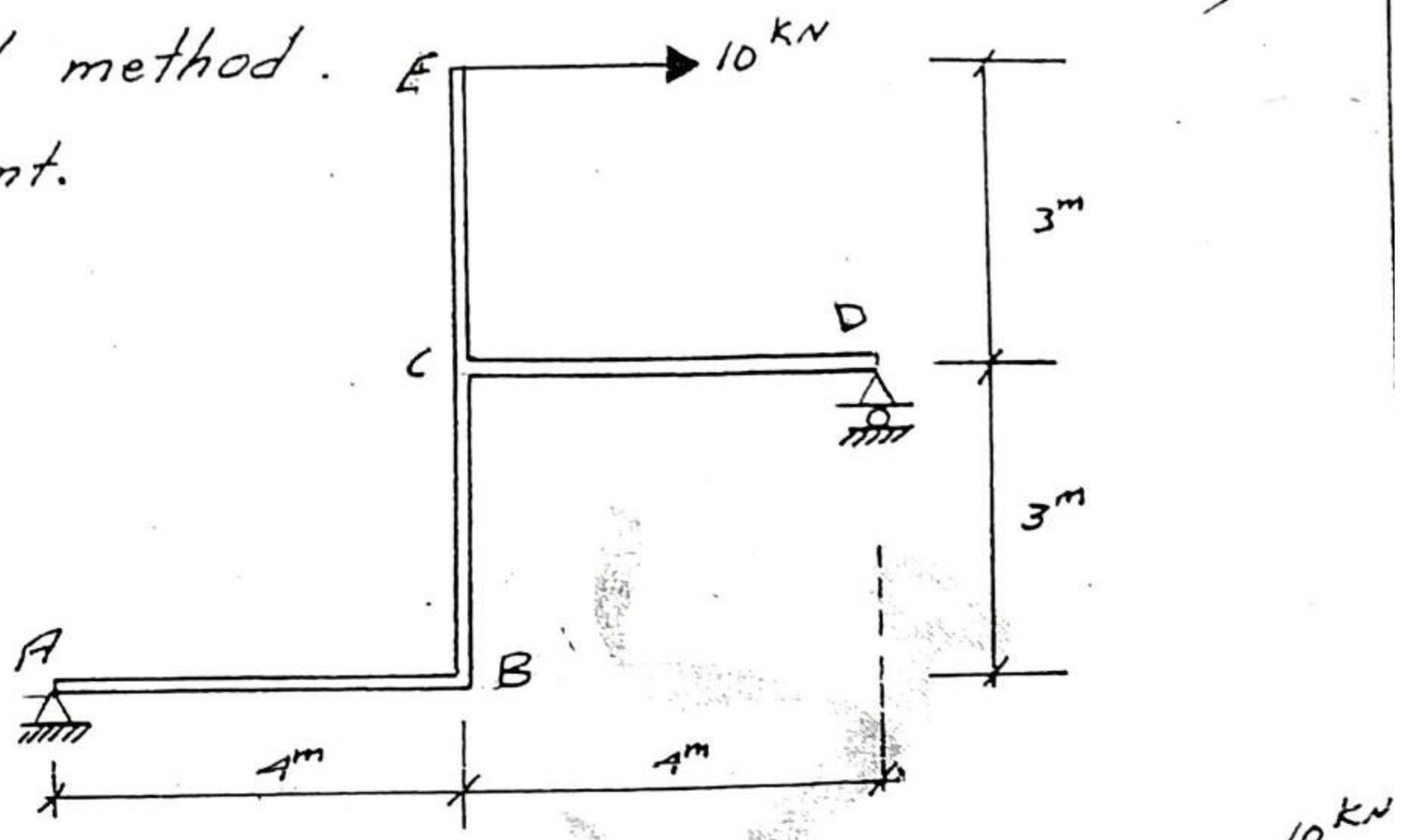
$$\therefore (\Delta H)_D = \Sigma \int \frac{Mm\,dx}{EI}$$

$$= 0 + 0 + \int_0^5 \frac{(6x)(0.4x)\,dx}{EI}$$

$$= \frac{1}{EI}\left[0.8x^3\right]_0^5 \Rightarrow \boxed{(\Delta H)_D = \frac{100}{EI} \rightarrow}$$

Ex:- Compute (ΔE) Horiz. of the structure shown using unit load method.

EI = Constant.



Sol.

* To find M:-

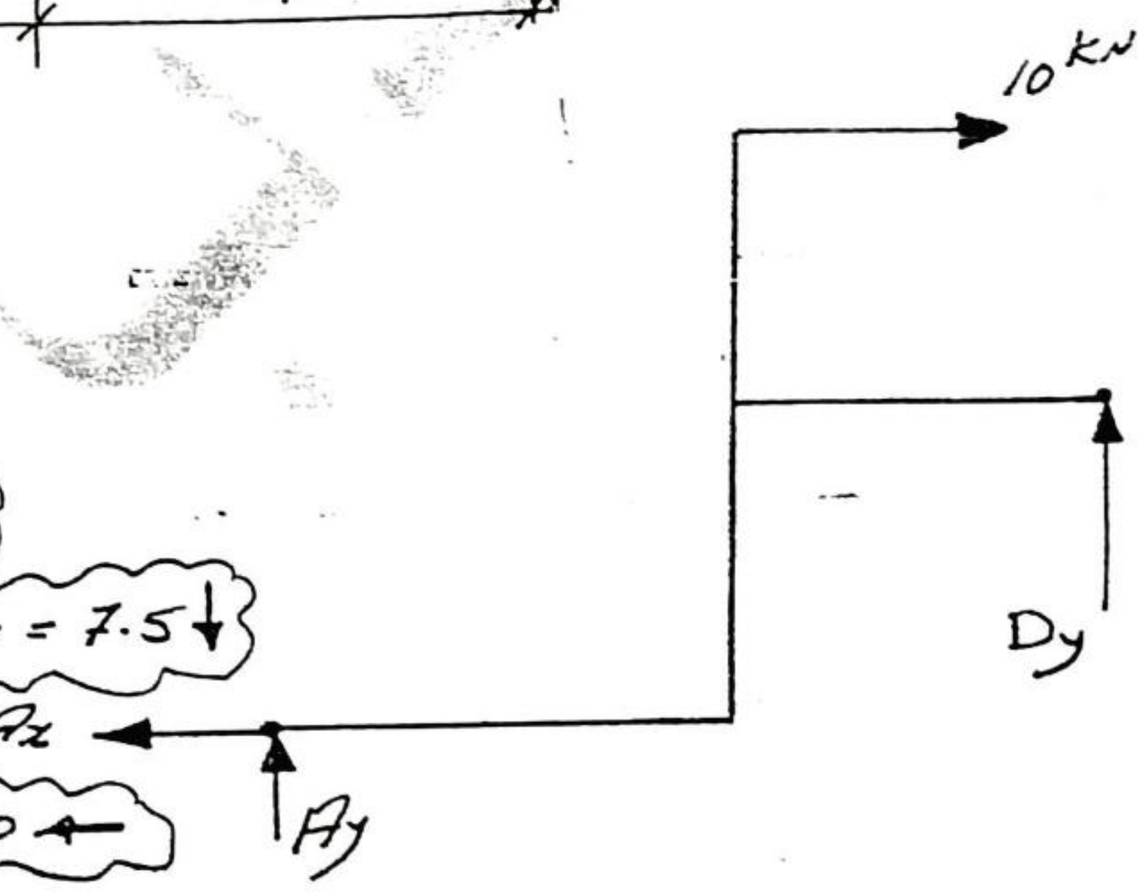
* To find reaction:-

$\Sigma M_A = 0$

$10 * 6 - D_y * 8 = 0 \Rightarrow D_y = 7.5 \uparrow$

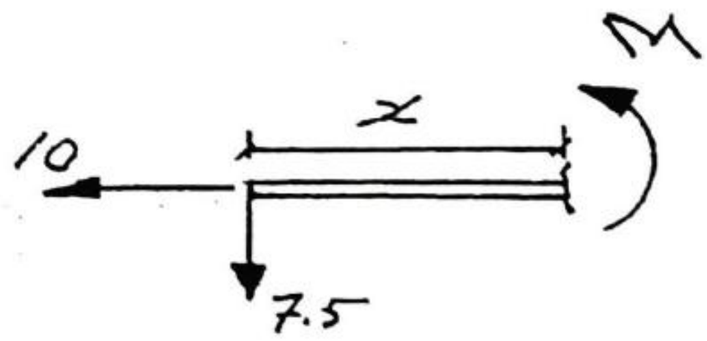
$\Sigma f_y = 0 \Rightarrow A_y + 7.5 = 0 \Rightarrow A_y = 7.5 \downarrow$

$\Sigma f_x = 0 \Rightarrow -A_x + 10 = 0 \Rightarrow A_x = 10 \leftarrow$



① - for part AB:-

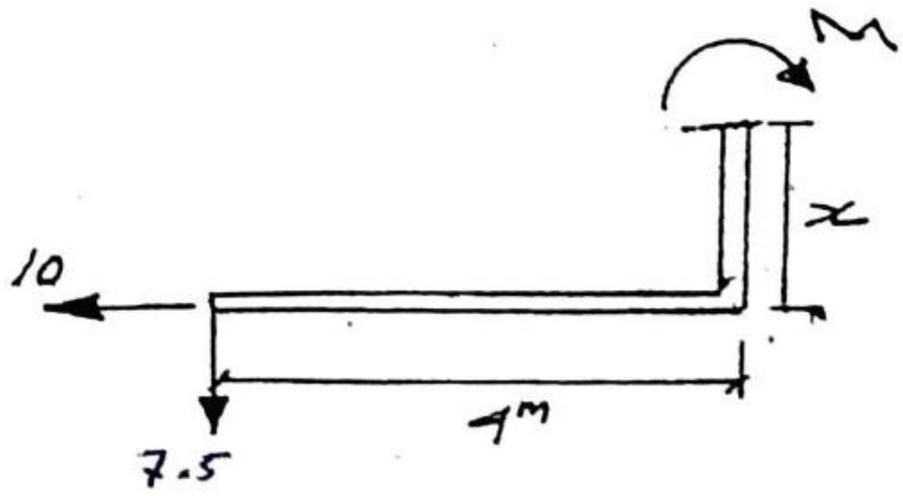
$M = -7.5X$



② - for part BC:-

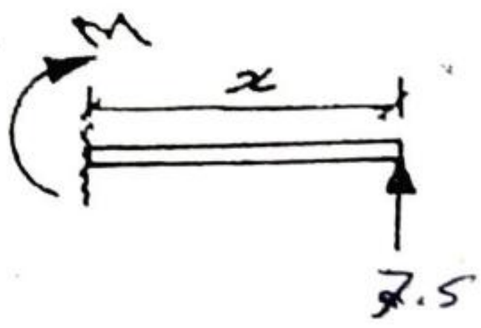
$M = 7.5 * 4 - 10(X)$

$M = 30 - 10X$



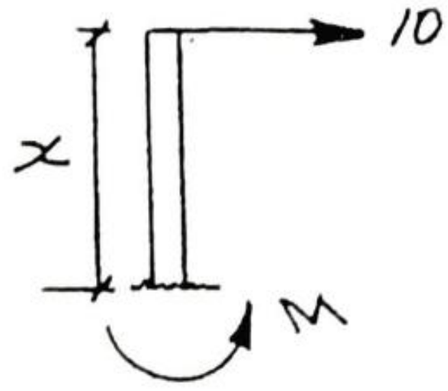
③ - for part CD:-

$M = 7.5X$



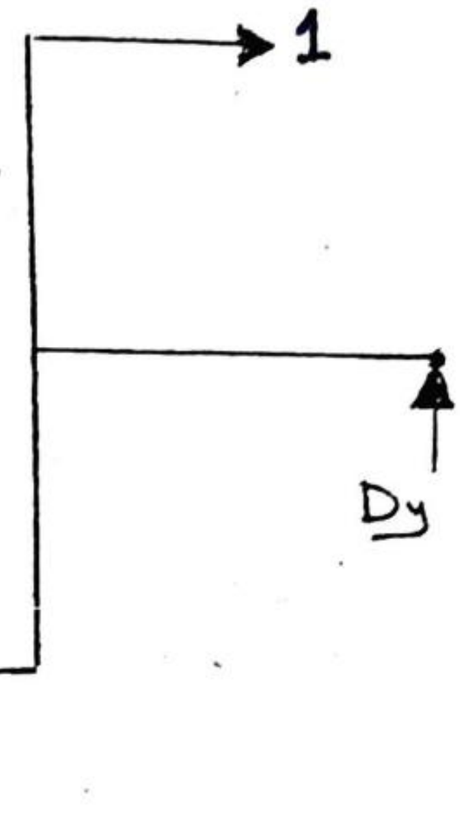
⊕- for part CE:-

$M = 10X$



* To find m:-

* بما أن هذا الشكل يشبه الشكل السابق ولكنه الفرق هو في القوة (10) لذلك يمكنه استخراج (m) بالطريقة السريعة وهي :-



$m = \frac{M}{10}$ ← (لكل جزء مع الإشارة)

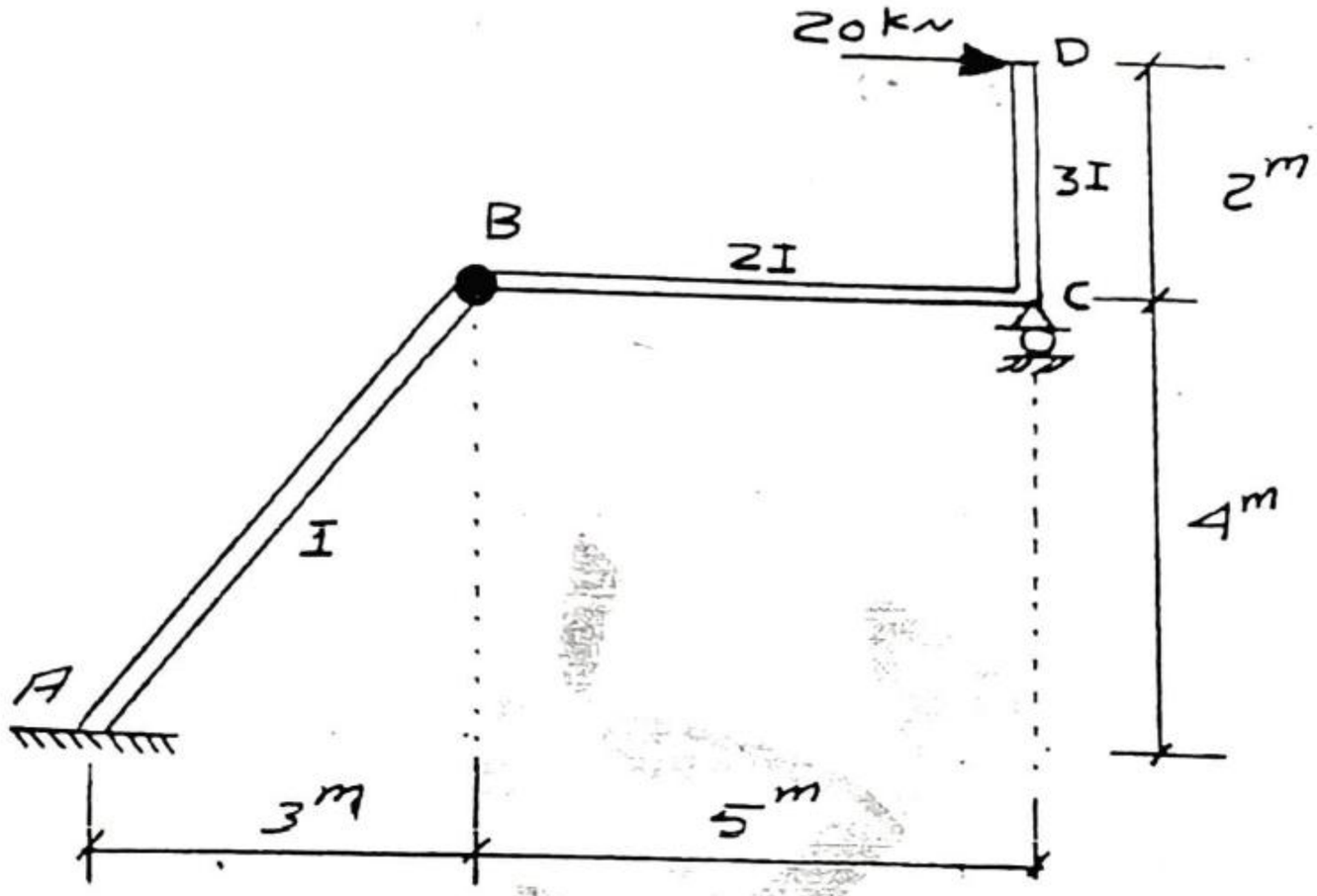
member	origin	Limit	EI	M	$m = \frac{M}{10}$
AB	A	$0 \to 4$	1	$-7.5X$	$-0.75X$
BC	B	$0 \to 3$	1	$30-10X$	$3-X$
CD	D	$0 \to 4$	1	$7.5X$	$0.75X$
CE	E	$0 \to 3$	1	$10X$	X

$$(\Delta E)_H = \int_0^4 \frac{(-7.5X)(-0.75X)dx}{EI} + \int_0^3 \frac{(30-10X)(3-X)dx}{EI}$$

$$+ \int_0^4 \frac{(7.5X)(0.75X)dx}{EI} + \int_0^3 \frac{(10X)(X)dx}{EI}$$

$= r$

Ex:- Find the vertical deflection at B of the structure shown in terms of (EI).

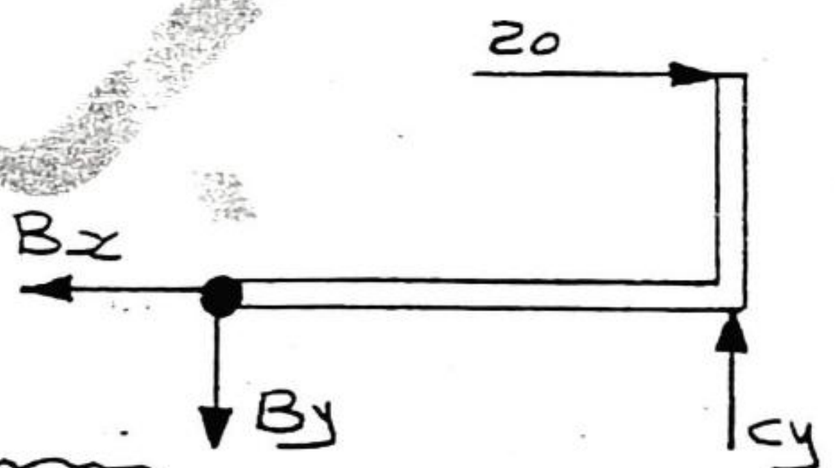


Sol:

* To find M :-
* To find reaction
* for part BCD :-

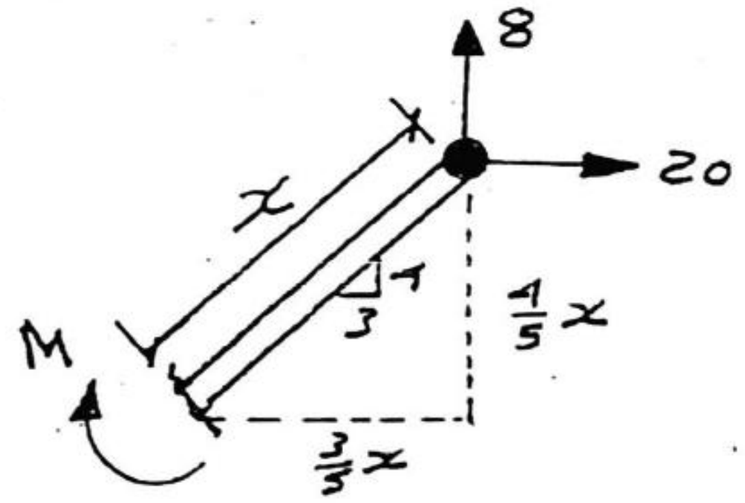
$\Sigma M_C = 0$ ↻

$20 * 2 - B_y * 5 = 0 \Rightarrow$ **$B_y = 8$ ↓**



$\Sigma F_x = 0 \Rightarrow -B_x + 20 = 0 \Rightarrow$ **$B_x = 20$ ←**

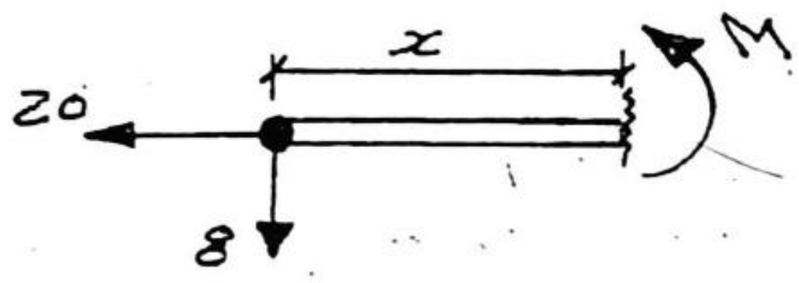
① - For part AB :-



$M + 8 * \frac{3}{5} X - 120 * \frac{4}{5} X$

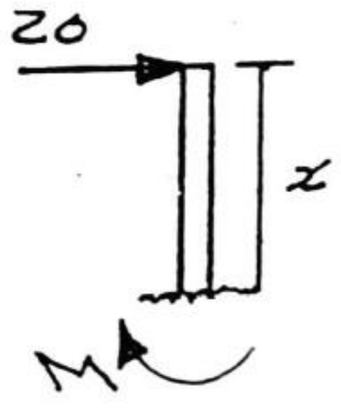
$M = -11.2 X$

② - For part BC :-



$M = -8X$

③ - For part CD :-

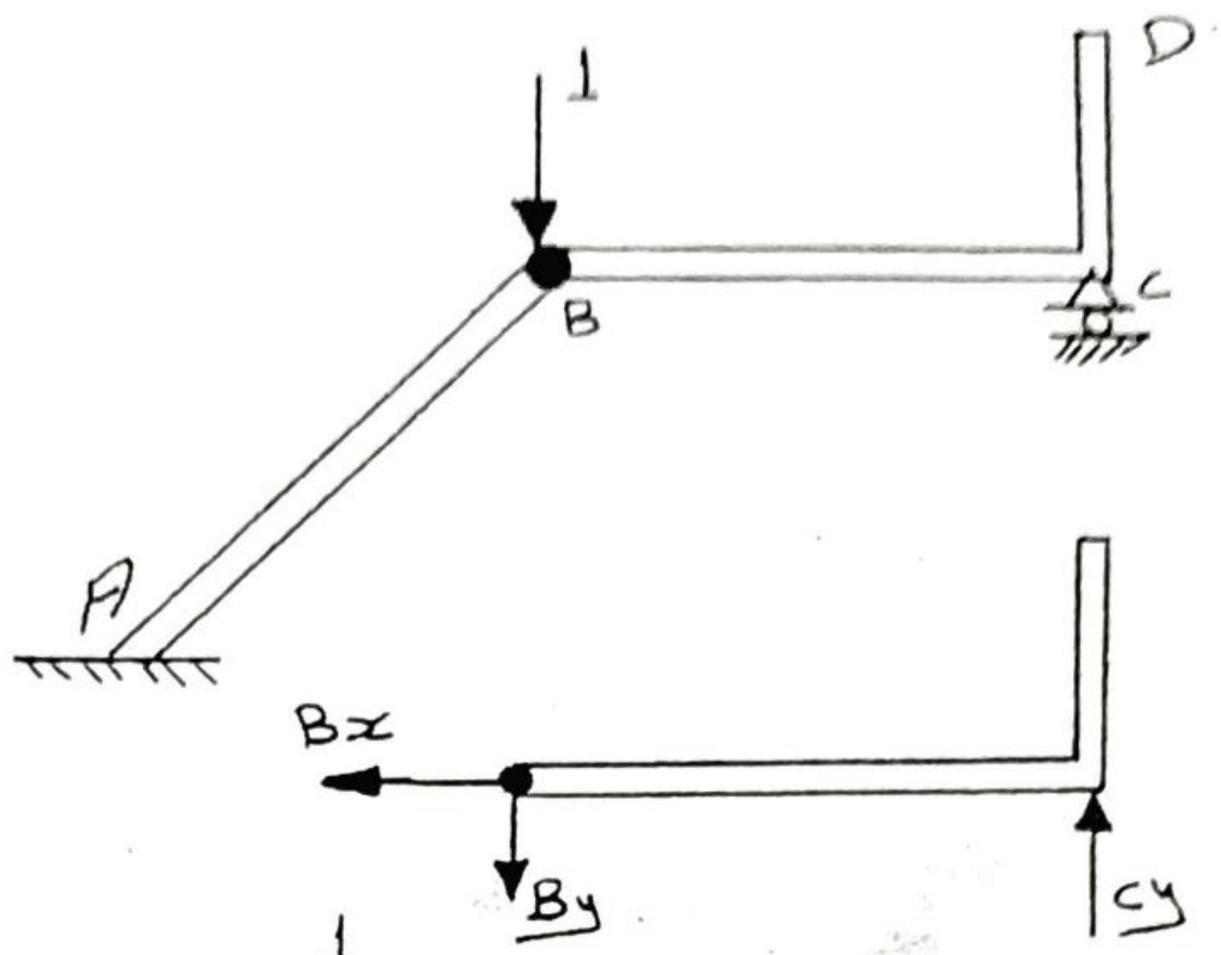


$M = -20X$

* To find m :-

* To find reaction :-

* for part BCD :-



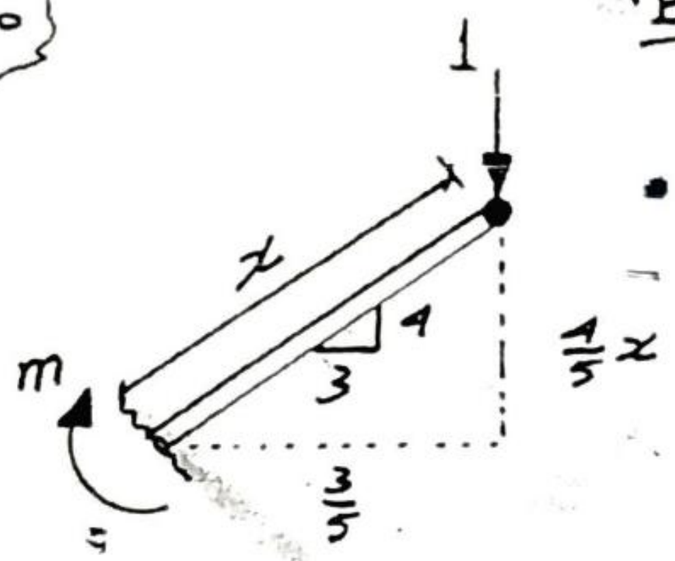
$\Sigma M_C = 0$

$B_y = 0$

$\Sigma F_x = 0 \Rightarrow B_x = 0$

$\Sigma F_y = 0 \Rightarrow C_y = 0$

① - for part AB :-

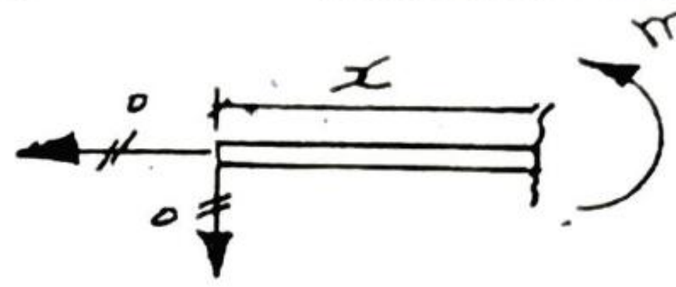


$m = -\frac{3}{5}x$

$m = -0.6x$

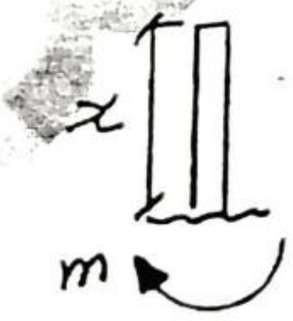
② - for part Bc :-

$m = 0$



③ - for part CD :-

$m = 0$



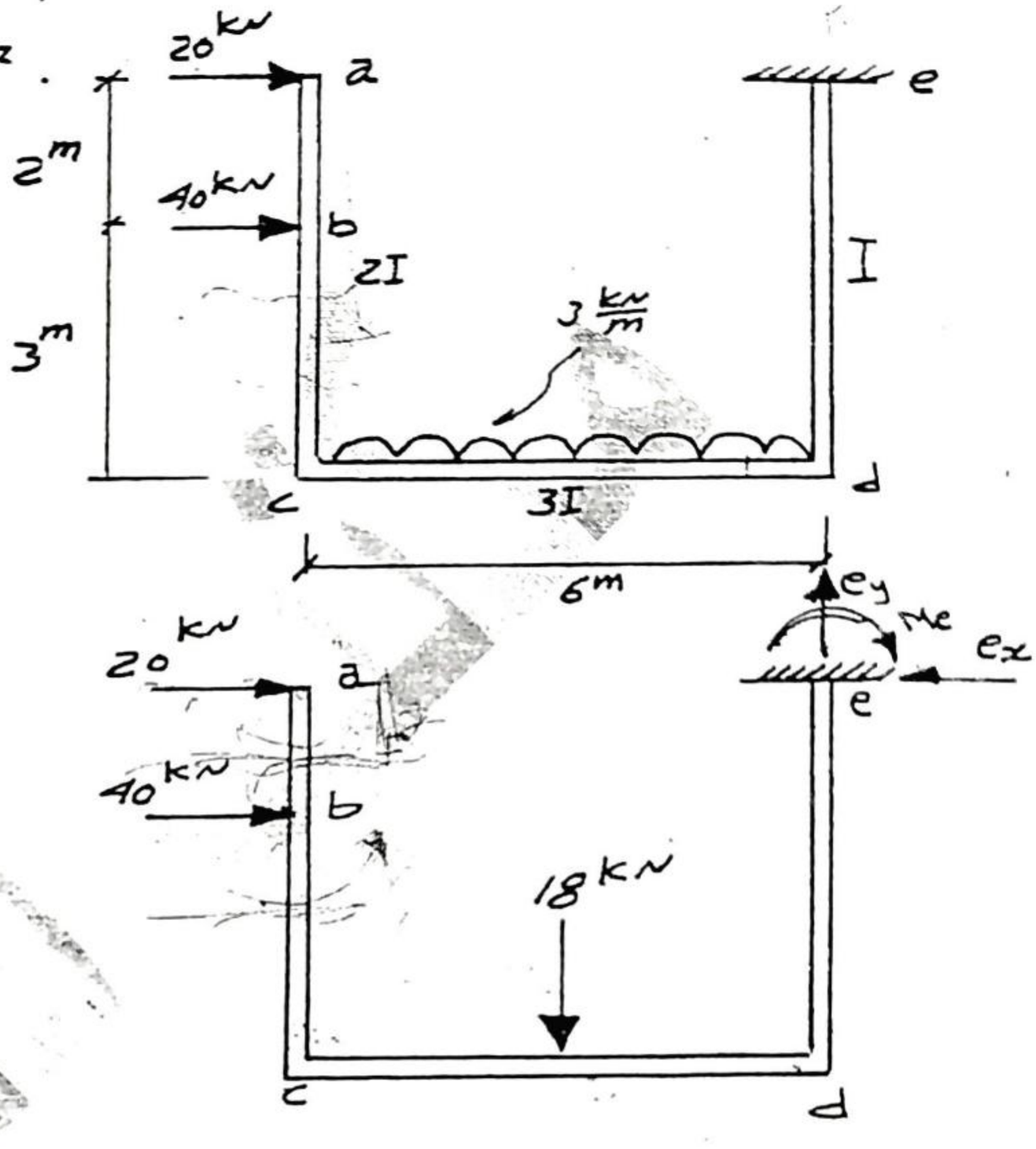
member	origin	Limit	EI	M	m
AB	B	$0 \to 5$	1	$-11.2x$	$-0.6x$
Bc	B	$0 \to 5$	2	$-8x$	0
CD	D	$0 \to 2$	3	$-20x$	0

$$\therefore (\Delta b)_V = \int_0^5 \frac{(-11.2x)(-0.6x)\,dx}{EI} + 0 + 0$$

$$= \frac{1}{EI}\left[2.24x^3\right]_0^5 \Rightarrow (\Delta b)_V = \frac{280}{EI} \downarrow$$

Ex:- Find the deflection and rotation in degree at point (a) ?

$EI = 1 * 10^4\ kN/m^2$.



Sol.

$\Sigma F_x = 0 \Rightarrow ex = 60$

$\Sigma F_y = 0 \Rightarrow ey = 18$

$\Sigma Me = 0$

$40 * 2 + 18 * 3 = Me$

$\rightarrow Mc = 134$

* To find M

part ab:-

$M = 20x$

part bc:-

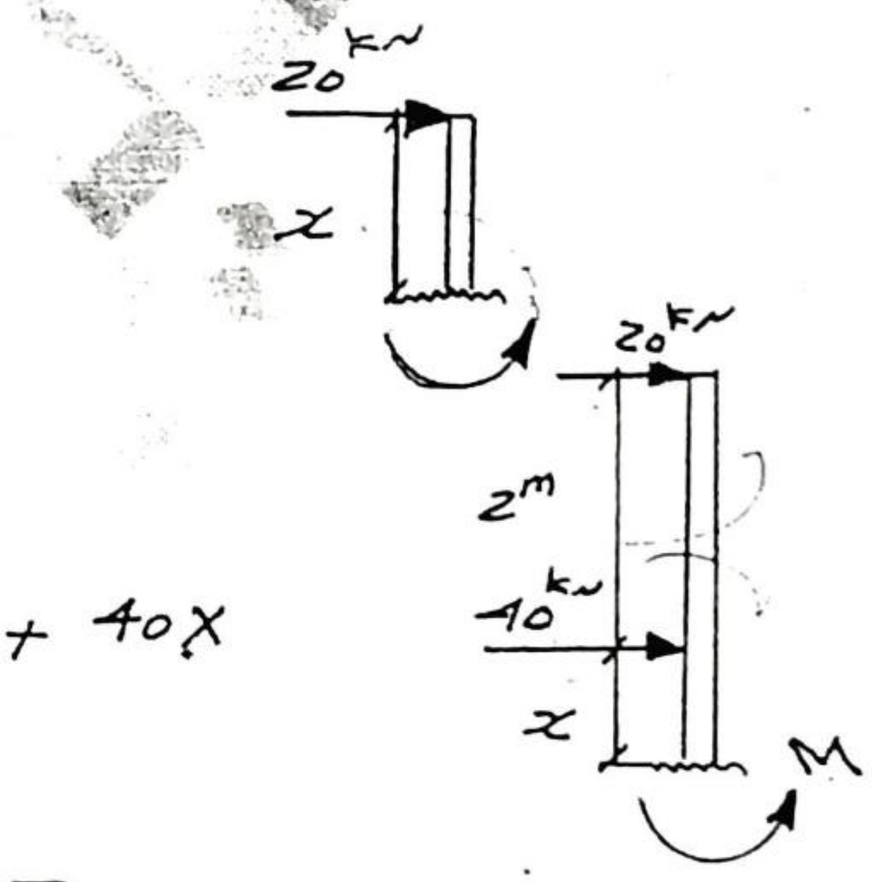
$M = 20(2 + x) + 40x$

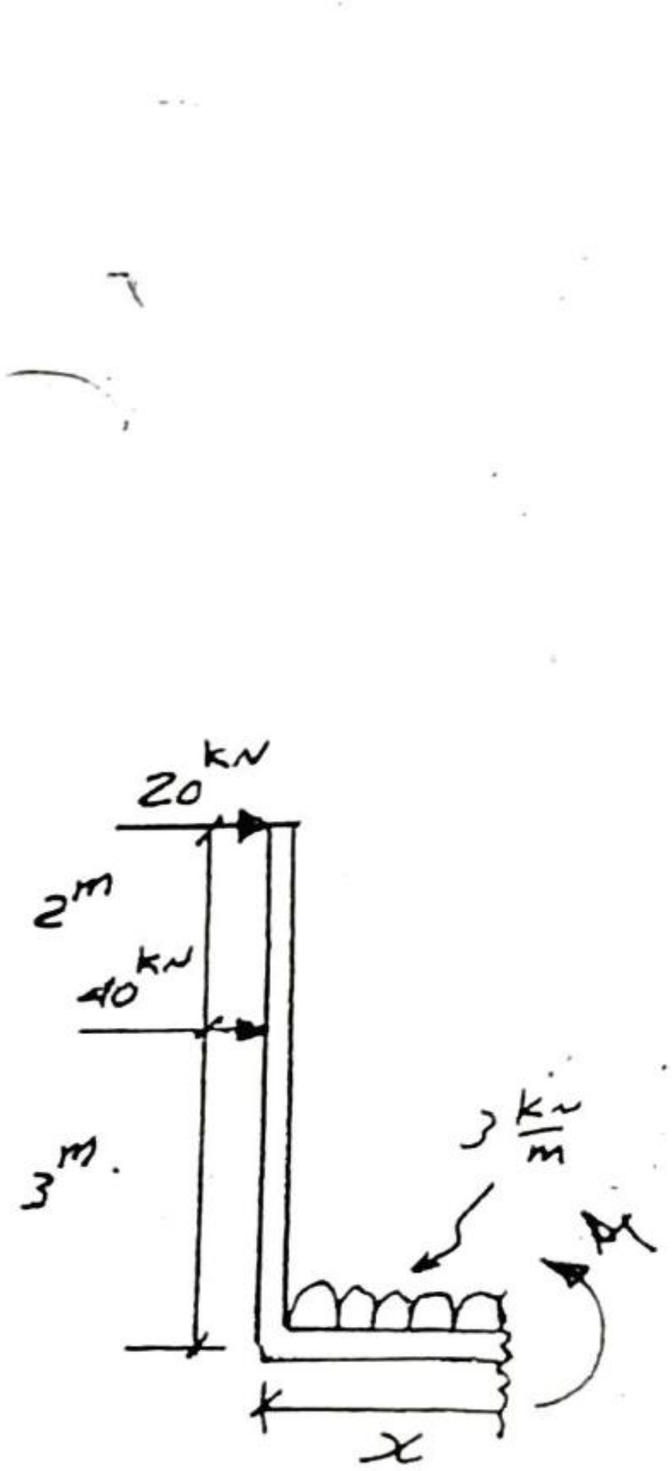
$M = 40 + 60x$

part cd:-

$M = 20 * 5 + 40 * 3 - \frac{3x^2}{2}$

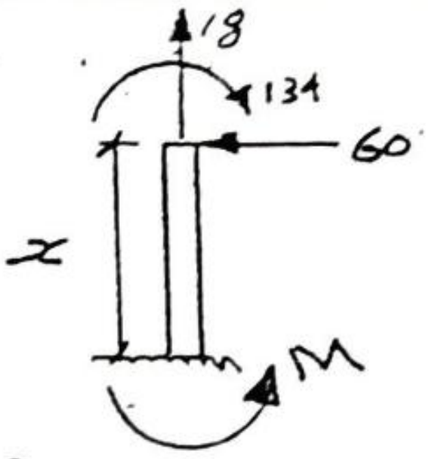
$M = 220 - 1.5x^2$





parted:-

$M = 134 - 60x$

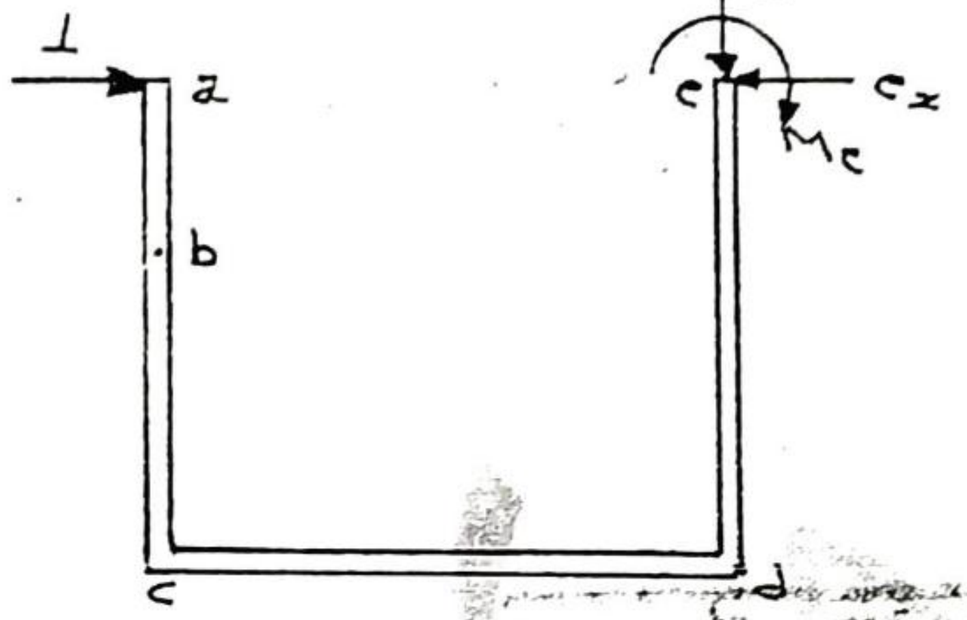


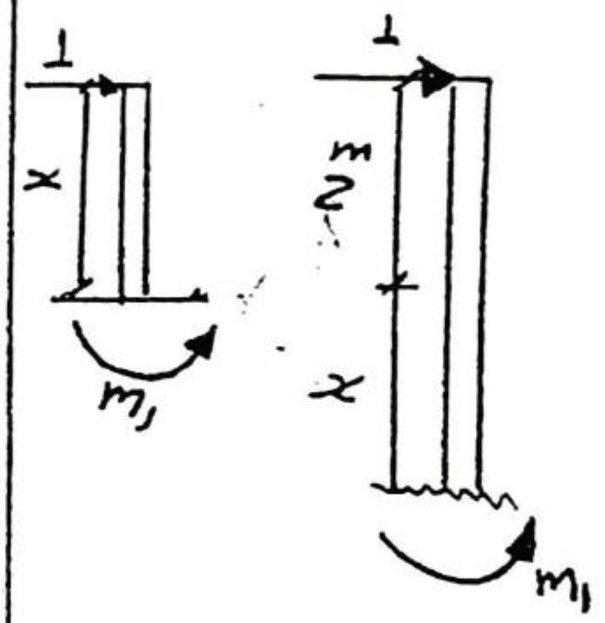
* To find m_1 : $(\Delta a)H$

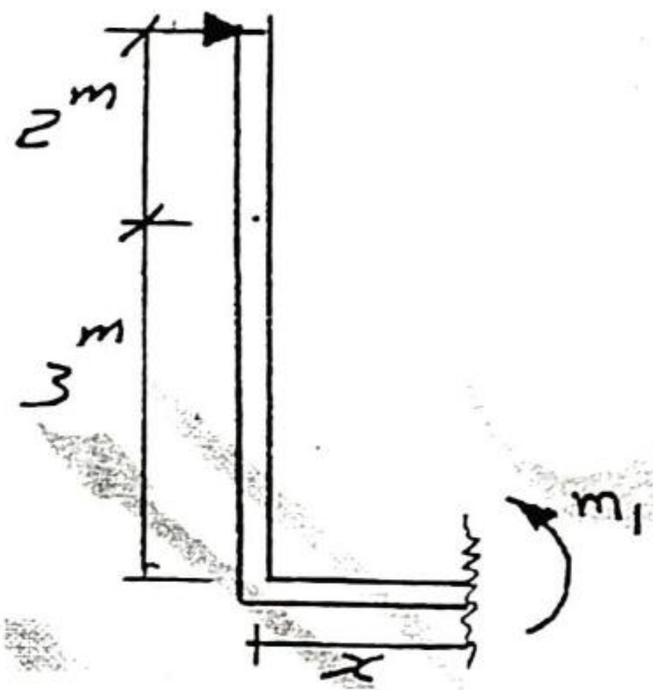
$\Sigma fx = 0 \Rightarrow ex = 1$

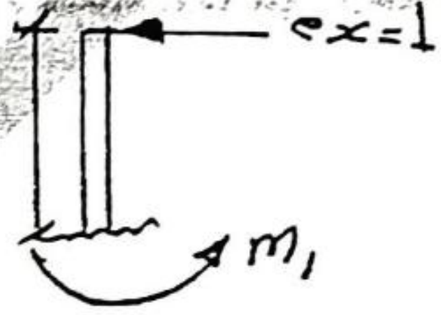
$\Sigma fy = 0 \Rightarrow ey = 0$

$\Sigma Me = 0 \Rightarrow Me = 0$









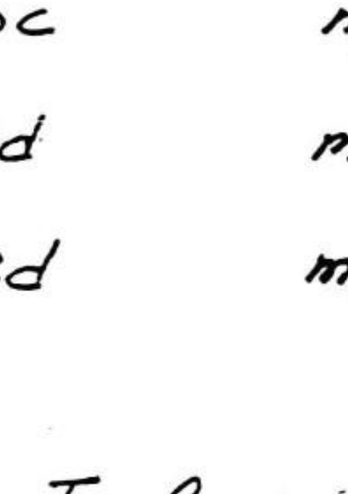
Part	m_1
ab	$m_1 = x$
bc	$m_1 = 2 + x$
cd	$m_1 = -5$
ed	$m_1 = -x$

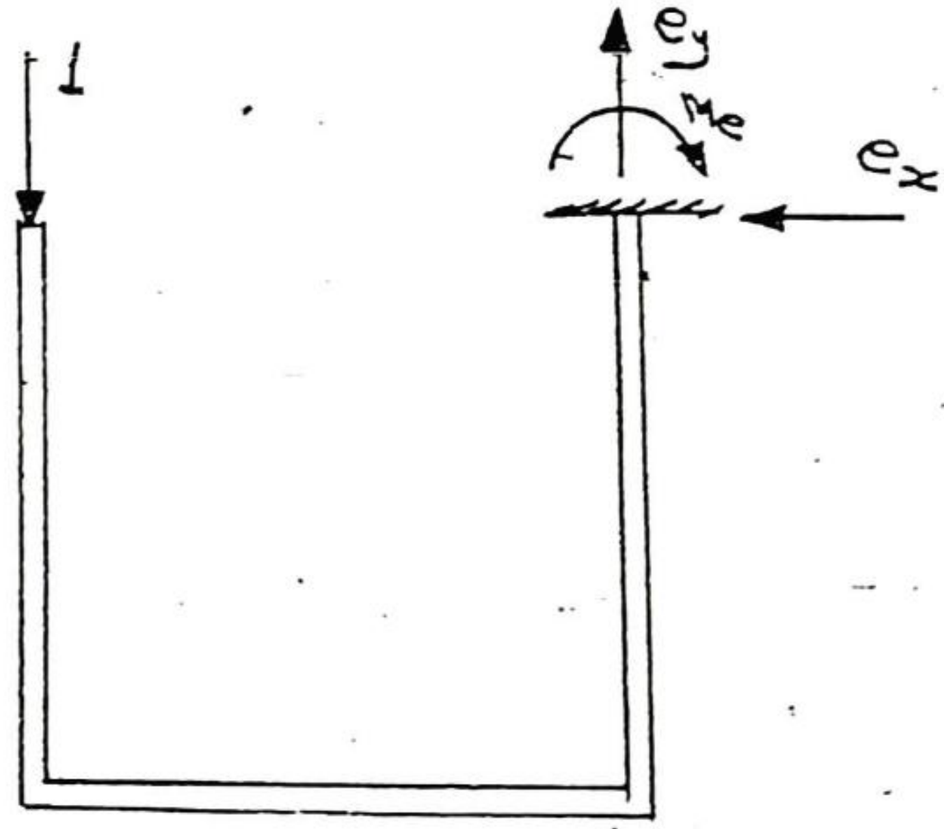
* To find m_2 : $(\Delta a)V$:

$\Sigma fx = 0 \Rightarrow ex = 0$

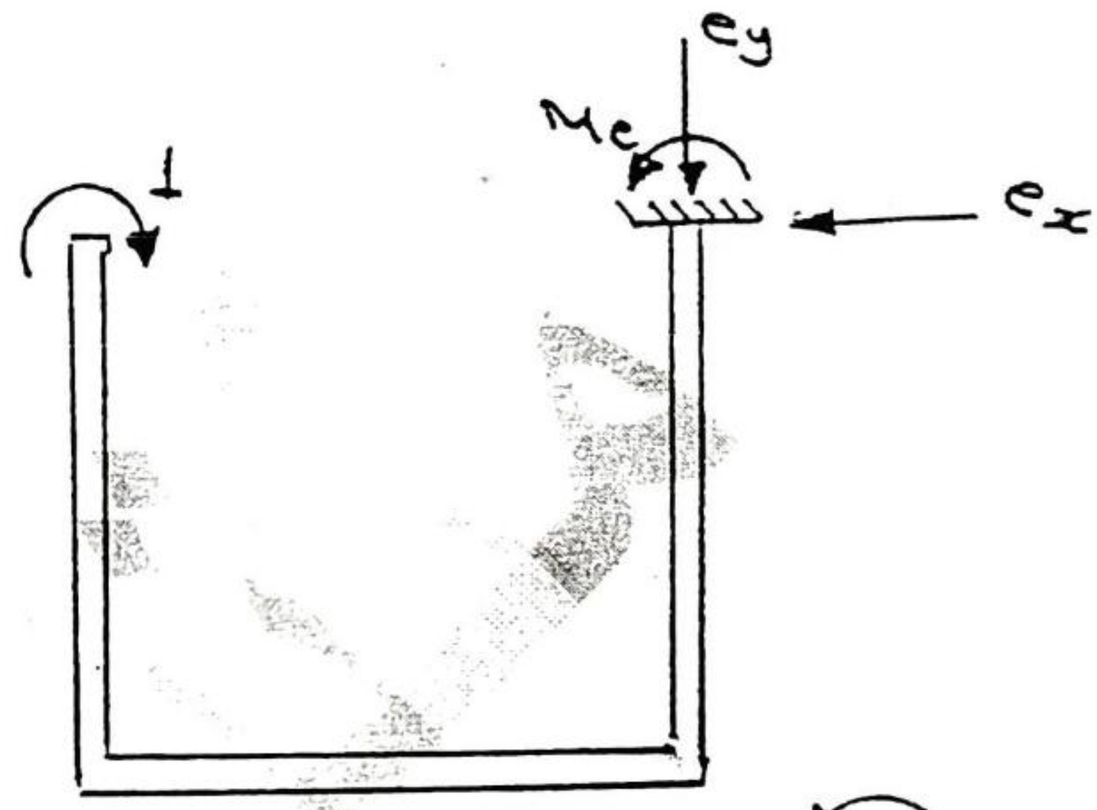
$\Sigma fx = 0 \Rightarrow ey = 1$

$\Sigma Me = 0 \Rightarrow Me = 6$





part	m_2
ab	0
bc	0
cd	X
de	6

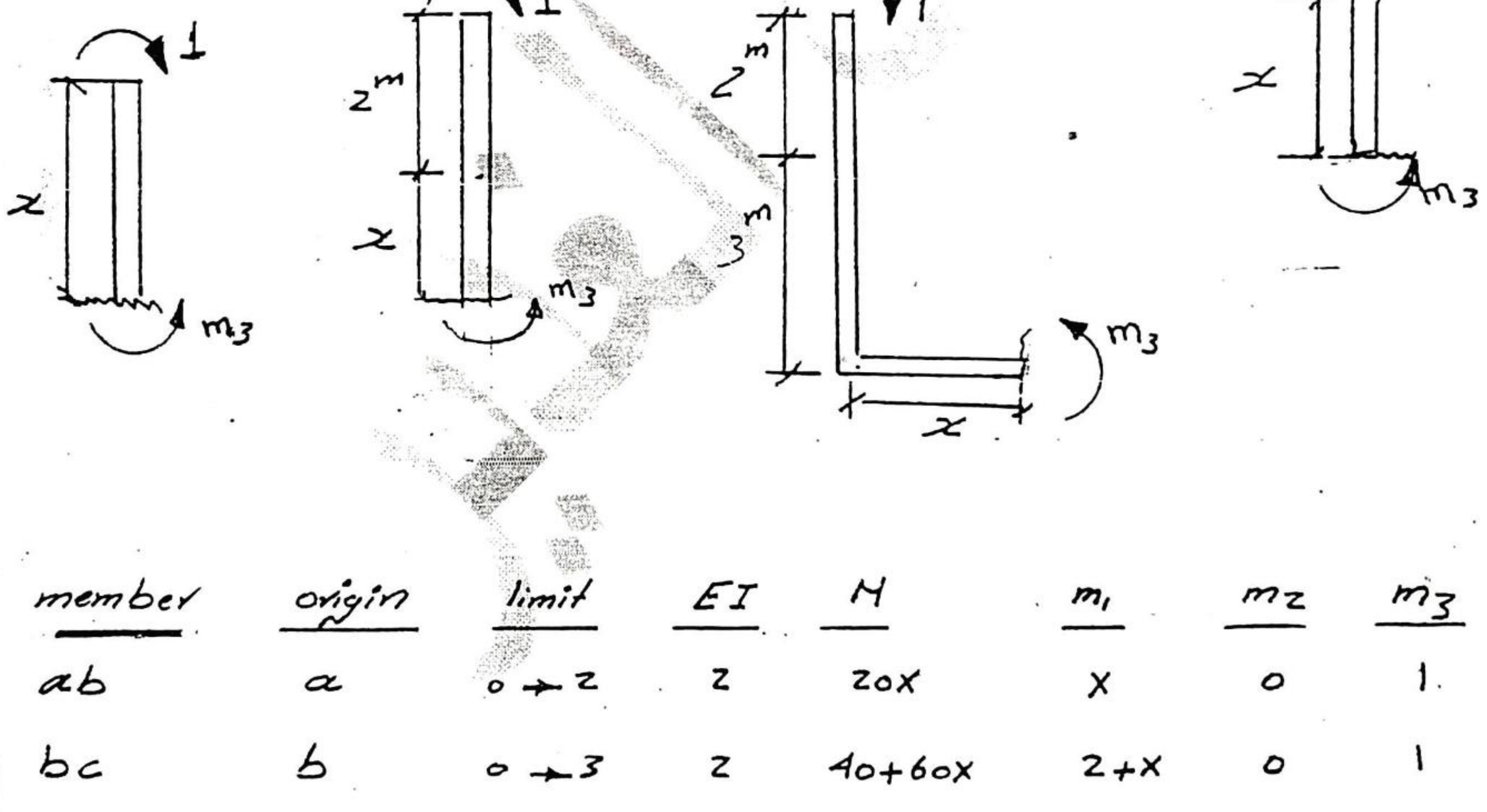


* To find m_3 :- (@a)

$\Sigma F_x = 0 \rightarrow ex = 0$

$\Sigma F_y = 0 \rightarrow ey = 0$

$\Sigma M_e = 0 \rightarrow Me = 1$



member	origin	limit	EI	M	m_1	m_2	m_3
ab	a	$0 \rightarrow 2$	2	20x	x	0	1
bc	b	$0 \rightarrow 3$	2	40+60x	2+x	0	1
cd	c	$0 \rightarrow 6$	3	$220+1.5x^2$	-5	x	-1
de	e	$0 \rightarrow 5$	1	134-60x	-x	6	-1

$$\therefore (\Delta a)H = \int_0^2 \frac{(20x)(x)dx}{2EI} + \int_0^3 \frac{(40+60x)(2+x)dx}{2EI}$$

$$+ \int_0^6 \frac{(220-1.5x^2)(-5)dx}{3EI} + \int_0^6 \frac{(134-60x)(-x)dx}{EI}$$

$$= \frac{1}{EI}\left[\left[\frac{10}{3}x^3\right]_0^2 + \frac{1}{2}\left[(80x + \frac{160}{2}x^2 + \frac{60}{3}x^3\right]_0^3\right.$$

$$\left. - \frac{5}{3}\left[220x - \frac{1.5}{5}x^3\right]_0^6 + \left[-\frac{134}{2}x^2 + \frac{60}{3}x^3\right]_0^6\right]$$

$$= \frac{-418.33}{EI} = \frac{-418.33}{1*10^4} = \boxed{0.041833\,m} \leftarrow$$

$$\therefore (\Delta a)V = 0 + 0 + \int_0^6 \frac{(220-1.5x^2)(x)dx}{3EI} + \int_0^5 \frac{(134-60x)(6)dx}{EI}$$

$$= \frac{1}{EI}\left[\left[\frac{(110x^2 - \frac{1.5}{4}x^4\right]_0^6}{3} + 6\left[134 - 30x^2\right]_0^5\right]$$

$$= \frac{678}{EI} = \frac{678}{1*10^4} = 0.0678$$

$$\therefore \Delta a = \sqrt{(-0.041835)^2 + (0.0678)^2} = 0.0797m$$

$$\theta a = \int_0^2 \frac{(20x)(1)\,dx}{2EI} + \int_0^3 \frac{(40+60x)}{2EI} + \int_0^6 \frac{(220-1.5x^2)(-1)dx}{3EI} + \int_0^5 \frac{(134-60x)(-1)dx}{EI}$$

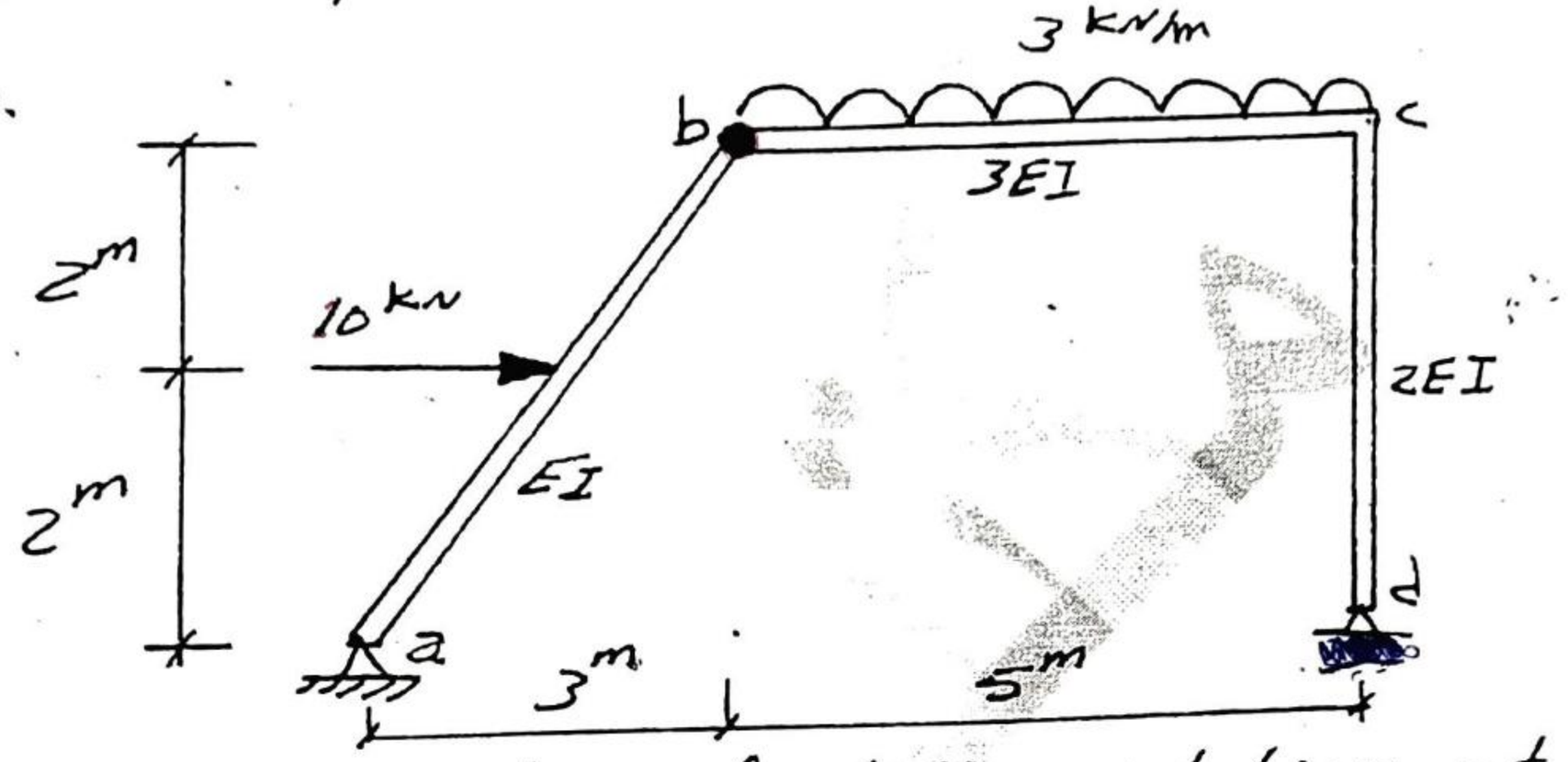
$$= \frac{1}{EI}\left[\left[5x^2\right]_0^2 + \left[20x - 15x^2\right]_0^3 + \frac{1}{3}\left[-220x + 0.5x^3\right]_0^3 + \left[-134x + 30x^2\right]_0^5\right]$$

$$= -\frac{109}{1*10^4} = -0.0109\ rad$$

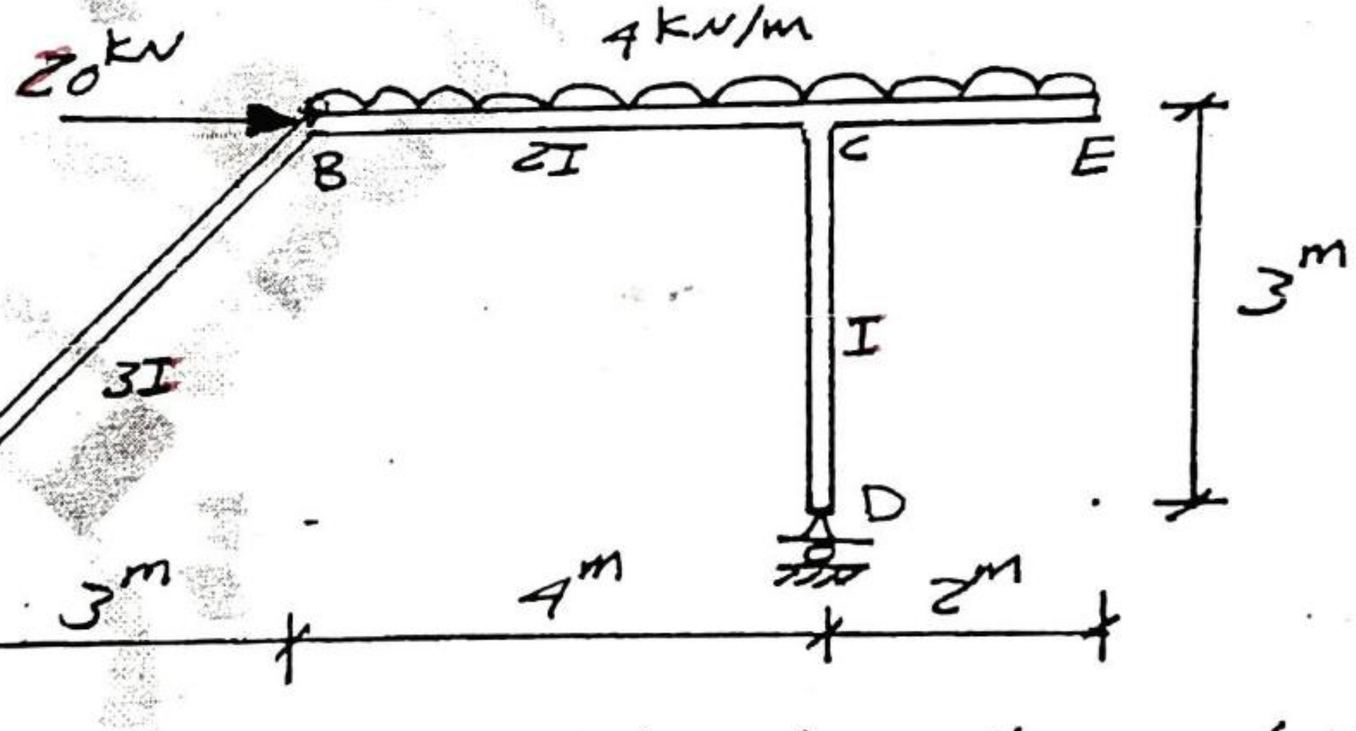
$$\rightarrow \theta 2 = -0.0109 * \frac{180}{\pi} = -0.624^\circ$$

H.W :-

① - Using unit load method find the ratation of end (b) of member (ba) for the frame shown.

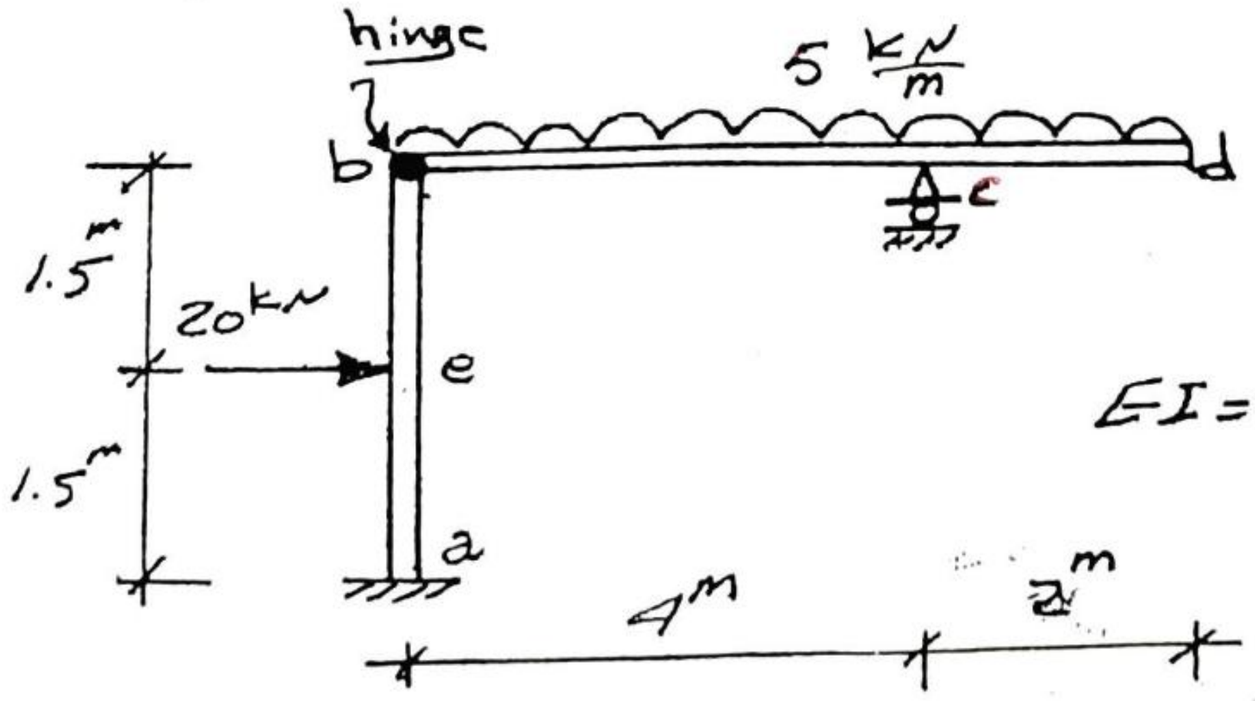


② - Using unit load method find the rotation at support (A) for the frame shown, then find (ΔD)?



③ - Using unit load method determine the rotation of end (b) of member (ba) and deformation of support

(c).



EI = const.