Note: You can write as many independent material balances as there are species involved in the system.

6. Material balance: in = out or in – out = 0

$$\text{NaOH balance:} \quad 1000 = P_{\text{NaOH}} \quad \text{or} \quad 1000 - P_{\text{NaOH}} = 0 \quad (1)$$

$$H_2O \text{ balance:} \quad W = P_{H_2O} \quad \text{or} \quad W - P_{H_2O} = 0 \quad (2)$$

$$\text{Given ratio:} \quad W = 0.9P \quad \text{or} \quad W - 0.9P = 0 \quad (3)$$

$$\text{Sum of components in } P\text{:} \quad P_{\text{NaOH}} + P_{H_2O} = P \quad \text{or} \quad P_{\text{NaOH}} + P_{H_2O} - P = 0 \quad (4)$$

Could you substitute the total mass balance 1000 + W = P for one of the two component mass balances? Of course In fact, you could calculate P by solving just two equations:

$$\text{Total balance:} \quad 1000 + W = P$$

$$\text{Given ratio:} \quad W = 0.9P$$

7. Solve equations:

W = 0.9 P substitute in total balance 1000 + 0.9 P = P

∴ P = 10000 kg & W = 0.9 * 10000 = 9000 kg (The basis is still 1 hr (F_{NaOH} = 1000 kg))

From these two values you can calculate the amount of H_2O and NaOH in the product

$$\text{From the} \begin{cases} \text{NaOH balance:} \\ \\ H_2O \text{ balance:} \end{cases} \quad \text{you get} \begin{cases} P_{\text{NaOH}} = 1000 \text{ kg} \\ \\ P_{H_2O} = 9000 \text{ kg} \end{cases}$$

Then

$$\omega^{P}_{\text{NaOH}} = \frac{1000 \text{ kg NaOH}}{10{,}000 \text{ kg Total}} = 0.1$$

$$\omega^{P}_{H_2O} = \frac{9{,}000 \text{ kg } H_2O}{10{,}000 \text{ kg Total}} = 0.9$$

Note

$$\boxed{\omega^{P}_{\text{NaOH}} + \omega^{P}_{H_2O} = 1}$$

8. The total balance would have been a redundant balance, and could be used to check the answers

$$P_{\text{NaOH}} + P_{H2O} = \dot{P}$$

$$1{,}000 + 9{,}000 = 10{,}000$$

Note: After solving a problem, use a redundant equation to check your values.

Degree of Freedom Analysis

The phrase degrees of freedom have evolved from the design of plants in which fewer independent equations than unknowns exist. The difference is called the degrees of freedom available to the designer to specify flow rates, equipment sizes, and so on. You calculate the number of degrees of freedom (N_D) as follows:

Degrees of freedom = number of unknowns — number of independent equations

$$N_D = N_U - N_E$$

✶ When you calculate the number of degrees of freedom you ascertain the solve ability of a problem. **Three** outcomes exist:

Case	N_D	Possibility of Solution
$N_U = N_E$	0	Exactly specified (determined); a solution exists

$N_U > N_E$	>0	Under specified (determined); more independent equations required
$N_U < N_E$	<0	Over specified (determined)

لايوجد حل هنا .. اما اضيف عدد من المعادلات او افرض قسم من المجاهيل

For the problem in **Example 6**,

$N_U = 4$

$N_E = 4$

So that

$N_D = N_U - N_E = 4 - 4 = 0$

And a **unique** solution exists for the problem.

Example 7

A cylinder containing CH_4, C_2H_6, and N_2 has to be prepared containing a CH_4 to C_2H_6 mole ratio of 1.5 to 1. Available to prepare the mixture is (l) a cylinder containing a mixture of 80% N_2 and 20% CH_4, (2) a cylinder containing a mixture of 90% N_2 and 10% C_2H_6, and (3) a cylinder containing pure N_2. What is the number of degrees of freedom, i.e., the number of independent specifications that must be made, so that you can determine the respective contributions from each cylinder to get the desired composition in the cylinder with the three components?

Solution

A sketch of the process greatly helps in the analysis of the degrees of freedom. Look at Figure 11.

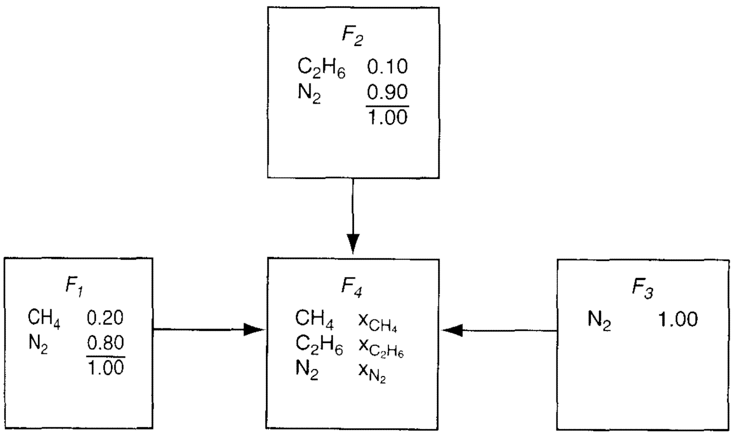

Figure 11

Do you count **seven unknowns** — **three** values of $\mathbf{x_i}$ and **four** values of $\mathbf{F_i}$? How many independent equations can be written?

- Three material balances: CH_4, C_2H_6, and N_2
- One specified ratio: moles of CH_4 to C_2H_6 equal 1.5 or $(X_{CH4}/X_{C2H6}) = 1.5$
- One summation of mole fractions: $\sum x_i^{F_4} = 1$

Thus, there are **seven minus five equals two degrees of freedom** ($N_D = N_U - N_E = 7 - 5 = 2$). If you pick a basis, such as $F_4 = 1$, one other value has to be specified to solve the problem to calculate composition of F_4.

Example 8

In the growth of biomass $CH_{1.8}O_{0.5}\ N_{0.16}S_{0.0045}P_{0.0055}$, with the system comprised of the biomass and the substrate, the substrate contains the carbon source for growth, $C_\alpha H_\beta O_\gamma$, plus NH_3, O_2, H_2O, CO_2, H_3PO_4, and H_2SO_4. The relations between the elements and the compounds in the system are:

	$CH_{1.8}O_{0.5}N_{0.16}S_{0.0045}P_{0.0055}$	$C_\alpha H_\beta O_\gamma$	NH_3	O_2	CO_2	H_2O	H_2SO_4	H_3PO_4
C	1	α	0	0	1	0	0	0
H	1.8	β	3	0	0	2	2	3
O	0.5	γ	0	2	2	1	4	4
N	0.16	0	1	0	0	0	0	0
S	0.0045	0	0	0	0	0	1	0
P	0.0055	0	0	0	0	0	0	1

How many degrees of freedom exist for this system (assuming that the values of α, β, and γ are specified)?

Solution

Based on the given data **six element** balances exist for the **8 species** present, hence the system has **two degrees of freedom**.

Questions

1. What does the concept "solution of a material balance problem" mean?
2. (a) How many values of unknown variables can you compute from one independent material balance?
   (b) From three independent material balance equations?
   (c) From four material balances, three of which are independent?
3. If you want to solve a set of independent equations that contain fewer unknown variables than equations (the over specified problem), how should you proceed with the solution?
4. What is the major category of implicit constraints (equations) you encounter in material balance problems?
5. If you want to solve a set of independent equations that contain more unknown variable than equations (the underspecified problem), what must you do to proceed with the solution?

Answers:

1. A solution means a (possibly unique) set of values for the unknowns in a problem that satisfies the equations formulated in the problem.
2. (a) one; (b) three; (c) three.
3. Delete nonpertinent equations, or find additional variables not included in the analysis.
4. The sum of the mass or mole fraction in a stream or inside a system is unity.
5. Obtain more equations or specifications, or delete variables of negligible importance.

Problems

1. A water solution containing 10% acetic acid is added to a water solution containing 30% acetic acid flowing at the rate of 20 kg/min. The product P of the combination leaves the rate of 100 kg/min. What is the composition of P? For this process,
   a. Determine how many independent balances can be written.
   b. List the names of the balances.
   c. Determine how many unknown variables can be solved for.
   d. List their names and symbols.
   e. Determine the composition of P.
2. Can you solve these three material balances for F, D, and P? Explain why not.

$$0.1F + 0.3D = 0.2P$$
$$0.9F + 0.7D = 0.8P$$
$$F + D = P$$

3. How many values of the concentrations and flow rates in the process shown in Figure SAT7.2P3 are unknown? List them. The streams contain two components, 1 and 2.

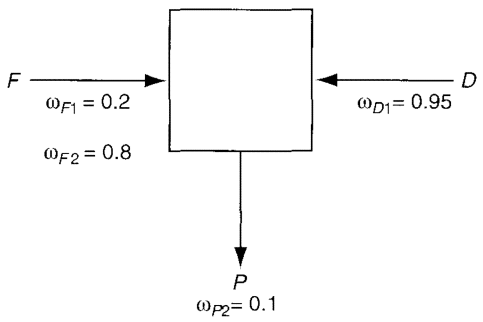

Figure SAT7.2P3

4. How many material balances are needed to solve problem 3? Is the number the same as the number of unknown variables? Explain.

Answers:

1. (a) Two; (b) two of these three: acetic acid, water, total; (c) two; (d) feed of the 10% solution (say F) and mass fraction ω of the acetic acid in P; (e) 14% acetic acid and 86% water
2. Not for a unique solution because only two of the equations are independent.
3. F, D, P, ω_{D2}, ω_{P1}
4. Three unknowns exist. Because only two independent material balances can be written for the problem, one value of F, D, or P must be specified to obtain a solution. Note that specifying values of ω_{D2} or ω_{P1} will not help.

2.3 Solving Material Balance Problems for Single Units without Reaction

The use of material balances in a process allows you **(a)** to calculate the values of the total flows and flows of species in the streams that enter and leave the plant equipment, and **(b)** to calculate the change of conditions inside the equipment.

(أ) لحساب قيم التدفقات الإجمالية وتدفق الأنواع في الجداول التي تدخل وتخرج من معدات المصنع.
(ب) حساب تغير الظروف داخل المعدات.

Example 9

Determine the mass fraction of Streptomycin in the exit organic solvent assuming that no water exits with the solvent and no solvent exits with the aqueous solution. Assume that the density of the aqueous solution is 1 g/cm^3 and the density of the organic solvent is 0.6 g/cm^3. Figure E8. 1 shows the overall process.

Solution

This is an **open** (flow), **steady-state** process without reaction. Assume because of the low concentration of Strep. in the aqueous and organic fluids that the **flow rates** of the **entering** fluids **equal** the flow rates of the **exit** fluids.