the atmosphere as shown in Fig. (2.5). In this manometer, we can measure positive pressure (gauge pressure) and negative pressure (vacuum).

Let $B$ is the point at which pressure is to be measured ( $p$ ). The datum line is $\mathbf{A}-\mathbf{A}$.


Fig. (2.5) U- tube Manometer.
If we want to measure the pressure ( $p$ ) at point $B$.
There are two legs in the manometer, if there is equilibrium between two legs (right and left legs) over the datum ( $\mathrm{A}-\mathrm{A}$ ), i. e the pressure at each leg over the datum are equal.

Mathematically,
( a ) For gauge pressure :
Pressure at left leg $=$ Pressure at right leg

$$
\begin{align*}
& \mathbf{P}+\gamma_{1} \mathbf{h}_{1}=\gamma_{2} \mathbf{h}_{2} \\
& \mathbf{P}=\gamma_{2} \mathbf{h}_{2}-\gamma_{1} \mathbf{h}_{1} \quad \mathbf{N} / \mathbf{m}^{2} \tag{2.6}
\end{align*}
$$

(b) For vacuum (negative) pressure:

Pressure at left leg = pressure at right leg

$$
\begin{align*}
& \mathbf{P}+\gamma_{1} \mathbf{h}_{1}+\gamma_{2} \mathbf{h}_{2}=\mathbf{0} \\
& \quad \mathbf{P}=-\gamma_{1} \mathbf{h}_{1}-\gamma_{2} \mathbf{h}_{2} \quad \mathbf{N} / \mathbf{m}^{2} \tag{2.7}
\end{align*}
$$

## 2.6/ Differential Manometers :

Differential manometer are the devices used for measuring the difference of pressures between between two points in a pipe or in two different pipes. A differential manometer consists of a $\mathbf{U}$ - tube, containing a heavy liquid (liquid manometer), frequently is mercury ( $\mathbf{H g}$ ). Most commonly types of differential manometers are:

1. U-tube differential manometer., 2 - Inverted U-tube differential manometer. Fig. (2.6) shows the differential manometer of U-tube type.


Fig. (2.6) U-tube differential manometer.
In Fig. (2.6) (a), let the two points A \& B are at different level and also contains liquids at different specific gravity (S) (sp. gr.).

Level X - X, level of equilibrium, the pressures in the left leg equal the pressures in right leg:

$$
\begin{align*}
& \mathbf{p}_{\mathrm{A}}+\gamma_{\mathrm{A}}(\mathbf{x}+\mathbf{h})=\mathbf{p}_{\mathrm{B}}+\gamma_{\mathrm{B}} \mathbf{y}+\gamma_{\mathrm{m}} \mathbf{h} \\
& \mathbf{p}_{\mathrm{A}}-\mathbf{p}_{\mathrm{B}}=\gamma_{\mathrm{B}} \mathbf{y}+\gamma_{\mathrm{m}} \mathbf{h}-\gamma_{\mathrm{A}}(\mathbf{x}+\mathbf{h}) \tag{2.8}
\end{align*}
$$

In Fig. (2.6) (b),

$$
\begin{align*}
& \mathbf{p}_{A}+\gamma_{A}(\mathbf{x}+\mathbf{h})=\mathbf{p}_{\mathrm{B}}+\gamma_{\mathrm{B}} \mathbf{x}+\gamma_{\mathrm{m}} \mathbf{h} \\
& \mathbf{p}_{\mathrm{A}}-\mathbf{p}_{\mathrm{B}}=\gamma_{\mathrm{B}} \mathbf{x}+\gamma_{\mathrm{m}} \mathbf{h}-\gamma_{\mathrm{A}}(\mathbf{x}+\mathbf{h}) \tag{2.9}
\end{align*}
$$

## 2.Inverted U- tube differential manometer:

It consists of an inverted $U$-tube. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measured difference of low pressure. Fig. (2.7) shows an inverted U- tube differential manometer connected to the two points $A \& B$. Let the pressure at $A$ is more than the pressure at $B$.


Fig. (2.7)
Taking $\mathbf{X}-\mathbf{X}$ as datum line, then,
Pressure at the left leg below the $X-X=\mathbf{p}_{\mathbf{A}}-\gamma_{1} \mathbf{h}_{1}$
Pressure at the right leg below the $X-X=p_{B}-\gamma_{2} h_{2}-\gamma_{m} h$
Pressure at the left leg = pressure at the right leg
$\mathbf{P}_{\mathrm{A}}-\gamma_{1} \mathbf{h}_{1}=\mathbf{p}_{\mathrm{B}}-\gamma_{2} \mathbf{h}_{2}-\gamma_{\mathrm{m}} \mathbf{h}$
$\mathbf{P}_{A}-p_{B}=\gamma_{1} h_{1}-\gamma_{2} h_{2}-\gamma_{m} h$

## 1.7 / Inclined Single Column Manometer:

Fig. (2.8) shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right side will be more.


Fig. (2.8) Inclined manometer
Let $\quad L$ - length of heavy liquid moved in right side from $X-X$
$\Theta$ - inclination of right leg with horizontal.
$h_{2}$ - vertical rise of heavy liquid in right leg from $X-X$

$$
\begin{align*}
& \quad(L \sin \theta) \\
& \mathbf{P}_{A}=\gamma_{2} h_{2}-\gamma_{1} h_{1} \\
& \mathbf{P}_{A}=\gamma_{2} L \sin \theta-\gamma_{1} h_{1} \tag{2.11}
\end{align*} \quad\left(\text { but } h_{2}=L \sin \theta\right)
$$

## 2.8/ Micromanometer:

It is used for determine small differences in pressure. With two gage liquids, immiscible in each other and in the fluid to be measured, a large gage difference $R$, as shown in Fig.(2.9) can be produced for a small pressure difference. The heaver gage liquid fills the lower $U$-tube up to $\mathbf{O - O}$ then the lighter gage liquid is added to both sides, filling the larger reservoir up to $1-1$. The gas or liquid in the system fills the space above 1 - 1 . When the pressure at $C$ is slightly greater than at $D$, the menisci move as indicated in Fig. (2.9). The volume of liquid displaced in each reservoir equals the displacement in the $\mathbf{U}$-tube, thus,


Fig. (2.9) Micromanometer

$$
\Delta y \cdot A=\frac{R}{2} \cdot a, \Delta y=\frac{R a}{2 A}
$$

In which, $\mathbf{A}$ is area of reservoir, $\mathbf{a}$ is area of $\mathbf{U}-$ tube.
The manometer equation may be written starting at $C$,

$$
\mathbf{P}_{\mathbf{c}}+\left(\mathbf{k}_{1}+\Delta \mathbf{y}\right) \gamma_{1}+\left(\mathbf{k}_{2}-\Delta \mathbf{y}+\frac{R}{2}\right) \gamma_{2}-\mathbf{R} \gamma_{3}-\left(\mathbf{k}_{2}-\frac{R}{2}+\Delta \mathbf{y}\right) \gamma_{2}-
$$

$$
\left(\mathbf{k}_{1}-\Delta \mathbf{y}\right) \gamma_{1}=\mathbf{p}_{\mathbf{D}}
$$

In which $\gamma_{1}, \gamma_{2} \gamma_{3}$ are the specific weights . Simplifying and substituting for $\Delta y$ gives:
$\mathbf{p}_{\mathrm{C}}-\mathbf{p}_{\mathrm{D}}=\mathbf{R}\left[\gamma_{3}-\gamma_{2}\left(1-\frac{a}{A}\right)-\gamma_{1} \frac{a}{A}\right]$
The quantity in bracket is a constant for specified gage and fluids, hence, the pressure difference is directly proportional to $R$.

## 1.8 / Bourdon Gage (Mechanical):

The bourdon pressure gage as shown in Fig. (2.10) is typical of the devices used for measuring gage pressure.

The bourdon gage (shown schematically) in Fig. (2.11). In the gage, a bent tube (A) of elliptical cross section is held rigidly at (B) and its free end is connected to a pointer (C) by a link (D). When pressure is admitted to the tube, its cross section tends to become circular, causing the tube to straighten and move the pointer to the right over the graduated scale.


Fig. (2.10) typical of Bourdon gage


Fig. (2.11) Schematically shown of Bourdon gage
The pointer rests at zero on the scale, when the gauge is disconnected, in this condition the pressure inside and outside of the tube are the same.

