Eccentricity and concentricity problems in foundations design

If we have a system of moments and forces act on a footing, but the resultant of all is acting in the center of the footing. This system is called concentric system. The resulting pressures on the soil beneath the footing are regular as in Fig.1

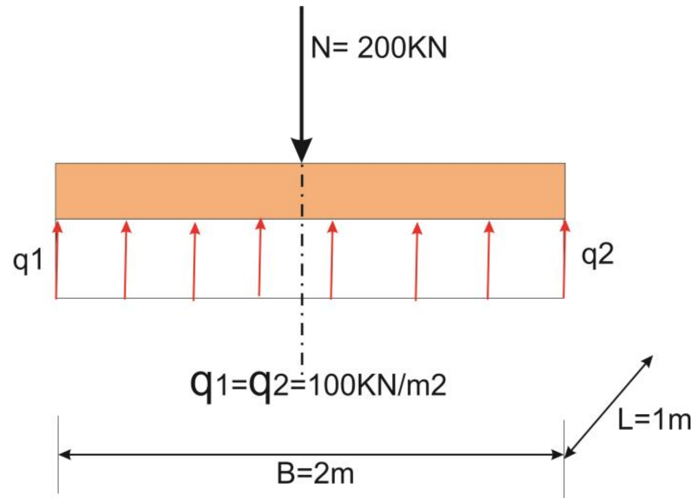

Fig.1 Concentric footing

If there is one or more forces and one or more torques acting on a foundation, it may cause that the resultant forces act in a place far from the center of gravity of the foundation as shown in Fig.2. This phenomena is called eccentricity Thus, the pressures caused by the foundation on the underlying soil will be irregular or unequal, and thus the design procedures for foundations will differ accordingly.

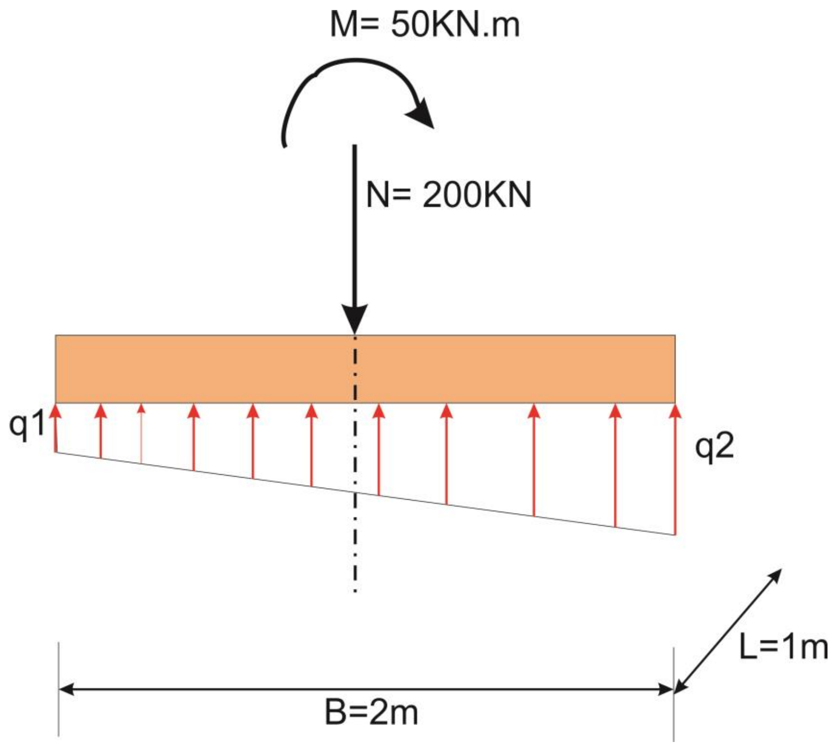

Fig.2 Eccentric Force . moment system

If a moment ($M = 50\ KN.m$) and a load ($\mathbf{N = 200\ KN}$) shown in Fig.2 act on the footing, certainly, the location of the resultant will be away from the center of the base at a distance called Eccentricity (e) which equals:-

$$e = \frac{M}{N} = \frac{50KN.m}{200\ KN} = 0.25\ m$$

This value of eccentricity should be less than $\frac{B}{6}$, in order to secure the foundation, it does not put negative pressure on the underlying soil. In general the value of the eccentricity (e) may be as

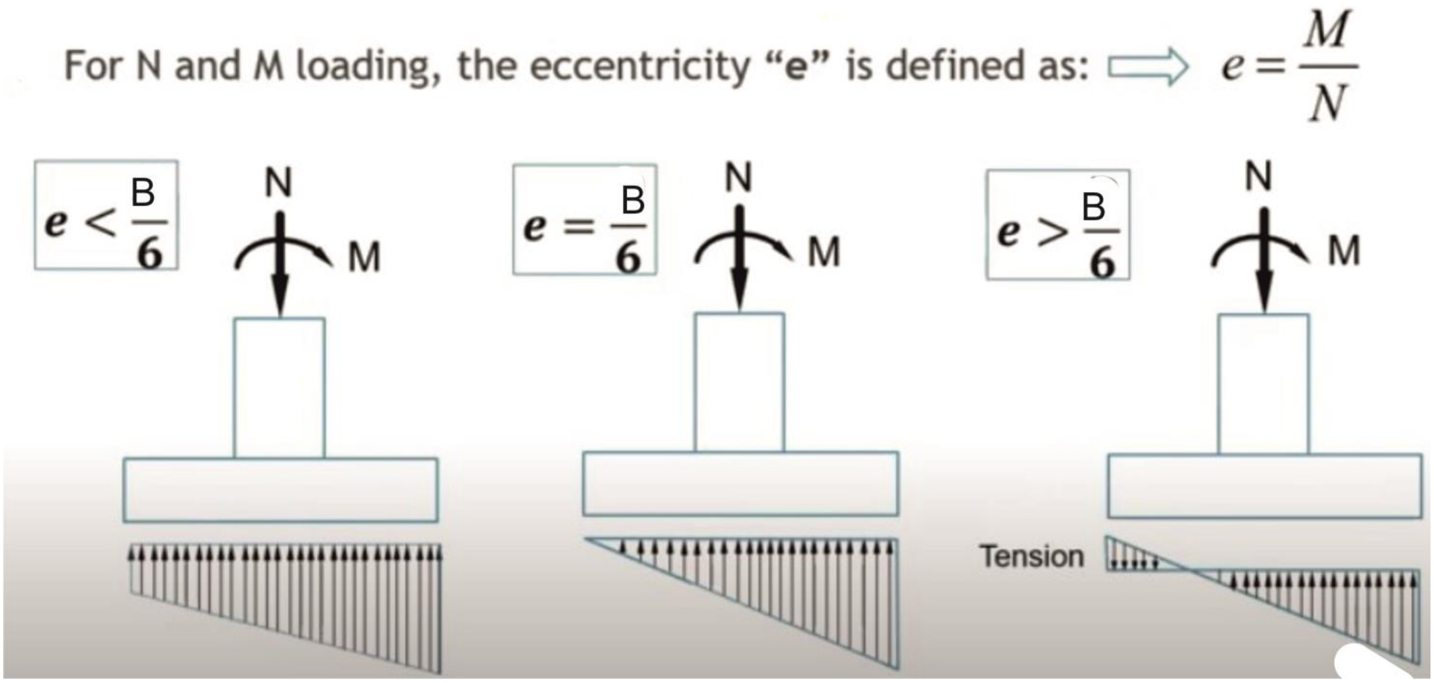

Fig.3 Eccentricity causes irregular pressure on the soil

Cases (1 and 2) are allowed to be used for foundation design but case (3) is not allowed because of the resulting negative pressure. Therefore, the engineer should find a solution to this problem. One may change the system of forces as shown in Fig.3

How is the eccentricity acts?

Now to check (e) in the Example of Fig.2

$\frac{B}{6} = \frac{2m}{6} = 0.333m$, so

$(e = 0.25) < (\frac{B}{6} = 0.333)$, therefore we have case1 of Fig.2. To see that let us try the following:

By the equations of strength of materials, the values of the applied pressures on the soil is

$$q_1 = \frac{N}{BL} - \frac{6M}{lB^2} = \frac{200}{2*1} - \frac{6*50}{1*2^2} = 25\ KN/m^2 \ ,$$ and

$$q_2 = \frac{N}{BL} + \frac{6M}{lB^2} = \frac{200}{2*1} + \frac{6*50}{1*2^2} = 75\ KN/m^2 \ ,$$ or

From the static equations of equilibrium and by inspection of Fig.4;

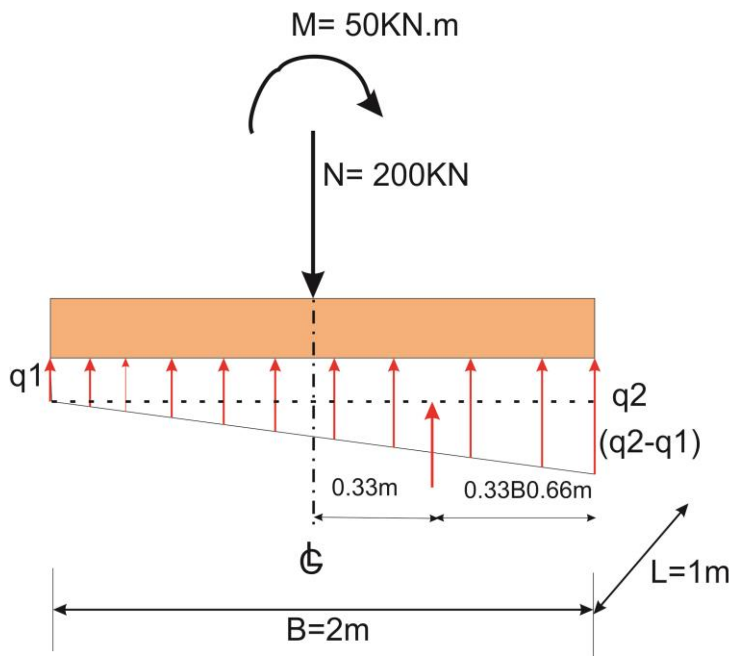

Fig.4 Force-Moment system

$\sum F_y = 0$

$\frac{q_1+q_2}{2} * 2 = 200$ more simplification gives

$q_1 + q_2 = 200$ ………………………………………………… (1)

Take the $\sum M_{Center} = 0$ to eliminate larger number of the applied forces

$50 \circlearrowright - \frac{(q_2-q_1)*2}{2} * 0.33 \circlearrowleft = 0$ …………………………………………………..(2)

By solving these simultaneous these two equations, one also obtains the same results:-

$q_1 = 25\ KN$, $q_2 = 75\ KN$

Where are the resultant acts?

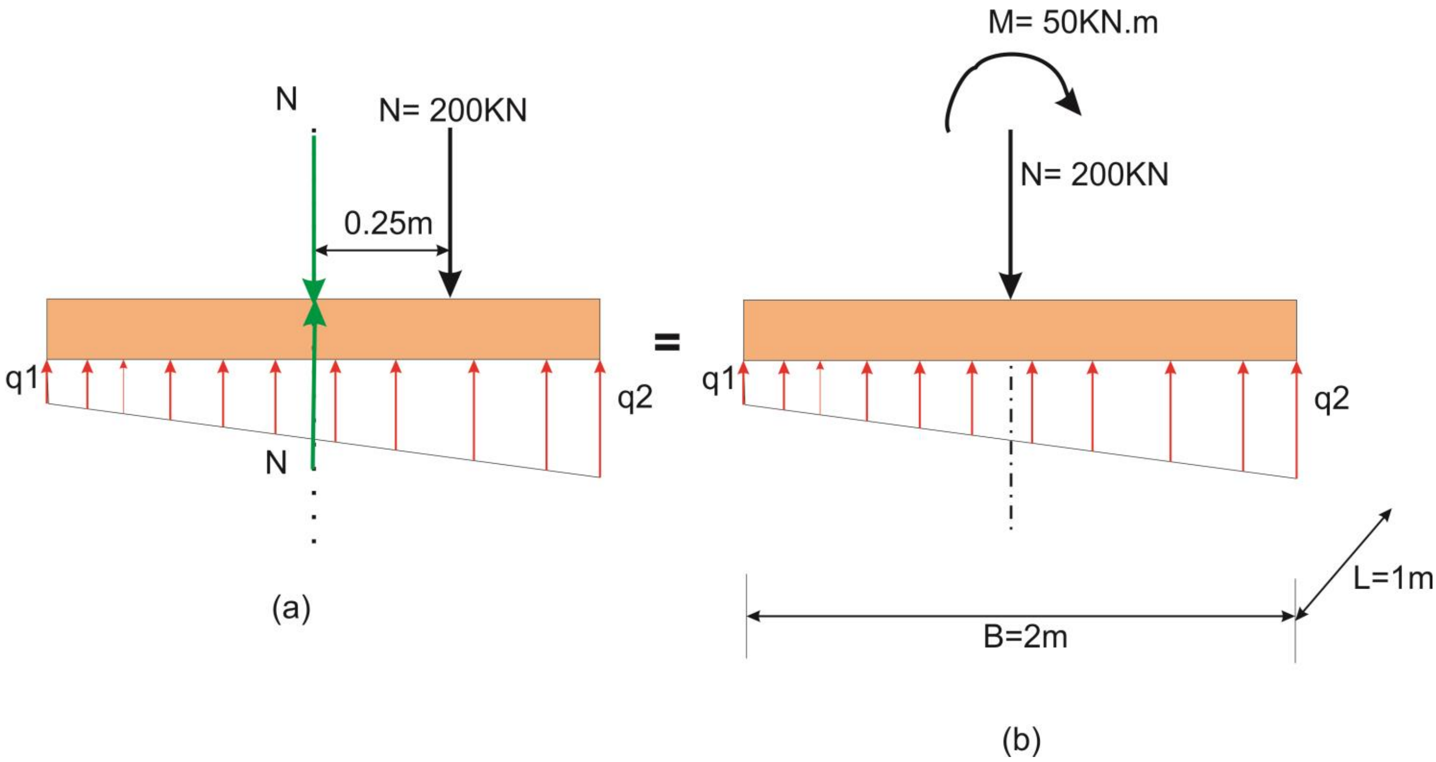

Fig.5 Changing of eccentric forces system

The analysis of the system of eccentric footing in Fig.5b should lead to same results of the applies stresses q1 and q2 of Fig.5a because it is equivalent system. This can be confirmed by the following estimations.

$\sum F_y = 0$

$\frac{q_1+q_2}{2} * 2 = 200$ or

$q_1 + q_2 = 200 \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$

$\sum M_{at\ force\ location} = 0$, the effect of the force N is ignored, thus

$q_1 * 2 * 0.25 \circlearrowright - \frac{2(q_2-q_1)}{2} * (1 - \frac{B}{3} - 0.25) \circlearrowleft = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots.. (2)$

More simplifications offers

$0.5q_1 - 0.0834(q_2 - q_1) = 0$ or

$0.5834q_1 - 0.0834q_2 = 0 \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots. (2)$

By solving these simultaneous equations (1) and (2), one also obtains:-

$q_1 = 25\ KN$, $q_2 = 75\ KN$

Solving the problem of eccentricity

In the system above we have an $e = 0.25$ to check the eccentricity of the footing in this case, it should be compared with the standard value $(\frac{B}{6})$,

$\frac{B}{6} = \frac{2}{6} = 0.333$ m

Since e = 0.25 m< 0.333m therefore we have case 1 of Fig.3.

Now suppose $B = 1.5m\ this\ give\ \frac{B}{6} = 0.25m\ case2$. If we let B < 1.5 therefore the eccentricity e becomes > 0.25 which leads to case 3. So as we mentioned before this case is not permitted because it produces negative stresses on the soils. If this case encountered, the engineer should change the system forces or increase the footing width if it is possible

Problem:

The eccentric force N is applied on a footing as shown in Fig.6a. Estimate the moment in the equivalent system Fig.6b. Also estimate $q_1 and q_2$.

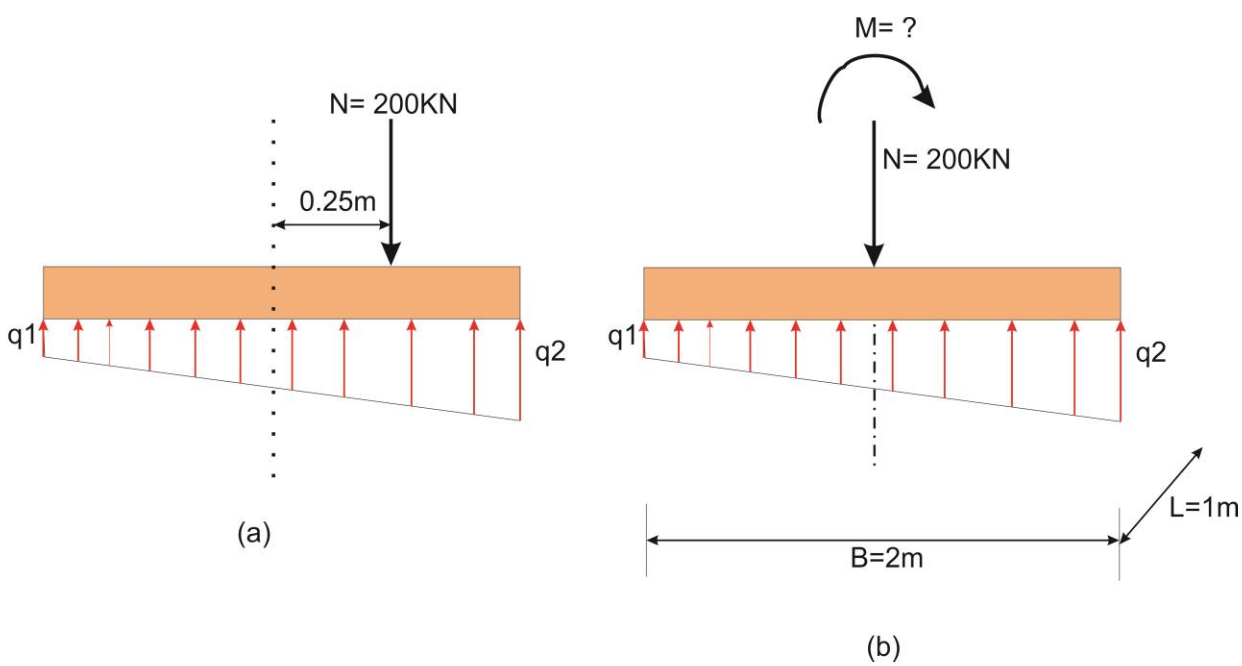

Fig.6

Design consideration of eccentricity in foundation

1- Transmit the resultant of eccentric force-moment system to the footing center.
2- Check the eccentricity and make the necessary arrangements for footing width. Such as increasing or reducing the width taking into account that the distributed loads not to exceed the allowable bearing capacity
3- Estimate the resulting distributed pressures on the underlying soil by using the following equations.

$$q_1 = \frac{N}{BL} - \frac{6M}{lB^2} \quad (1)$$

$$q_2 = \frac{N}{BL} + \frac{6M}{lB^2} \quad (2)$$

4- Design the footing according to the resulting applied distribution loads and the allowable bearing capacity.

Problem: Estimate the concentric resultant (which acts at the center of footing). Also estimated the resulting applied pressures on the underlying soil of the force system of Fig.7

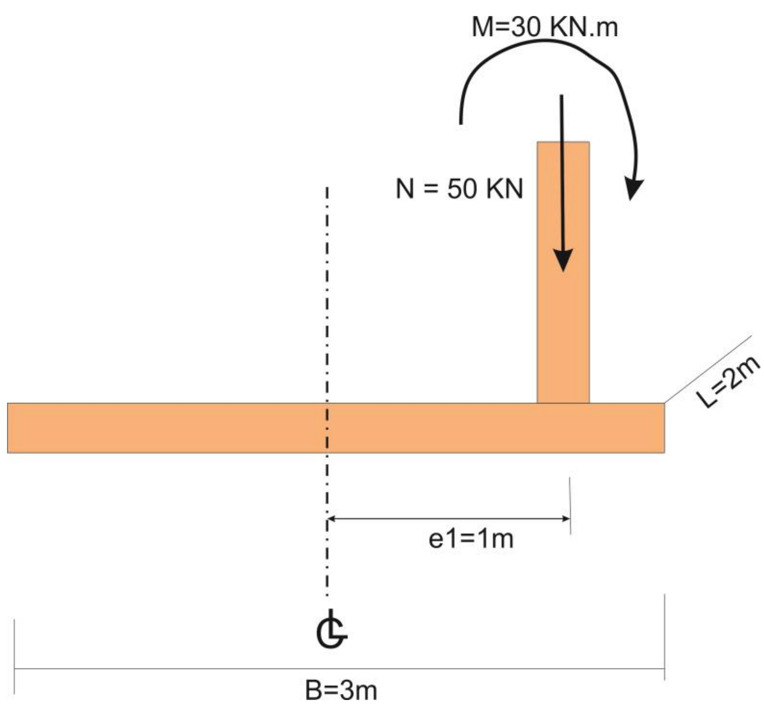

Fig.7 Eccentric problem

Example2: Estimate the applied distributed pressures on the underlying soil of the eccentric force-moment system as shown in Fig.8.

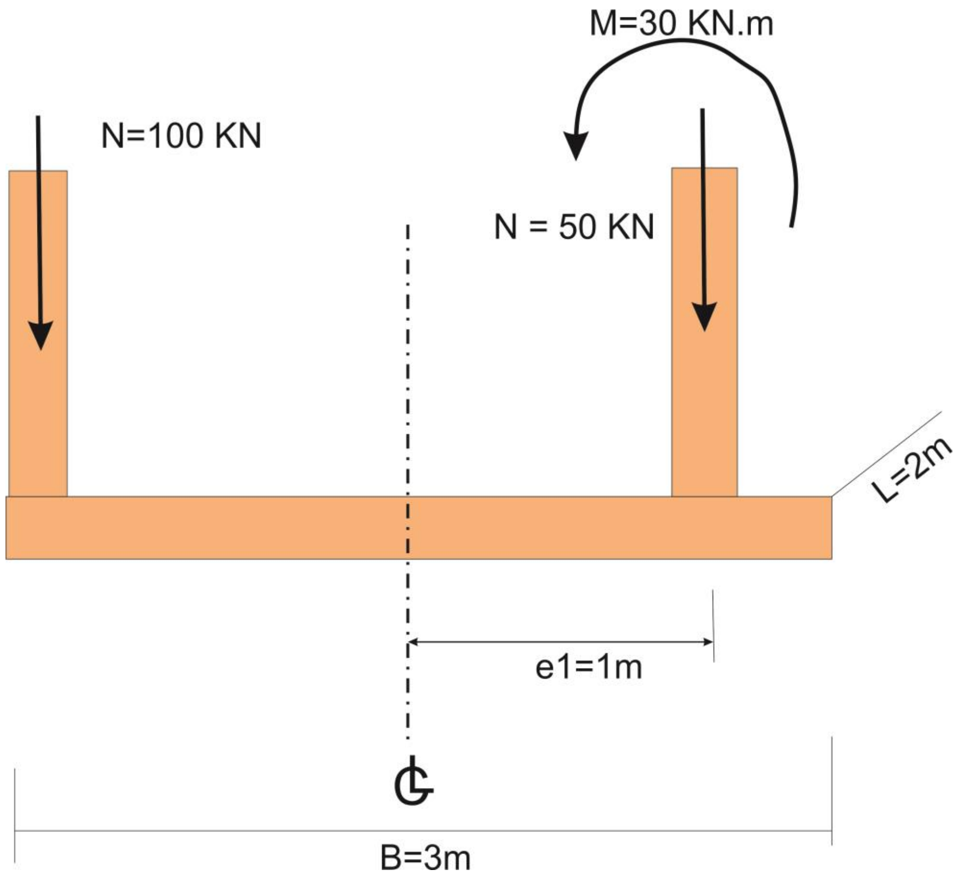

Fig.8 Eccentric combined footing