Cauhy -Riemann Equation

Let f (z) =u(x,y) +vi (x,y), the necessary condition that f(z) be intuition in region Rishat u (x,y)sqtisfyi

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{———} \quad 1$$

$$\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x} \quad \text{———} \quad 2$$

Ex/ check the following function if they are analytic.

1) f (z) =x -iy

u+vi =x-iy

u = x

v = - y

$$\frac{\partial u}{\partial x} = 1 \quad , \frac{\partial v}{\partial y} = -1$$

Since $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$ $then\ f(z) is\ not\ analytic$

2) $f(z) = z^2 + 2z - 1$

$u + vi = (x + yi)^2 + 2(x + yi) - 1$

$u + vi = x^2 + 2xyi - y^2 + 2x + 2yi - 1$

$u + vi = x^2 - y^2 + 2x + 2xyi + 2yi - 1$

$u = x^2 - y^2 + 2x - 1$

$v = 2xy + 2y$

$\frac{\partial u}{\partial x} = 2x + 2$, $\frac{\partial u}{\partial y} = 2x + 2$

$\frac{\partial u}{\partial y} = -2y$, $\frac{\partial u}{\partial x} = -2y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = \frac{-\partial u}{\partial x}$$

Then f (z) is analytic

H . W

Check the following functions if analytic

1 – f (z) = z

2 – f (z) = z+z

3 – f (z) = e^z

4 – f (z) = $\frac{1}{z}$

Harmonic function

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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Ex/ prof that u (x , y) = $e^y \cos x$ is harmonic

$\acute{u}_x = e^y \sin x \qquad \bar{\bar{u}}_{xx} = -e^y \cos x$

$\acute{u}_y = e^y \cos x \qquad \bar{\bar{u}}_{yy} = e^y \cos x$

$\bar{\bar{u}}_{xx} + \bar{\bar{u}}_{yy} = 0$

$-e^y \cos x + e^y \cos x = 0$

2 – u (x, y) = ln (x^2+y^2)

$\acute{u}_x = \frac{2x}{x^2+y^2} \quad , \quad \bar{\bar{u}}_{xx} = \frac{2(x^2+y^2)-2\,(2x)}{(x^2+y^2)^2}$

$= \frac{2x^2+2y^2-4x}{(x^2+y^2)^2}$

$\acute{u}_y = \frac{2y}{x^2+y^2} \qquad \bar{\bar{u}}_{yy} = \frac{2(x^2+y^2)-2y\,(2y)}{(x^2+y^2)^2}$

$= \frac{2x^2+2y^2-4y^2}{(x^2+y^2)^2}$

$\bar{\bar{u}}_{xx} + \bar{\bar{u}}_{yy} = 0$

Ex/ Find harmonic f (z)that real foot is

u (x ,y) = sin hx sin y .

$\frac{\partial u}{\partial x}$ = cos hx sin y

$\frac{\partial u}{\partial^2 x}$ = sin hx sin y

$\frac{\partial u}{\partial y}$ = - sin hx sin y

$\frac{\partial u}{\partial^2 y}$ = - sin hx sin y

$\frac{\partial u}{\partial^2 x} + \frac{\partial u}{\partial^2 y}$ = 0

$\frac{\partial u}{\partial x^2} = \frac{\partial v}{\partial y}$

cos hx sin x $\partial_y = \partial_v$

$\int dv$ = cos hx $\int sin$ y dy

v = -cos hx cos y + $\varphi\ (x)$

$\frac{\partial v}{\partial x}$ -sin x cos y + $\acute{\varphi}\ (x)$

$\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

Sin hx cos y = sin hx cos -$\acute{\varphi}\ (x)$

$\int \varphi\ (x) = \int \theta$ ⟶ $\varphi\ (x)$ → sub in (1)

v = -cos hx cos y +c

f (z) = u + vi

H.W /Prof that u (x , y) = $y^3 - 3xy^2$ harmonic function and find heroinic conjugate .