13/11/2023
Mon.
GROUP:A

$(s^2 + 9s + 8) = 0$

$s_1 =$

$s_2 =$

$(s + a_1)(s + b_1) = s^2 + (a_1 + b_1)s + a_1 b_1$

$= \boxed{as^2 + bs + c} = 0$

$1s^2 + 9s + 8$

$c = 8$

$b = 9$

$a = 1$

$$s_1, s_2 = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \mp \sqrt{81 - 4*1*8}}{2}$$

$$= \frac{-9 \mp \sqrt{49}}{2}$$

$$= \frac{-9 \mp 7}{2}$$

$$s_1 = \frac{-16}{2} = -8$$

$$s_2 = \frac{-2}{2} = -1$$

81
32
49

$(s_1 + 8)(s + 1)$

$$Y(s) = \frac{1}{s^2+9s+8} = \frac{A}{s+8} + \frac{B}{s+1}$$

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 8y = \delta(t) \qquad y(t) = ?$$

$$s^2 Y(s) + 9s\,Y(s) + 8\,Y(s) = 1$$

$$Y(s)\left[s^2+9s+8\right] = 1 \Rightarrow Y(s) = \frac{1}{s^2+9s+8}$$

$$Y(s) = \frac{1}{(s+8)(s+1)}$$

$$y(t) = \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1}\frac{1}{(s+8)(s+1)} = \mathcal{L}^{-1}\left[\frac{A}{s+8}\right] + \mathcal{L}^{-1}\left[\frac{B}{s+1}\right]$$

Hint: $\mathcal{L}\left[e^{-at}\right] = \frac{1}{s+a}$

$$\frac{1}{s^2+9s+8} = \frac{A}{s+8} + \frac{B}{s+1}$$

$$= \frac{A(s+1)+B(s+8)}{(s+8)(s+1)}$$

$$= \frac{As+A+Bs+8B}{(s+8)(s+1)}$$

$A + 8B = 1$ — (1)

$A + B = 0$ — (2)

$A = -B$

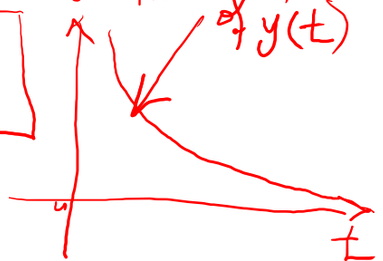
$-B + 8B = 1 \qquad B = \frac{1}{7}\ ;\ A = -\frac{1}{7}$

$$Y(s) = \frac{-1/7}{s+8} + \frac{1/7}{s+1}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{-1/7}{s+8}\right] + \mathcal{L}^{-1}\left[\frac{1/7}{s+1}\right]$$

$$y(t) = -\tfrac{1}{7}e^{-8t} + \tfrac{1}{7}e^{-t}$$

$$= \tfrac{1}{7}\left[e^{-t} - e^{-8t}\right]$$

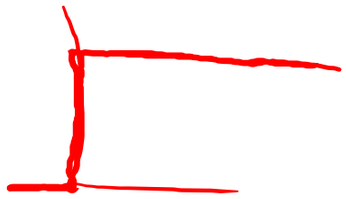


$y(t) = ?$

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 8y(t) = 12 \longleftarrow \text{step Function}$$

$$\mathcal{L}[A] = \frac{A}{s}$$

$$\mathcal{L}[12] = \frac{12}{s}$$



$$\left[s^2 + 9s + 8\right] Y(s) = \frac{12}{s} \qquad X_i = 12\,; t \geq 0$$

$$Y(s) = \frac{12}{s(s^2+9s+8)} = \frac{12}{s(s+8)(s+1)}$$

$$= \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+1}$$

$$A = \left.\frac{12}{(s+8)(s+1)}\right|_{s=0} = \frac{12 \times 3}{8 \times 2} = \frac{3}{2} = 1.5$$

$$B = \left.\frac{12}{s(s+1)}\right|_{s=-8} = \frac{12}{-8(-7)} = \frac{12}{56}$$

$$C = \left.\frac{12}{s(s+8)}\right|_{s=-1} = \frac{12}{(-1)(7)} = -\frac{12}{7}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{A}{s}\right] + \mathcal{L}^{-1}\left[\frac{B}{s+8}\right] + \mathcal{L}^{-1}\frac{C}{s+1}$$

$$= 1.5 + \frac{12}{56}e^{-8t} - \frac{12}{7}e^{-t}$$