## EQUILIBRIUM

we defined equilibrium as the condition in which the result ant of all forces and moments acting on a body is zero.

$$
\Sigma F_{x}=0 \quad \Sigma F_{y}=0 \quad \Sigma M_{O}=0
$$

| CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS |  |  |
| :---: | :---: | :---: |
| Force System | Free-Body Diagram | Independent Equations |
| 1. Collinear |  | $\Sigma F_{x}=0$ |
| 2. Concurrent at a point |  | $\begin{aligned} & \Sigma F_{x}=0 \\ & \Sigma F_{y}=0 \end{aligned}$ |
| 3. Parallel |  | $\Sigma F_{x}=0 \quad \Sigma M_{z}=0$ |
| 4. General |  | $\begin{aligned} & \Sigma F_{x}=0 \quad \Sigma M_{z}=0 \\ & \Sigma F_{y}=0 \end{aligned}$ |

## Problem 1

Determine the magnitudes of the forces C and T , which, along with the other three forces shown, act on the bridge truss joint.


Solution
$\left[\Sigma \mathrm{F}_{\mathrm{x}}=0\right] \quad 8+\mathrm{T} \cos 40+\mathrm{C} \sin 20-16=0$
$0.766 \mathrm{~T}+0.342 \mathrm{C}=8$
$\left[\Sigma \mathrm{F}_{\mathrm{y}}=0\right] \quad T \sin 40-\mathrm{C} \cos 20-3=0$
$0.643 \mathrm{~T}-0.940 \mathrm{C}=3$

Simultaneous solution of Equations (1) and (2) produces

$$
\mathrm{T}=9.09 \mathrm{kN} \quad \mathrm{C}=3.03 \mathrm{kN}
$$

## Problem 2

Calculate the tension $T$ in the cable which supports the 1000 lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.


Solution
$M_{o}=0$
$\mathrm{T}_{1} \mathrm{r}-\mathrm{T}_{2} \mathrm{r}=0$
$\mathrm{T}_{1}=\mathrm{T} 2$
$\Sigma \mathrm{F}_{\mathrm{y}}=0 \quad \mathrm{~T}_{1}+\mathrm{T}_{2}-1000=0 \quad 2 \mathrm{~T}_{1}=1000 \quad \mathrm{~T}_{1}=\mathrm{T}_{2}=500 \mathrm{Ib}$
$\mathrm{T}_{3}=\mathrm{T}_{4}=\mathrm{T}_{2} / 2=250 \mathrm{Ib}$
$\mathrm{T}=\mathrm{T}_{3}$ or $\mathrm{T}=250 \mathrm{Ib}$

## Problem 3

The uniform 100 -kg I-beam is supported initially by its end roller s on the horizontal surface at $A$ and $B$. By mean s of the cable at $C$ it is desired to elevate end $B$ to a position 3 m above end A . Determine the required tension P , the reaction at A , and the angle 8 made by the beam with the horizontal in the elevated position.


Solution


Moment equilibrium about A eliminates force $R$ and gives
$\Sigma \mathrm{M}_{0}=0 \quad \mathrm{P}(6 \cos \theta)-981(4 \cos \theta)=0 \quad \mathrm{P}=654 \mathrm{~N}$
Equilibrium of vertical forces requires
$\Sigma \mathrm{F}_{\mathrm{y}}=0$
$654+\mathrm{R}-981=0$
$\mathrm{R}=327 \mathrm{~N}$
The angle $\theta$ depends only on the specified geometry and is
$\sin \theta=3 / 8 \quad \theta=22^{\circ}$

## Problem 4

Determine the magnitude $T$ of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam $A B$ is a standard 0.5 m I-beam with a mass of 95 kg per mete r of length.


Solution


Free-body diagram

$$
\sum M_{A}=0
$$

$\left(\mathrm{T} \cos 25^{\circ}\right) 0.25+\left(\mathrm{T} \sin 25^{\circ}\right)(5-0.12)-10(5-1.5-0.12)-4.66(2.5-0.12)=0$
$\mathrm{T}=19.61 \mathrm{kN}$
Equating the sums of forces in the x - and y -directions to zero gives
$\Sigma \mathrm{F}_{\mathrm{x}}=0$
$\mathrm{A}_{\mathrm{x}}-19.61 \cos 25^{\circ}=0$
$\mathrm{A}_{\mathrm{x}}=17.77 \mathrm{kN}$
$\Sigma \mathrm{F}_{\mathrm{y}}=0$
$\mathrm{A}_{\mathrm{y}}+19.61 \sin 25^{\circ}-4.66-10=0$
$\mathrm{A}_{\mathrm{y}}=6.37 \mathrm{kN}$
$A=\sqrt{A_{x}^{2}+A_{y}^{2}} \quad A=\sqrt{17.77^{2}+6.37^{2}}$
$\mathrm{A}=18.88 \mathrm{KN}$

## Problem 5

The mass center G of the $1400-\mathrm{kg}$ rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.


Solution


$$
\begin{aligned}
& +\uparrow F=0: 2 N_{f}+2 N_{r}-1400(9.81)=0 \\
& +\Sigma M_{F}=0:-1400(9.81)(1386)+2 N_{r}(1386+964)=0 \\
& \text { Solution: }\left\{\begin{array}{l}
N_{f}=2820 \mathrm{~N} \\
N_{r}=4050 \mathrm{~N}
\end{array}\right.
\end{aligned}
$$

