Answers:

1. 33.3 kg
2. $178 \mathrm{~kg} / \mathrm{hr}$
3. (a) $28 \% \mathrm{Na}_{2} \mathrm{SO}_{4}$; (b) 33.3
4. Salt: 0.00617; Oil: 0.99393 .

### 2.4 The Chemical Reaction Equation and Stoichiometry

## Stoichiometry

- The stoichiometric coefficients in the chemical reaction equation

$$
\left.\mathrm{C}_{7} \mathrm{H}_{16}(\ell)+11 \mathrm{O}_{2}(\mathrm{~g}) \rightarrow 7 \mathrm{CO}_{2}(\mathrm{~g})+8 \mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \mathrm{C}_{7} \mathrm{H}_{16}, 11 \text { for } \mathrm{O}_{2} \text { and so on }\right) .
$$

- Another way to use the chemical reaction equation is to indicate that $\mathbf{1}$ mole of $\mathbf{C O}_{2}$ is formed from each (1/7) mole of $\mathrm{C}_{7} \mathrm{H}_{16}$, and $\mathbf{1}$ mole of $\mathrm{H}_{2} \mathrm{O}$ is formed with each (7/8) mole of $\mathrm{CO}_{2}$. The latter ratios indicate the use of stoichiometric ratios in determining the relative proportions of products and reactants.

For example how many kg of $\mathrm{CO}_{2}$ will be produced as the product if 10 kg of $\mathrm{C}_{7} \mathrm{H}_{16}$ react completely with the stoichiometric quantity of $\mathrm{O}_{2}$ ? On the basis of 10 kg of $\mathrm{C}_{7} \mathrm{H}_{6}$

$$
10 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}\left|\frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{7} \mathrm{H}_{16}\right| \frac{7 \mathrm{~kg} \mathrm{~mol} \mathrm{CO}}{2} \text { }\left|\frac{44.0 \mathrm{~kg} \mathrm{CO}_{2}}{100.1 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}}\right| \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{7} \text { H } \mathrm{H}_{16} 6.8 \mathrm{~kg} \mathrm{CO}_{2}
$$

## Example 15

The primary energy source for cells is the aerobic catabolism (oxidation) of glucose $\left(\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}\right.$, a sugar). The overall oxidation of glucose produces $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ by the following reaction

$$
\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}+a \mathrm{O}_{2} \rightarrow b \mathrm{CO}_{2}+c \mathrm{H}_{2} \mathrm{O}
$$

Determine the values of $a, b$, and $c$ that balance this chemical reaction equation.

## Solution

## Basis: The given reaction

By inspection, the carbon balance gives $b=6$, the hydrogen balance gives $c=6$, and an oxygen balance

$$
6+2 a=6 * 2+6
$$

Gives $\mathrm{a}=6$. Therefore, the balanced equation is

$$
\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}+6 \mathrm{O}_{2} \rightarrow 6 \mathrm{CO}_{2}+6 \mathrm{H}_{2} \mathrm{O}
$$

## Example 16

In the combustion of heptane, $\mathrm{CO}_{2}$ is produced. Assume that you want to produce 500 kg of dry ice per hour, and that $50 \%$ of the $\mathrm{CO}_{2}$ can be converted into dry ice, as shown in Figure E9.2. How many kilograms of heptane must be burned per hour? (MW: $\mathrm{CO}_{2}=44$ and $\mathrm{C}_{7} \mathrm{H}_{16}=100.1$ )


Figure E9.2

## Solution

The chemical equation is

$$
\mathrm{C}_{7} \mathrm{H}_{16}+11 \mathrm{O}_{2} \rightarrow 7 \mathrm{CO}_{2}+8 \mathrm{H}_{2} \mathrm{O}
$$

## Basis: 500 kg of dry ice (equivalent to $\mathbf{1 ~ h r}$ )

The calculation of the amount of $\mathrm{C}_{7} \mathrm{H}_{16}$ can be made in one sequence:

$$
\left.\begin{gathered}
500 \mathrm{~kg} \text { dry ice }
\end{gathered}\left|\frac{1 \mathrm{~kg} \mathrm{CO}_{2}}{0.5 \mathrm{~kg} \text { dry ice }}\right| \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{CO}}{2}\left|\frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}_{7} \mathrm{H}_{16}}{44.0 \mathrm{~kg} \mathrm{CO}_{2}}\right| \frac{100.1 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}}{7 \mathrm{~kg} \mathrm{~mol} \mathrm{CO}_{2}} \right\rvert\, \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{7} \text { H } \mathrm{H}_{16} \quad 325 \mathrm{~kg} \mathrm{C} \mathrm{C}_{7} \mathrm{H}_{16}
$$

## Example 17

A limestone analyses (weight \%): $\mathrm{CaCO}_{3} 92.89 \%, \mathrm{MgCO}_{3} 5.41 \%$ and Inert 1.70\%
By heating the limestone you recover oxides known as lime.
(a) How many pounds of calcium oxide can be made from 1 ton of this limestone?
(b) How many pounds of $\mathrm{CO}_{2}$ can be recovered per pound of limestone?
(c) How many pounds of limestone are needed to make 1 ton of lime?

Mol. Wt.: $\mathrm{CaCO}_{3}(100.1) \quad \mathrm{MgCO}_{3}(84.32) \quad \mathrm{CaO}(56.08) \quad \mathrm{MgO}(40.32) \quad \mathrm{CO}_{2}(44.0)$

## Solution



Chemical Equation:

$$
\begin{aligned}
\mathrm{CaCO}_{3} & \rightarrow \mathrm{CaO}+\mathrm{CO}_{2} \\
\mathrm{MgCO}_{3} & \rightarrow \mathrm{MgO}+\mathrm{CO}_{2}
\end{aligned}
$$

Basis: 100 lb of limestone

| Limestone |  |  | Solid Products |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Component | $l b=$ percent | lb mol | Compound | lb mol | lb |
| $\mathrm{CaCO}_{3}$ | 92.89 | 0.9280 | CaO | 0.9280 | 52.04 |
| $\mathrm{MgCO}_{3}$ | 5.41 | 0.0642 | MgO | 0.0642 | 2.59 |
| Inert | 1.70 |  | Inert |  | 1.70 |
| Total | 100.00 | 0.95920 | Total | 0.9920 | 56.33 |

The quantities listed under Products are calculated from the chemical equations. For example, for the last column:

The production of $\mathbf{C O}_{2}$ is:
0.9280 lb mol CaO is equivalent to $0.9280 \mathrm{lb} \mathrm{mol} \mathrm{CO} \mathbf{C O}_{2}$
$0.0642 \mathrm{lb} \mathbf{~ m o l ~} \mathbf{~ M g O}$ is equivalent to $\mathbf{0 . 0 6 4 2} \mathbf{~ l b ~ m o l ~ C O} \mathbf{C O}_{2}$
Total lb mol CO $2=0.9280+0.0642=\mathbf{0 . 9 9 2} \mathbf{l b ~ m o l ~ C O} 2$

$$
\xlongequal{0.992 \mathrm{lb} \mathrm{~mol} \mathrm{CO}} 2 \left\lvert\, \frac{44.0 \mathrm{lb} \mathrm{CO}_{2}}{1 \mathrm{lb} \mathrm{~mol} \mathrm{CO}} 2 \mathrm{Cl}\right.
$$

Alternately, you could have calculated the lb $\mathrm{CO}_{2}$ from a total balance: $\mathbf{1 0 0} \mathbf{- 5 6 . 3 3}=\mathbf{4 4 . 6 7}$.
Now, to calculate the quantities originally asked for:
(a) CaO produced $=\quad \frac{52.04 \mathrm{lb} \mathrm{CaO}}{100 \mathrm{lb} \text { limestone }} \left\lvert\, \frac{2000 \mathrm{lb}}{1 \mathrm{ton}}=1041 \mathrm{lb} \mathrm{CaO} / \mathrm{ton}\right.$
(b) $\mathrm{CO}_{2}$ recovered $=\frac{43.65 \mathrm{lb} \mathrm{CO}_{2}}{100 \mathrm{lb} \text { limestone }}=0.437 \mathrm{lb} \mathrm{CO}_{2} / \mathrm{lb}$ limestone
(c) Limestone required $=\quad \frac{100 \mathrm{lb} \text { limestone }}{56.33 \mathrm{lb} \text { lime }} \left\lvert\, \frac{2000 \mathrm{lb}}{1 \mathrm{ton}}=\begin{aligned} & 3550 \mathrm{lb} \text { limestone/ } \\ & \text { ton lime }\end{aligned}\right.$

## Terminology for Applications of Stoichiometry

## Extent of Reaction

The extent of reaction, $\xi$, is based on a particular stoichiometric equation, and denotes how much reaction occurs.
The extent of reaction is defined as follows:

$$
\xi=\frac{n_{i}-n_{i o}}{v_{i}}
$$

Where:
$n_{i}=$ moles of species $i$ present in the system after the reaction occurs
$n_{i o}=$ moles of species $i$ present in the system when the reaction starts
$v_{i}=$ coefficient for species $i$ in the particular chemical reaction equation (moles of species $i$ produced or consumed per moles reacting)
$\xi=$ extent of reaction (moles reacting)

- The coefficients of the products in a chemical reaction are assigned positive values and the reactants assigned negative values. Note that $\left(n_{i}-n_{i o}\right)$ is equal to the generation or consumption of component $i$ by reaction.

Equation (9.1) can be rearranged to calculate the number of moles of component $i$ from the value of the extent of reaction

$$
n_{i}=n_{i 0}+\xi v_{i}
$$

## Example 18

Determine the extent of reaction for the following chemical reaction

$$
\mathrm{N}_{2}+3 \mathrm{H}_{2} \rightarrow 2 \mathrm{NH}_{3}
$$ given the following analysis of feed and product:

|  | $N_{2}$ | $H_{2}$ | $\mathrm{NH}_{3}$ |
| :--- | :--- | :--- | :--- |
| Feed | 100 g | 50 g | 5 g |
| Product |  |  | 90 g |

Also, determine the $g$ and $g$ mol of $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$ in the product.

## Solution

The extent of reaction can be calculated by applying Equation 9.1 based on $\mathrm{NH}_{3}$ :

$$
\begin{aligned}
& \left.n_{i}=\frac{90 \mathrm{~g} \mathrm{NH}_{3}}{} \right\rvert\, \frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{NH}}{3} \\
& 17 \mathrm{~g} \mathrm{NH}_{3}
\end{aligned}=5.294 \mathrm{~g} \mathrm{~mol} \mathrm{NH}_{3} .
$$

Equation 9.2 can be used to determine the g mol of $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$ in the products of the reaction

$$
\begin{aligned}
\mathrm{N}_{2}: \quad n_{i 0} & =\frac{100 \mathrm{~g} \mathrm{~N}_{2}}{} \left\lvert\, \frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}}{28 \mathrm{~g} \mathrm{~N}_{2}}=3.57 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}\right. \\
n_{\mathrm{N}_{2}} & =3.57+(-1)(2.5)=1.07 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2} \\
m_{\mathrm{N}_{2}} & =\frac{1.07 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}}{} \left\lvert\, \frac{28 \mathrm{~g} \mathrm{~N}_{2}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}}=30 \mathrm{~g} \mathrm{~N}_{2}\right. \\
\mathrm{H}_{2}: \quad n_{i 0} & =\frac{50 \mathrm{~g} \mathrm{H}_{2}}{} \left\lvert\, \frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{H}_{2}}{2 \mathrm{~g} \mathrm{H}_{2}}=25 \mathrm{~g} \mathrm{~mol} \mathrm{H}_{2}\right. \\
\mathrm{n}_{\mathrm{H} 2} & =25+(-3)(2.5)=17.5 \mathrm{~g} \mathrm{~mol} \mathrm{H}
\end{aligned}
$$

Note: If several independent reactions occur in the reactor, say $k$ of them, $\xi$ can be defined for each reaction, with $v_{k i}$ being the stoichiometric coefficient of species $i$ in the $k$ th reaction, the total number of moles of species $i$ is

$$
n_{i}=n_{i 0}+\sum_{k=1}^{R} v_{k i} \xi_{k}
$$

Where $R$ is the total number of independent reactions.

## Limiting and Excess Reactants

* The excess material comes out together with, or perhaps separately from, the product, and sometimes can be used again.
* The limiting reactant is the species in a chemical reaction that would theoretically run out first (would be completely consumed) if the reaction were to proceed to completion according to the chemical equationeven if the reaction does not proceed to completion! All the other reactants are called excess reactants.

* For example, using the chemical reaction equation in Example 9.2,

$$
\mathrm{C}_{7} \mathrm{H}_{16}+11 \mathrm{O}_{2} \rightarrow 7 \mathrm{CO}_{2}+8 \mathrm{H}_{2} \mathrm{O}
$$

If $\mathbf{1 g ~ m o l}$ of $\mathrm{C}_{\mathbf{7}} \mathrm{H}_{\mathbf{1 6}}$ and $\mathbf{1 2} \mathbf{g ~ m o l}$ of $\mathrm{O}_{\mathbf{2}}$ are mixed.

As a straightforward way of determining the limiting reactant, you can determine the maximum extent of reaction, $\xi^{\max }$, for each reactant based on the complete reaction of the reactant. The reactant with the smallest maximum extent of reaction is the limiting reactant. For the example, for $\mathbf{g ~ m o l}$ of $\mathbf{C}_{7} \mathbf{H}_{16}$ plus $\mathbf{1 2 g}$ mole of $\mathrm{O}_{2}$, you calculate

$$
\begin{aligned}
& \xi^{\max }\left(\text { based on } \mathrm{O}_{2}\right)=\frac{0 \mathrm{~g} \operatorname{mol~\mathrm {O}_{2}}-12 \mathrm{~g} \operatorname{mol} \mathrm{O}_{2}}{-11 \mathrm{~g} \mathrm{~mol} \mathrm{O}_{2} / \text { moles reacting }=1.09 \text { moles reacting }} \\
& \xi^{\max }\left(\text { based on } \mathrm{C}_{7} \mathrm{H}_{16}\right)=\frac{0 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{7} \mathrm{H}_{16}-1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{7} \mathrm{H}_{16}}{-1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{7} \mathrm{H}_{16} / \text { moles reacting }}=1.00 \text { moles reacting }
\end{aligned}
$$

Therefore, heptane is the limiting reactant and oxygen is the excess reactant.

As an alternate to determining the limiting reactant,

$$
\frac{\mathrm{O}_{2}}{\frac{\text { Ratio in feed }}{\mathrm{C}_{7} \mathrm{H}_{16}}:} \frac{12}{1}=12 \quad>\quad \frac{\text { Ratio in chemical equation }}{1}=11
$$

$$
\text { * Consider the following reaction } \quad \mathrm{A}+3 \mathrm{~B}+2 \mathrm{C} \rightarrow \text { Products }
$$

If the feed to the reactor contains $\mathbf{1 . 1}$ moles of $\mathbf{A}, \mathbf{3 . 2}$ moles of $\mathbf{B}$, and $\mathbf{2 . 4}$ moles of $\mathbf{C}$. The extents of reaction based on complete reaction of $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are

$$
\begin{aligned}
& \xi^{\max }(\text { based on } \mathrm{A})=\frac{-1.1 \mathrm{~mol} \mathrm{~A}}{-1}=1.1 \\
& \xi^{\max }(\text { based on } \mathrm{B})=\frac{-3.2 \mathrm{~mol} \mathrm{~B}}{-3}=1.07 \\
& \xi^{\max }(\text { based on } \mathrm{C})=\frac{-2.4 \mathrm{~mol} \mathrm{C}}{-2}=1.2
\end{aligned}
$$

As a result, $\mathbf{B}$ is identified as the limiting reactant in this example while $\mathbf{A}$ and $\mathbf{C}$ are the excess reactants.
 We choose A as the reference substance and calculate Limiting اذا كانت النببة في ال معايلة الكيميائية اكبز فالبسط هـ

$$
\begin{array}{ccc} 
& \frac{\text { Ratio in feed }}{3.2}=2.91 & < \\
\frac{\mathrm{B}}{\mathrm{~A}}: & \frac{\text { Ratio in chemical equation }}{1.1}=\frac{3}{1}=3 \\
\frac{\mathrm{C}}{\mathrm{~A}}: & \frac{2.4}{1.1}=2.18 & >
\end{array}
$$

We conclude that $\mathbf{B}$ is the limiting reactant relative to $\mathbf{A}$, and that $\mathbf{A}$ is the limiting reactant relative to $\mathbf{C}$, hence $\mathbf{B}$ is the limiting reactant among the set of three reactant. In symbols we have $\mathbf{B}<\mathbf{A}, \mathbf{C}>\mathbf{A}$ (i.e., $\mathbf{A}<\mathbf{C}$ ), so that $\mathbf{B}<\mathbf{A}<$ C.

## Example 19

If you feed 10 grams of $\mathrm{N}_{2}$ gas and 10 grams of $\mathrm{H}_{2}$ gas into a reactor:
a. What is the maximum number of grams of $\mathrm{NH}_{3}$ that can be produced?
b. What is the limiting reactant?
c. What is the excess reactant?

## Solution

|  |  |  | $\mathrm{NH}_{3}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 4 |  |  |
|  | $\begin{gathered} \mathrm{N}_{2}(\mathrm{~g}) \\ 10 \mathrm{~g} \end{gathered}$ |  | Reac |  | $\begin{aligned} & \mathrm{H}_{2}(\mathrm{~g}) \\ & 10 \mathrm{~g} \end{aligned}$ |
|  | $\mathrm{N}_{2}(\mathrm{~g})$ | + | $3 \mathrm{H}_{2}(\mathrm{~g})$ | $\rightarrow$ | $2 \mathrm{NH}_{3}(\mathrm{~g})$ |
| Given g : | 10 | 10 |  | 0 |  |
| MW: | 28 | 2.016 |  | 17.02 |  |
| Calculated g mol: | 0.357 | 4.960 |  | 0 |  |

(b) $\mathrm{N}_{2}$ is the limiting reactant, and that (c) $\mathrm{H}_{2}$ is the excess reactant.

The excess $\mathrm{H}_{2}=4.960-3(0.357)=3.89 \mathrm{~g}$ mol. To answer question (a), the maximum amount of $\mathrm{NH}_{3}$ that can be produced is based on assuming complete conversion of the limiting reactant

$$
\underline{0.357 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}} \left\lvert\, \frac{2 \mathrm{~g} \mathrm{~mol} \mathrm{NH}}{3}+\frac{17.02 \mathrm{~g} \mathrm{NH}_{3}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}} 2 \mathrm{l}\right.
$$

## Conversion and degree of completion

$\boxtimes$ Conversion is the fraction of the feed or some key material in the feed that is converted into products.
Q Conversion is related to the degree of completion of a reaction namely the percentage or fraction of the limiting reactant converted into products.

Thus, percent conversion is

$$
\% \text { conversion }=\frac{\text { moles (or mass) of feed (or a compound in the feed) that react }}{\text { moles (or mass) of feed (or a component in the feed) introduced }} \times 100
$$

For example, for the reaction equation described in Example 16, if 14.4 kg of $\mathrm{CO}_{2}$ are formed in the reaction of 10 kg of $\mathrm{C}_{7} \mathrm{H}_{16}$, you can calculate what percent of the $\mathrm{C}_{7} \mathrm{H}_{16}$ is converted to $\mathrm{CO}_{2}$ (reacts) as follows:

$$
\mathrm{C}_{7} \mathrm{H}_{16}+11 \mathrm{O}_{2} \rightarrow 7 \mathrm{CO}_{2}+8 \mathrm{H}_{2} \mathrm{O}
$$

$\mathrm{C}_{7} \mathrm{H}_{16}$ equivalent to $\mathrm{CO}_{2}$ in the product
الكمية من المعاينلة
$\mathrm{C}_{7} \mathrm{H}_{16}$ in the reactants

$$
\frac{10 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}}{\left\lvert\, \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{7} \mathrm{H}_{16}\right.} \frac{100.1 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}}{10.0999 \mathrm{~kg} \mathrm{~mol} \mathrm{C}_{7} \mathrm{H}_{16}}
$$

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$$
\% \text { conversion }=\frac{0.0468 \mathrm{~mol} \text { reacted }}{0.0999 \mathrm{~kg} \mathrm{~mol} \mathrm{fed}} 100=46.8 \% \text { of the } \mathrm{C}_{7} \mathrm{H}_{16}
$$

$$
\begin{aligned}
& \xi^{\max }\left(\text { based on } \mathrm{N}_{2}\right)=\frac{-0.357 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2}}{-1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}_{2} / \text { moles reacting }}=0.357 \text { moles reacting } \\
& \xi^{\max }\left(\text { based on } \mathrm{H}_{2}\right)=\frac{-4.960 \mathrm{~g} \mathrm{~mol} \mathrm{H}_{2}}{-3 \mathrm{~g} \mathrm{~mol} \mathrm{H}} \mathrm{H}_{2} / \text { moles reacting } \quad=1.65 \text { moles reacting }
\end{aligned}
$$

® The conversion can also be calculated using the extent of reaction as follows:
Conversion is equal to the extent of reaction based on $\mathrm{CO}_{2}$ formation (i.e., the actual extent of reaction) divided by the extent of reaction assuming complete reaction of $\mathrm{C}_{7} \mathrm{H}_{16}$ (i.e., the maximum possible extent of reaction).

## Conversion $=\frac{\text { extent of reaction that actually occurs }}{\text { extent of reaction that would occur if complete reaction took place }}$

$$
=\frac{\xi}{\xi^{\max }}
$$

## Selectivity

Selectivity is the ratio of the moles of a particular (usually the desired) product produced to the moles of another (usually undesired or by-product) product produced in a set of reactions.

For example, methanol $\left(\mathrm{CH}_{3} \mathrm{OH}\right)$ can be converted into ethylene $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)$ or propylene $\left(\mathrm{C}_{3} \mathrm{H}_{6}\right)$ by the reactions

$$
\begin{aligned}
& 2 \mathrm{CH}_{3} \mathrm{OH} \rightarrow \mathrm{C}_{2} \mathrm{H}_{4}+2 \mathrm{H}_{2} \mathrm{O} \\
& 3 \mathrm{CH}_{3} \mathrm{OH} \rightarrow \mathrm{C}_{3} \mathrm{H}_{6}+3 \mathrm{H}_{2} \mathrm{O}
\end{aligned}
$$

What is the selectivity of $\mathrm{C}_{2} \mathrm{H}_{4}$ relative to the $\mathrm{C}_{3} \mathrm{H}_{6}$ at $80 \%$ conversion of the $\mathrm{CH}_{3} \mathrm{OH}$ ? At $80 \%$ conversion: $\mathrm{C}_{2} \mathrm{H}_{4} 19$ mole $\%$ and for $\mathrm{C}_{3} \mathrm{H}_{6} 8$ mole $\%$. Because the basis for both values is the same, the selectivity $=\mathbf{1 9 / 8}=\mathbf{2} .4 \mathbf{~ m o l} \mathbf{C}_{2} \mathbf{H}_{4}$ per $\mathrm{mol} \mathrm{C}_{3} \mathrm{H}_{6}$.

## Yield

No universally agreed-upon definitions exist for yield-in fact, quite the contrary. Here are three common ones:

- Yield (based on feed) - the amount (mass or moles) of desired product obtained divided by the amount of the key (frequently the limiting) reactant fed.
- Yield (based on reactant consumed) -the amount (mass or moles) of desired product obtained divided by amount of the key (frequently the limiting) reactant consumed.
- Yield (based on theoretical consumption of the limiting reactant)-the amount (mass or moles) of a product obtained divided by the theoretical (expected) amount of the product that would be obtained based on the limiting reactant in the chemical reaction equation if it were completely consumed.


## Example 20

The following overall reaction to produce biomass, glycerol, and ethanol

$$
\begin{aligned}
\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\text { glucose }) & +0.118 \mathrm{NH}_{3} \rightarrow 0.59 \mathrm{CH}_{1.74} \mathrm{~N}_{0.2} \mathrm{O}_{0.45}(\text { biomass }) \\
& \left.+0.43 \mathrm{C}_{3} \mathrm{H}_{8} \mathrm{O}_{3}(\text { glycerol })+1.54 \mathrm{CO}_{2}+1.3 \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH} \text { (ethanol }\right)+0.03 \mathrm{H}_{2} \mathrm{O}
\end{aligned}
$$

Calculate the theoretical yield of biomass in $g$ of biomass per $g$ of glucose. Also, calculate the yield of ethanol in $g$ of ethanol per $g$ of glucose.

## Solution

Basis: 0.59 g mol of biomass

$$
\begin{aligned}
& \frac{0.59 \mathrm{~g} \text { mol biomass }}{1 \mathrm{~g} \mathrm{~mol} \text { glucose }}\left|\frac{23.74 \mathrm{~g} \text { biomass }}{1 \mathrm{~g} \mathrm{~mol} \text { biomass }}\right| \frac{1 \mathrm{~g} \text { mol glucose }}{180 \mathrm{~g} \text { glucose }}=0.0778 \mathrm{~g} \text { biomass } / \mathrm{g} \text { glucose } \\
& \frac{1.3 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}}{1 \mathrm{~g} \mathrm{~mol} \text { glucose }}\left|\frac{46 \mathrm{~g} \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}}\right| \frac{1 \mathrm{~g} \mathrm{~mol} \text { glucose }}{180 \mathrm{~g} \text { glucose }}=0.332 \mathrm{~g} \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH} / \mathrm{g} \text { glucose }
\end{aligned}
$$

## Example 21

For this example, large amounts of single wall carbon nanotubes can be produced by the catalytic decomposition of ethane over Co and Fe catalysts supported on silica

$$
\begin{gather*}
\mathrm{C}_{2} \mathrm{H}_{6} \rightarrow 2 \mathrm{C}+3 \mathrm{H}_{2}  \tag{a}\\
\searrow \mathrm{C}_{2} \mathrm{H}_{4}+\mathrm{H}_{2} \tag{b}
\end{gather*}
$$

If you collect 3 g mol of $\mathrm{H}_{2}$ and 0.50 g mol of $\mathrm{C}_{2} \mathrm{H}_{4}$, what is the selectivity of C relative to $\mathrm{C}_{2} \mathrm{H}_{4}$ ?

## Solution

> Basis: $3 \mathrm{~g} \mathrm{~mol} \mathrm{H} \mathrm{H}_{2}$ by Reaction (a)
> $\quad 0.50 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{2} \mathrm{H}_{4}$ by Reaction (b)

The 0.5 g mol of $\mathrm{C}_{2} \mathrm{H}_{4}$ corresponds to 0.50 g mol of $\mathrm{H}_{2}$ produced in Reaction (b).
The $\mathrm{H}_{2}$ produced by Reaction (a) $=3-0.50=2.5 \mathrm{~g} \mathrm{~mol}$.

The nanotubes (the C) produced by Reaction $(\mathrm{a})=(2 / 3)(2.5)=1.67 \mathrm{~g} \mathrm{~mol} \mathrm{C}$
The selectivity $=1.67 / 0.50=3.33 \mathrm{~g} \mathrm{~mol} \mathrm{C} / \mathrm{g} \mathrm{mol} \mathrm{C}_{2} \mathrm{H}_{4}$

## Example 22

The two reactions of interest for this example are

$$
\begin{align*}
& \mathrm{Cl}_{2}(\mathrm{~g})+\mathrm{C}_{3} \mathrm{H}_{6}(\mathrm{~g}) \rightarrow \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}(\mathrm{~g})+\mathrm{HCl}(\mathrm{~g})  \tag{a}\\
& \mathrm{Cl}_{2}(\mathrm{~g})+\mathrm{C}_{3} \mathrm{H}_{6}(\mathrm{~g}) \rightarrow \mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}(\mathrm{~g}) \tag{b}
\end{align*}
$$

$\mathrm{C}_{3} \mathrm{H}_{6}$ is propylene (propene) $(\mathrm{MW}=42.08)$
$\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}$ is allyl chloride (3-chloropropene) ( $\mathrm{MW}=76.53$ )
$\mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}$ is propylene chloride (1,2-dichloropropane) ( $\mathrm{MW}=112.99$ )
The species recovered after the reaction takes place for some time are listed in Table E9.8.

| species | $\mathrm{Cl}_{2}$ | $\mathrm{C}_{3} \mathrm{H}_{6}$ | $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ | $\mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}$ | HCl |
| :--- | :--- | :--- | :--- | :--- | :--- |
| g mol | 141 | 651 | 4.6 | 24.5 | 4.6 |

Based on the product distribution assuming that no allyl chlorides were present in the feed, calculate the following:
a. How much $\mathrm{Cl}_{2}$ and $\mathrm{C}_{3} \mathrm{H}_{6}$ were fed to the reactor in g mol?
b. What was the limiting reactant?
c. What was the excess reactant?
d. What was the fraction conversion of $\mathrm{C}_{3} \mathrm{H}_{6}$ to $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ ?
e. What was the selectivity of $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ relative to $\mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}$ ?
f. What was the yield of $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ expressed in $g$ of $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ to the g of $\mathrm{C}_{3} \mathrm{H}_{6}$ fed to the reactor?
g. What was the extent of reaction of the first and second reactions?

## Solution

Figure E9.8 illustrates the process as an open-flow system. A batch process could alternatively be used.


Figure E9.8
A convenient basis is what is given in the product list in Table E9.8.

## Reaction (a)

$$
\underline{4.6 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}} \left\lvert\, \frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}}{2}-1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl} ~=4.6 \mathrm{~g} \mathrm{~mol} \mathrm{Cl} l_{2}\right. \text { reacts }
$$

## Reaction (b)

$$
\xrightarrow[24.5 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}]{\mid} \left\lvert\, \frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}_{2}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{C} \mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}}=24.5 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}_{2}\right. \text { reacts }
$$

Total $=4.6+24.5=29.1 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}_{2}$ reacts
$\mathrm{Cl}_{2}$ in product $=141.0$ from Table E9.8
(a) Total $\mathrm{Cl}_{2}$ fed $=141.0+29.1=170.1 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}_{2}$

Total $\mathrm{C}_{3} \mathrm{H}_{6}$ fed $=651.0+29.1=680.1 \mathrm{~g} \mathrm{~mol}$ of $\mathrm{C}_{3} \mathrm{H}_{6}$
(b) and (c) Since both reactions involve the same value of the respective reaction stoichiometric coefficients, both reactions will have the same limiting and excess reactants

$$
\begin{aligned}
& \xi^{\max }\left(\text { based on } \mathrm{C}_{3} \mathrm{H}_{6}\right)=\frac{-680.1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6}}{-1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6} / \text { moles reacting }}=680.1 \text { moles reacting } \\
& \xi^{\max }\left(\text { based on } \mathrm{Cl}_{2}\right)=\frac{-170.1 \mathrm{~g} \text { mole } \mathrm{Cl}_{2}}{-1 \mathrm{~g} \mathrm{~mol} \mathrm{Cl}} / \text { moles reacting }=170.1 \text { moles reacting }
\end{aligned}
$$

Thus, $\mathbf{C}_{3} \mathbf{H}_{\mathbf{6}}$ was the excess reactant and $\mathbf{C l}_{\mathbf{2}}$ the limiting reactant.
(d) The fraction conversion of $\mathrm{C}_{3} \mathrm{H}_{6}$ to $\mathrm{C}_{3} \mathrm{H}_{5} \mathrm{C} 1$ was

$$
\frac{4.6 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6} \text { that reacted }}{680.1 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6} \text { fed }}=6.76 \times 10^{-3}
$$

(e) The selectivity was

$$
\frac{4.6 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}}{24.5 \mathrm{~g} \mathrm{~mol} \mathrm{C}_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}}=0.19 \frac{\mathrm{~g} \mathrm{~mol} \mathrm{C} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}}{\mathrm{~g} \mathrm{~mol} \mathrm{C}}{ }_{3} \mathrm{H}_{6} \mathrm{Cl}_{2}
$$

(f) The yield was

$$
\frac{(76.53)(4.6) \mathrm{g} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}}{(42.08)(680.1) \mathrm{g} \mathrm{C}_{3} \mathrm{H}_{6}}=0.012 \frac{\mathrm{~g} \mathrm{C}_{3} \mathrm{H}_{5} \mathrm{Cl}}{\mathrm{~g} \mathrm{C}_{3} \mathrm{H}_{6}}
$$

(g) Because $\mathbf{C}_{\mathbf{3}} \mathbf{H}_{\mathbf{5}} \mathbf{C 1}$ is produced only by the first reaction, the extent of reaction of the first reaction is

$$
\xi_{1}=\frac{n_{i}-n_{i o}}{v_{i}}=\frac{4.6-0}{1}=4.6
$$

Because $\mathbf{C}_{3} \mathbf{H}_{\mathbf{6}} \mathbf{C}_{\mathbf{1 2}}$ is produced only by the second reaction, the extent of reaction of the second reaction is

$$
\xi_{2}=\frac{n_{i}-n_{i o}}{v_{i}}=\frac{24.5-0}{1}=24.5
$$



## Example 23

Five pounds of bismuth ( $\mathrm{MW}=209$ ) is heated along with one pound of sulfur $(M W=32)$ to form $\mathrm{Bi}_{2} \mathrm{~S}_{3}(\mathrm{MW}=514)$. At the end of the reaction, the mass is extracted and the free sulfur recovered is $5 \%$ of the reaction mass. Determine

$$
2 \mathrm{Bi}+3 \mathrm{~S} \longrightarrow \mathrm{Bi}_{2} \mathrm{~S}_{3}
$$

5\% من كثلة الثفاعل الكلية

1. The limiting reactant.
2. The percent excess reactant.
3. The percent conversion of sulfur to $\mathrm{Bi}_{2} \mathrm{~S}_{3}$

## Solution

a. Find the Limiting reactant

$$
\begin{aligned}
& \text { Ratio in the feed }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Ratio in the chemical equation }=\frac{2 \mathrm{lbmol} \mathrm{Bi}}{3 \mathrm{lbmol} \mathrm{~S}} \\
& =0.667
\end{aligned}
$$

b. \% Excess reactant

$$
\begin{aligned}
& \text { Bi required }=\begin{array}{l|l|l}
1 \mathrm{lb} \mathrm{~S} & 1 \mathrm{lb} \mathrm{~mol} \mathrm{~S} & 2 \mathrm{~mol} \mathrm{Bi} \\
\hline 32 \mathrm{lb} \mathrm{~S} & 3 \mathrm{~mol} \mathrm{~S}
\end{array}=\quad 0.0208 \mathrm{lb} \mathrm{~mol} \mathrm{Bi} \\
& \% \text { excess } \mathrm{Bi}=\frac{(0.0239-0.028)}{0.028} \times 100=\mathbf{1 4 . 9 \%}
\end{aligned}
$$

c. We will assume that no gaseous products are formed, so that the total mass of the reaction mixture is conserved at $6 \mathrm{lb}(5 \mathrm{lbBi}+1 \mathrm{lbS})$. The free sulfur at the end of the reaction $=$ $5 \%$.

$$
\begin{aligned}
& \begin{array}{r|c|c}
6.00 \mathrm{lb} \mathrm{rxn} \mathrm{mass} & 5.00 \mathrm{lb} \mathrm{~S} & 1 \mathrm{lb} \mathrm{~mol} \mathrm{~S} \\
100 \mathrm{lb} \text { rxn mass } & 32.0 \mathrm{lb} \mathrm{~S}
\end{array}=0.00938 \mathrm{lb} \mathrm{~mol} \mathrm{~S} \\
& \% \text { Conversion }
\end{aligned} \begin{array}{r}
\frac{\text { moles of feed that react }}{\text { moles of feed introduced }} \times 100 \\
= \\
=\frac{0.0313-0.00938}{0.0313} \times \quad 100=\mathbf{7 0 . 0 \%}
\end{array}
$$

## Questions

1. What is a limiting reactant?
2. What is an excess reactant?
3. How do you calculate the extent of reaction from experimental data?

## Answers:

Q. 3 Reactant present in the least stoichiometric quantity.
Q. 4 All other reactants than the limiting reactant.
Q. 5 For a species in

Open system: $\quad \xi=\frac{n_{\mathrm{out}, i}-n_{\mathrm{in}, i}}{v_{i}} \quad$ Closed system: $\quad \xi=\frac{n_{\text {final }, i}-n_{\text {initial }, i}}{v_{i}}$

## Problems

1. Write balanced reaction equations for the following reactions:
a. $\quad \mathrm{C}_{9} \mathrm{H}_{18}$ and oxygen to form carbon dioxide and water.
b. $\mathrm{FeS}_{2}$ and oxygen to form $\mathrm{Fe}_{2} \mathrm{O}_{3}$ and sulfur dioxide.
2. If 1 kg of benzene $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$ is oxidized with oxygen, how many kilograms of $\mathrm{O}_{2}$ are needed to convert all the benzene to $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ ?
3. The electrolytic manufacture of chlorine gas from a sodium chloride solution is carried out by the following reaction:

$$
2 \mathrm{NaCl}+2 \mathrm{H}_{2} \mathrm{O} \rightarrow 2 \mathrm{NaOH}+\mathrm{H}_{2}+\mathrm{Cl}_{2}
$$

How many kilograms of $\mathrm{Cl}_{2}$ can be produced from $10 \mathrm{~m}^{3}$ of brine solution containing $5 \%$ by weight of NaCl ? The specific gravity of the solution relative to that of water at $4^{\circ} \mathrm{C}$ is 1.07 .
4. Can you balance the following chemical reaction equation?

$$
\mathrm{a}_{1} \mathrm{NO}_{3}+\mathrm{a}_{2} \mathrm{HClO} \rightarrow \mathrm{a}_{3} \mathrm{HNO}_{3}+\mathrm{a}_{4} \mathrm{HCl}
$$

5. For the reaction in which stoichiometric quantities of the reactants are fed

$$
2 \mathrm{C}_{5} \mathrm{H}_{10}+15 \mathrm{O}_{2} \rightarrow 10 \mathrm{CO}_{2}+10 \mathrm{H}_{2} \mathrm{O}
$$

and the reaction goes to completion, what is the maximum extent of reaction based on $\mathrm{C}_{5} \mathrm{H}_{10}$ ? On $\mathrm{O}_{2}$ ? Are the respective values different or the same? Explain the result.
6. Calcium oxide $(\mathrm{CaO})$ is formed by decomposing limestone (pure $\mathrm{CaCO}_{3}$ ). In one kiln the reaction goes to $70 \%$ completion.
a. What is the composition of the solid product withdrawn from the kiln?
b. What is the yield in terms of pounds of $\mathrm{CO}_{2}$ produced per pound of limestone fed into the process?
7. Aluminum sulfate can be made by reacting crushed bauxite ore with sulfuric acid, according to the following chemical equation:

$$
\mathrm{Al}_{2} \mathrm{O}_{3}+3 \mathrm{H}_{2} \mathrm{SO}_{4} \rightarrow \mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3}+3 \mathrm{H}_{2} \mathrm{O}
$$

The bauxite ore contains $55.4 \%$ by weight of aluminum oxide, the remainder being impurities. The sulfuric acid solution contains $77.7 \%$ pure sulfuric acid, the remainder being water. To produce crude aluminum sulfate containing 1798 lb of pure aluminum sulfate, 1080 lb of bauxite ore and 2510 lb of sulfuric acid solution are reacted.
a. Identify the excess reactant.
b. What percentage of the excess reactant was consumed?
c. What was the degree of completion of the reaction?
8. Two well-known gas phase reactions take place in the dehydration of ethane:

$$
\begin{align*}
\mathrm{C}_{2} \mathrm{H}_{6} & \rightarrow \mathrm{C}_{2} \mathrm{H}_{4}+\mathrm{H}_{2}  \tag{a}\\
\mathrm{C}_{2} \mathrm{H}_{6}+\mathrm{H}_{2} & \rightarrow 2 \mathrm{CH}_{4} \tag{b}
\end{align*}
$$

Given the product distribution measured in the gas phase reaction of $\mathrm{C}_{2} \mathrm{H}_{6}$ as follows
$\mathrm{C}_{2} \mathrm{H}_{6} 27 \%, \quad \mathrm{C}_{2} \mathrm{H}_{4} 33 \%, \quad \mathrm{H}_{2} 13 \%$, and $\mathrm{CH}_{4} 27 \%$
a. What species was the limiting reactant?
b. What species was the excess reactant?
c. What was the conversion of $\mathrm{C}_{2} \mathrm{H}_{6}$ to $\mathrm{CH}_{4}$ ?
d. What was the degree of completion of the reaction?
e. What was the selectivity of $\mathrm{C}_{2} \mathrm{H}_{4}$ relative to $\mathrm{CH}_{4}$ ?
f. What was the yield of $\mathrm{C}_{2} \mathrm{H}_{4}$ expressed in kg mol of $\mathrm{C}_{2} \mathrm{H}_{4}$ produced per kg mol of $\mathrm{C}_{2} \mathrm{H}_{6}$ ?
g. What was the extent of reaction of $\mathrm{C}_{2} \mathrm{H}_{6}$.

## Answers:

1. (a)

$$
\mathrm{C}_{9} \mathrm{H}_{18}+\frac{27}{2} \mathrm{O}_{2} \rightarrow 9 \mathrm{CO}_{2}+9 \mathrm{H}_{2} \mathrm{O}
$$

$$
4 \mathrm{FeS}_{2}+11 \mathrm{O}_{2} \rightarrow 2 \mathrm{Fe}_{2} \mathrm{O}_{3}+8 \mathrm{SO}_{2}
$$

2. 3.08
3. 323
4. No
5. (a) 1 ,
(b) 1 ,
(c) The same,
(d) The extent of reaction depends on the reaction equation as a whole and not on one species in the equation.
6. $\mathrm{CaCO}_{3}: 43.4 \%, \mathrm{CaO}: 56.4 \%$; (b) 0.308
7. (a) $\mathrm{H}_{2} \mathrm{SO}_{4}$
(b) $79.2 \%$;
(c) 0.89
8. (a) $\mathrm{C}_{2} \mathrm{H}_{6}$ (the hydrogen is from reaction No.2, not the feed);
(b) None;
(c) Fraction conversion $=0.184$;
(d) 0.45 ;
(e) 1.22
(f) Based on reactant in the feed: 0.45 , based on reactant consumed: 0.84 , based on theory: 0.50 ;
(g) Reaction (a) is 33 mol reacting and reaction (b) is 13.5 mol reacting, both based on 100 mol product.

### 2.5 Material Balances for Processes Involving Reaction

## Species Material Balances

## Processes Involving a Single Reaction

The material balance for a species must be augmented to include generation and consumption terms when chemical reactions occur in a process.
$\left\{\begin{array}{c}\text { moles of } i \\ \text { at } t_{2} \\ \text { in the system }\end{array}\right\}-\left\{\begin{array}{c}\text { moles of } i \\ \text { at } t_{1} \\ \text { in the system }\end{array}\right\}=\left\{\begin{array}{l}\text { moles of } i \\ \text { entering } \\ \text { the system }\end{array}\right\}-\left\{\begin{array}{c}\text { moles of } i \\ \text { leaving } \\ \text { the system }\end{array}\right\}+\left\{\begin{array}{c}\text { moles of } i \\ \text { generated } \\ \text { by reaction }\end{array}\right\}-\left\{\begin{array}{c}\text { moles of } i \\ \text { consumed } \\ \text { by reaction }\end{array}\right\}$

