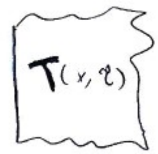
F_o → Fourier's Number. $F_o = \frac{\alpha \tau}{s^2} = \frac{K \tau}{\rho c s^2}$

$$\frac{\theta}{\theta_o} = e^{-Bi F_o}$$

* semi-infinite plate. (slab).

Is a solid with a single plane surface with it's other three surfaces being far enough to be ignored.



① semi-infinite plate, (very larg slab) (very thicknes) (larg thick layer)

(sudden change in temp).

$$* \quad \frac{T(x,\tau) - T_o}{T_i - T_o} = erf\left(\frac{x}{2\sqrt{\alpha \tau}}\right)$$

erf → Table A-1.

x → depth, τ → Time] → @ T

T_i → initial temp

T_o → final temp

خطوات الحل. ← ① نجد $\frac{x}{2\sqrt{\alpha \tau}}$

② نجد ما يقابله من الجدول.

$$* \quad q = \frac{-K A (T_i - T_o)}{\sqrt{\alpha \pi \tau}} e^{\left(\frac{-x^2}{4 \alpha \tau}\right)}$$

at surface $x = 0$

$$q = -\frac{K A (T_i - T_o)}{\sqrt{\pi \alpha \tau}}$$

4-21 A very large slab of copper is initially at a temperature of 300◦C. The surface temperature is suddenly lowered to 35◦C. What is the temperature at a depth of 7.5 cm 4 min after the surface temperature is changed?

very larg slab of copper. // suddenly lowered //.

$T_i = 300°C$, $T_o = 35$. find T | $x = 7.5\ cm = 0.075\ m$

$\tau = 4\ min = 240\ s$

$$\frac{T - T_o}{T_i - T_o} = erf\left(\frac{X}{2\sqrt{\alpha \tau}}\right)$$

$$\frac{X}{2\sqrt{\alpha \tau}} = \frac{0.075}{2\sqrt{11.234 \times 10^{-5} \times 240}} = 0.2284$$

$\frac{X}{2\sqrt{\alpha \tau}}$	$erf \frac{X}{2\sqrt{\alpha \tau}}$
0.22	0.24430
0.2284	??
0.24	0.2657

$erf \frac{X}{\ }$

$$\frac{T - 35}{300 - 35} = 0.2533 \Rightarrow T = 102.1249\ C^{o}$$

4-26 A large slab of copper is initially at a uniform temperature of 90◦C. Its surface temperature is suddenly lowered to 30◦C. Calculate the heat-transfer rate through a plane 7.5 cm from the surface 10 s after the surface temperature is lowered.

large slab. copper. $T_i = 90^oC$, $T_o = 30^oC$ (suddenly), find $\frac{q}{A}$ (Heat T. rate)

$X = 7.5$ cm. from the surface.

$\tau = 10$ s.

sol:- copper $K = 386$. $\alpha = 11.23 * 10^{-5}\ m^2/s$

$$q = \frac{-KA(T_i - T_o)}{\sqrt{\alpha \pi \tau}} e^{\left(-\frac{x^2}{4\alpha\tau}\right)} \Rightarrow \frac{q}{A} = \frac{-K(T_i - T_o)}{\sqrt{\alpha \pi \tau}} e^{\left(-\frac{x^2}{4\alpha\tau}\right)}$$

$$\frac{q}{A} = \frac{-386(90-30)}{\sqrt{11.23 * 10^{-5} * \pi \times 10}} e^{\left(\frac{-(0.075)^2}{4 * 11.23 * 10^{-5} * 10}\right)} = -111.46\ kW/m^2.$$

② constant H. Flux on semi- Infinite solid.

$$T - T_i = \frac{2 q_o}{AK}\sqrt{\frac{\alpha \tau}{\pi}}\exp\left(\frac{-x^2}{4\alpha\tau}\right) - \frac{q_o x}{KA}\left(1 - \operatorname{erf}\frac{x}{2\sqrt{\alpha\tau}}\right)$$

انتبه . erf لجميع القيم التي فوق 3.6 في $\frac{x}{2\sqrt{\alpha\tau}}$ يساوي 1 هذا معناه أن الحد الثاني من المعادلة قد اصبح صفر في هذه الحالة، فانتبه

4-27 A large slab of aluminum at a uniform temperature of 30◦C is suddenly exposed to a constant surface heat flux of 15 kW/m2. What is the temperature at a depth of 2.5 cm after 2 min?

large slab Aluminum. , $T_i = 30°C$, $\frac{q_o}{A} = 15\ kW/m^2$

find $T|_{x = 2.5\,cm,\ \tau = 120\,s}$

Sol:- from table $K = 204$. , $\alpha = 8.42 * 10^{-5}$

$$- X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.025}{2\sqrt{8.42*10^{-5}*120}} = 0.124355$$

$- \operatorname{erf}(0.124355) \Rightarrow$

x	erf
0.12	0.13476
0.124355	??
0.14	0.15695

$\Rightarrow \operatorname{erf}(X) = 0.13959$

$$T - 30 = \frac{2*15000}{204}\sqrt{\frac{8.42*10^{-5}*120}{\pi}}\exp\left(\frac{-(0.025)^2}{4*8.42*10^{-5}*120}\right) - \frac{15000*0.025}{204}(1 - 0.13959)$$

$$\boxed{T = 36.59°c}$$

③ Energy pulse @ surface.

$$T - T_i = \frac{Q_o}{A \rho C\sqrt{\pi\alpha\tau}}\exp\left(\frac{-x^2}{4\alpha\tau}\right)$$

energy: Q_o :- in (J)

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4-45 A semi-infinite solid of stainless steel (18% Cr, 8% Ni) is initially at a uniform temperature of 0◦C. The surface is pulsed with a laser with 10 MJ/m2 instantaneous energy. Calculate the temperature at the surface and depth of 1 cm after a time of 3 s.

semi-infinite solid of stainless steel (18% Cr, 8% Ni), $T_i = 0°C$

$\frac{Q_o}{A} = 10 \times 10^6 \ J/m^2$ Find $T \Big|_{\substack{x = 0.01 m \\ \tau = 3 s}}$?

sol:-

from Table. $\rho = 7817$, $C = 460$, $\alpha = 0.444 \times 10^{-5} \ m^2/s$

$$T - 0 = \frac{10 \times 10^6}{7817 \times 460 \times \sqrt{\pi \times 0.444 \times 10^{-5} \times 3}} \exp\left(\frac{-(0.01)^2}{4 \times 0.444 \times 10^{-5} \times 3}\right)$$

$T = 64.99°C$

see P.4.46 ~~P.4.48~~, 49, 50.

(4) semi-infinite slab with convection from the surface

$$\frac{T - T_i}{T_\infty - T_i} = 1 - \operatorname{erf} X - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha \tau}{k^2}\right)\right] \times \left[1 - \operatorname{erf}\left(X + \frac{h\sqrt{\alpha \tau}}{k}\right)\right]$$

$X = \frac{x}{2\sqrt{\alpha \tau}}$

T_i : initial temp of solid

T_∞ : environment temp.

4-24 A semi-infinite slab of material having k =0.1 W/m · ◦C and α=1.1×10−7m2/s is maintained at an initially uniform temperature of 20◦C. Calculate the temperature at a depth of 5 cm after 100 s if (a) the surface temperature is suddenly raised to 150◦C, (b) the surface is suddenly exposed to a convection source with h=40W/m2 ◦C and 150◦C, and (c) the surface is suddenly exposed to a constant heat flux of 350 W/m2.

P.4-24 semi-infinite slab, $K = 0.1$, $\alpha = 1.1 \times 10^{-7}$, $T_i = 20^\circ C$

find T | $x = 0.05\,m$, $\tau = 100\,s$

If
1. suddenly raised to $150^\circ C \rightarrow T_o$
2. suddenly exposed to convection source. $h = 40$, $T_\infty = 150^\circ C$
3. suddenly exposed to C.H.F. $\frac{q}{A} = 350$

Solu:-

① $$X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.05}{2\sqrt{1.1*10^{-7}*100}} = 7.53$$

$\text{erf}(7.53) = 1 \rightarrow$ Table A-1 (The error function)

$$\frac{T - T_o}{T_i - T_o} = \text{erf}(X) = 1$$

$$T - T_o = T_i - T_o \Rightarrow \boxed{T = T_i = 20^\circ C}$$ #

② $$\frac{T - T_i}{T_\infty - T_i} = 1 - \text{erf}\,X - \left[\exp\left(\frac{hx}{K} + \frac{h^2\alpha\tau}{K^2}\right)\right]\left[1 - \text{erf}\left(X + \frac{h\sqrt{\alpha\tau}}{K}\right)\right]$$

هنا واضح انه erf هنا لأنه $X = 7.53$ وسنرجع لها قيمة وستبقى erf = 1 لذلك الحد الأخير صفر.

$$\frac{T - T_i}{T_\infty - T_i} = 0 \Rightarrow T - T_i = 0 \quad \boxed{T = T_i = 20^\circ C}$$

③ $$T - 20 = \frac{2 * 350}{0.1} * \sqrt{\frac{1.1*10^{-7}*100}{\pi}} * \exp\left(\frac{-(0.05)^2}{4*1.1*10^{-7}*100}\right) - \frac{q_o x}{KA}\left(1 - \text{erf}\,X\right)$$

$\boxed{\text{erf}(X) = 1}$

$$T - 20 = 2.76*10^{-24} \Rightarrow \boxed{T = 20^\circ C}$$ #