Fluid Mechanics

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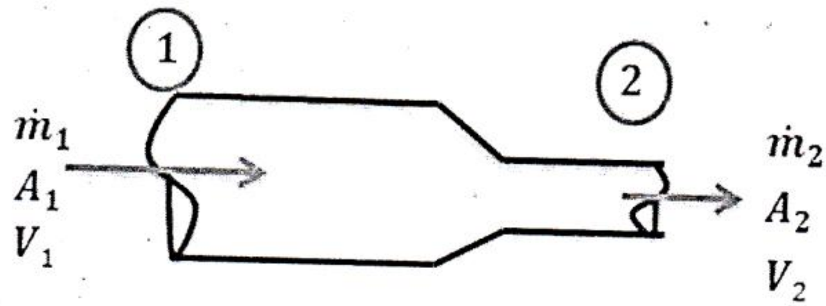
Basic equation of fluid motion:-

1. Continuity Equation.
2. Energy Equation.
3. Impulse–momentum equation.

Continuity Equation.

Continuity equation states that any quantity of fluid passing through any system at time must be the same (fixed). It depends on the conservation of mass law.

Consider the following figure:-



$\dot{m} = \dot{m}_1 = \dot{m}_2 =$ **constant**, where $\dot{m} = \rho V A$ (mass flow rate).
Units → ***Kg.m³/s***
$\therefore \rho_1 V_1 A_1 = \rho_2 V_2 A_2$ Continuity equation for compressible flow for ***Gasous***).

$Q = Q_1 = Q_2 =$ **constant**, where $Q = VA$ (volume flow rate or flow rate only)
$\therefore V_1 A_1 = V_2 A_2$ (Continuity equation for incompressible flow for ***Liquid*** only); where $(\rho_1 = \rho_2)$.
Units → m^3/s , ***A***= cross-section of pipe, m^2 , V= velocity, m/s

Energy Equation (Burnoulli's Equation).

Assumptions:-

1. Derived depending on Conservation of energy law.
2. Steady flow.
3. Uniform flow.
4. Ideal flow (frictionless).
5. Incompressible flow.

Burnoulli's Equation states as follow :-

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line and the total energy heads must be constant".

By considering the given figure:-

$$\frac{P}{\gamma}+\frac{V^2}{2g}+Z=constant=H$$

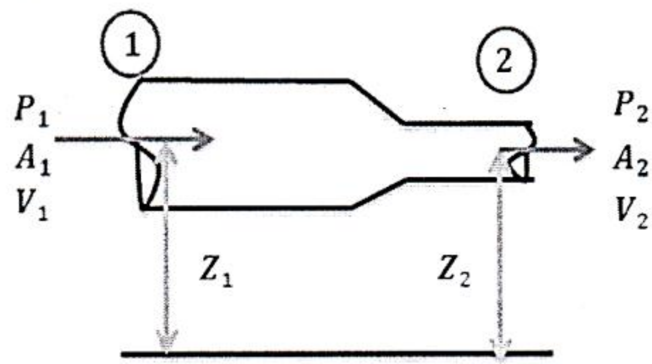
Where:-

H:- **total head, (m)**

$\frac{P}{\gamma}$:- **pressure head, (m)**

$\frac{V^2}{2g}$:- **velocity head, (m)**

Z :- **potential head, (m)**



Then :-

$$\frac{P_1}{\gamma}+\frac{V^2{}_1}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V^2{}_2}{2g}+Z_2$$ ..... **Burnoulli's Equation**

Energy Distribution:

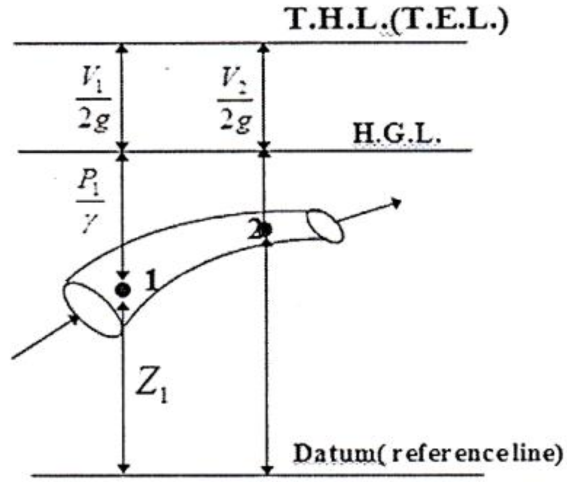
T.H.L : Total head line.

Or:

T.E.L.: Total energy line.

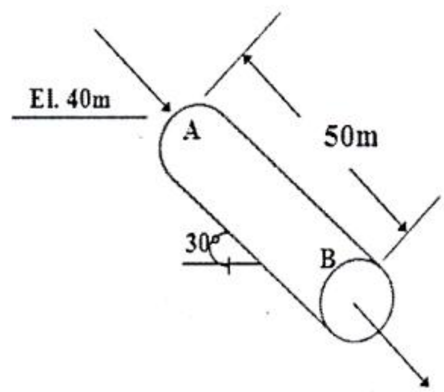
H.G.L.: Hydraulic grade line.

Or pezometric head.



Example **16** **Water flows through a cylindrical pipe, 150mmd. If the pressure at point A is 130Kpa, what is the pressure at B.**

Solution:-



$$Z_A = 40m$$

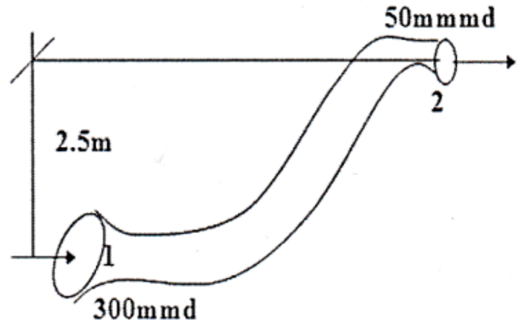
$$Z_B = 40 - 50\sin e30 = 40 - 25 = 15m$$

$$\frac{V_A^{\ 2}}{2g} + \frac{P_A}{\gamma} + Z_A = \frac{V_B^{\ 2}}{2g} + \frac{P_B}{\gamma} + Z_B$$

$$\frac{130*10^3}{9800} + 40 = \frac{P_B}{9800} + 15 \qquad (V_A = V_B)$$

$$P_B = 375Kpa \qquad Ans.$$

Example **17** **Find the discharge of Oil (r.d.=0.85) through the pipeline and the velocity at pint 2 for 55 Kpa pressure at point 1.**



Solution:-

$$V_1 = \frac{Q}{A_1} \qquad V_2 = \frac{Q}{A_2}$$

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} \frac{P_2}{\gamma} + Z_2$$

$$\frac{\left(\frac{Q}{A_1}\right)^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{\left(\frac{Q}{A_2}\right)^2}{2g} + \frac{P_2}{\gamma} + Z_2$$

$$P_2 = 0$$

$$\frac{\left(\frac{Q}{\frac{(0.3)^2}{4}\pi}\right)^2}{2g} + \frac{55 * 10^3}{0.85\,(9800)} + 0 = \frac{\left(\frac{Q}{\frac{(0.05)^2}{4}\pi}\right)^2}{2g} + 2.5$$

$$10.2Q^2 + 6.6 = 13220 \quad Q^2 + 2.5$$

$$Q = 0.0176 \quad m^3/S \qquad Ans.$$

$$V_2 = \frac{Q}{A_2} = \frac{0.0176}{\frac{(0.05)^2}{4}\pi} = 8.97 \quad m/S \qquad Ans.$$