جامعة المستقبل
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Lecture : 3
Logic: Part III

Converse, Contrapositive ,and Inverse

We can form some new conditional statements starting with a conditional statement

$P \rightarrow q$

There are three related conditional statements

(1) The converse of $p \rightarrow q$ is the proposition $q \rightarrow p$.
(2) The contrapositive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.
(3) The inverse of $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$

Note: only the contrapositive always has the same truth value as $P \rightarrow q$

P	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Example:

What are the contraposition ,the converse and the inverse of the conditional statement "The home team wins whenever it is raining"

Solution:

Because "q whenever p" is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as

"If it is raining , then the home team wins"

Consequently,

The contrapositive of this conditional statement is

"If the home team does not win, then it is not raining"

The inverse is

"If it is not raining ,then the home team does not win"

The converse is

"If the home team wins, then it is raining"

Only the contrapositive is equivalent to the original statement.

1.Propositions & Truth Tables

Let $P(p, q, \dots)$ denote an expression constructed from logical variables $p, q, \dots$, which take on the value TRUE (T) or FALSE (F), and the logical connectives $\wedge$, $\vee$, and $\neg$. Such an expression $P(p, q, \dots)$ will be called a *proposition*.

Example:

$\neg (p \wedge q) \vee r$ and $(\neg p \wedge q) \vee (r \vee \neg s)$

The main property of a proposition $P(p, q, \dots)$ is that its truth value depends exclusively upon the truth values of its variables.

Example, consider proposition $\neg(p \wedge \neg q)$. Its truth table is as follows:

P	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Note: the number of combinations of T and F depends on the number of variables and is given by the number:

n^2 (where n is the number of variables).

If n =1 we have 2 rows

n =2 we have 4 rows

2.Tautologies and Contradictions

Some propositions $P(p, q, \dots)$ contain only *T* in the last column of their truth tables or, in other words, they are true for any truth values of their variables. Such propositions are called *tautologies*. Analogously, a proposition $P(p, q, \dots)$ is called a *contradiction* if it contains only *F* in the last column of its truth table or, in other words, if it is false for any truth values of its variables.

Example:

From the truth tables, $p \vee \neg p$ is tautology and $p \wedge \neg p$ is a contradiction.

Tautology

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

contradiction

P	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Logical Equivalence

Two propositions *P(p, q, . . .)* and *q (p, q, . . .)* are said to be *logically equivalent*, or simply *equivalent* or *equal*, denoted by

$$P(p, q, \ldots) \equiv q(p, q, \ldots)$$

if they have identical truth tables.

For example, Consider the truth tables of $\neg(p \wedge q)$ and $\neg p \vee \neg q$:

Observe that both truth tables are the same, that is, both propositions are false in the first case and true in the other three cases.

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(a) $\neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(b) $\neg p \vee \neg q$

Accordingly, we can write: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

Example:

Let p be "Roses are red" and q be "Violets are blue." Let S be the statement:

"It is not true that roses are red and violets are blue."

Then S can be written in the form $\neg(p \wedge q)$. *However, as noted above,* $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Accordingly, S has the same meaning as the statement:

"Roses are not red, or violets are not blue."