$$\boxed{\mathcal{L}\, f(t) = \int_0^\infty f(t)\, e^{-st}\, dt}$$

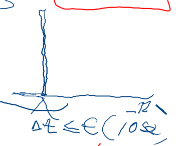
$f(t) = $ step Function $= A \;;\; t \geq 0$

$\mathcal{L}[A] = \dfrac{A}{s}$

$$\boxed{e^{-2s}\frac{A}{s}}$$

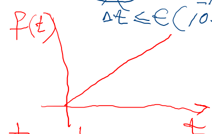
$f(t) = $ Impulse Function $\delta(t)$

$\mathcal{L}[\delta(t)] = 1$



$f(t) = $ Ramp Function $= t$

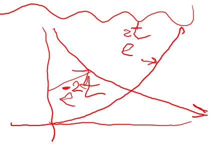
$\mathcal{L}[t] = \dfrac{1}{s^2}$



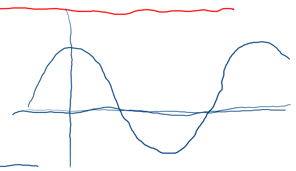
$$\boxed{f(t) = \mathcal{L}\, f(t) = \int_0^\infty f(t)\, e^{-st}\, dt}$$

$f(t) = e^{-at}$

$\mathcal{L}[e^{-at}] = \dfrac{1}{s+a}$



$f(t) = \cos(\omega t)$



$\mathcal{L}[\cos \omega t] = \dfrac{s}{s^2 + \omega^2}$

$\boxed{f(t) = 12\cos(3t)} \Rightarrow \dfrac{12 \cdot s}{s^2 + 3^2}$

$= \dfrac{12s}{s^2 + 9}$

~~Example~~

$$\boxed{f(t) = 3 - 2e^{-2t} + 7\cos(2t)}$$

$\mathcal{L}[f(t)] = \mathcal{L}[3] - 2\mathcal{L}[e^{-2t}] + 7\mathcal{L}[\cos 2t]$

$\mathcal{L}[3] = \dfrac{3}{s}$

$\mathcal{L}2[e^{-2t}] = \dfrac{2}{s+2}$

$\mathcal{L}7[\cos(2t)] = \dfrac{7s}{s^2 + 2^2} = \dfrac{7 \cdot s}{s^2 + 4}$

$F(s) = \dfrac{3}{s} - \dfrac{2}{s+2} + \dfrac{7s}{s^2+4}$

$= \dfrac{3(s+2)(s^2+4) - 2s(s^2+4) + 7s^2(s+2)}{s(s+2)(s^2+4)}$

Q/ Find the Laplace transform of the following Function?

$$\boxed{y(t) = e^{-7t}\cos(4t)}$$

$\mathcal{L}[\cos 4t] = \dfrac{s}{s^2 + 4^2}$

$\mathcal{L}[e^{-7t}\cos(4t)]$

$= \dfrac{(s+7)}{(s+7)^2 + 4^2} = \dfrac{s+7}{(s+7)^2 + 16}$

Find Laplace transform of the following Function:

$$f(t) = 5e^{-3t}\cos(2t)$$

$$\mathcal{L}[\cos(2t)] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$\mathcal{L}\, e^{-3t}\cos 2t = \frac{(s+3)}{(s+3)^2+4}$$

$$f(t) = t \Longrightarrow \mathcal{L}(t) = \frac{1}{s^2}$$

$$f(t) = \int t\,dt$$

$$\mathcal{L}\left[\int t\,dt\right] = \frac{1}{s} \cdot \frac{1}{s^2} = \frac{1}{s^3}$$

$$f(t) = \int e^{-3t}\,dt \qquad \mathcal{L}\, e^{-3t} = \frac{1}{s+3}$$

$$\boxed{F(s) = \frac{1}{s\,(s+3)}}$$

$$\boxed{\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y(t) = 7}$$

Order = 2 ; type = 0

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2\,Y(s)$$

$$\boxed{\mathcal{L}\left[6\frac{dy}{dt}\right] = 6\cdot s\,Y(s)}$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$\mathcal{L}[8y(t)] = 8\cdot Y(s)$$

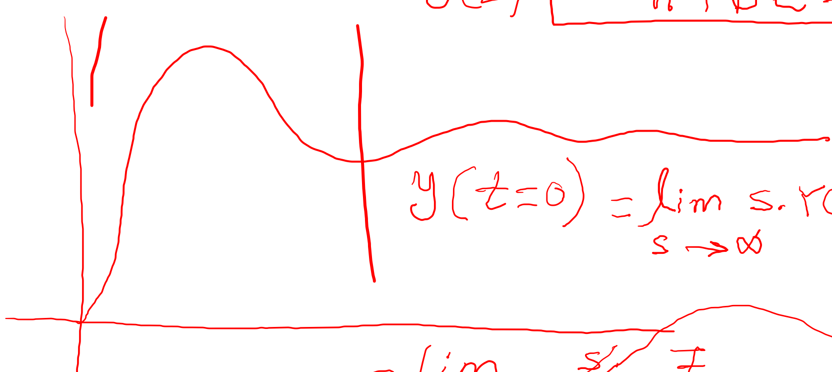
$$s^2Y(s) + 6sY(s) + 8Y(s) = \mathcal{L}(7)$$

$$\boxed{[s^2+6s+8]\;Y(s) = \frac{7}{s}}$$

$$Y(s) = \frac{7}{s(s^2+6s+8)}$$

$$Y(s) = \frac{7}{s(s+2)(s+4)} \Rightarrow \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$y(t) = \boxed{A + Be^{-2t} + Ce^{-4t}}$$



$$y(t=0) = \lim_{s\to\infty} s.Y(s)$$

$$= \lim_{s\to\infty} \frac{\cancel{s}\;7}{\cancel{s}(s^2+6s+8)}$$

$$y(t=0) = \emptyset$$

$$y(t=\infty) = \lim_{s\to 0} \frac{\cancel{s}\cdot 7}{\cancel{s}(s^2+6s+8)} = \frac{7}{8}$$