Electromagnetic waves

Lecture 5

Ampere's Law

Msc. Ali Jaafar

Tow stage
Department of medical physics

Al-Mustaqbal University

What is Ampere's Law?

question

According to Ampere's law, magnetic fields are related to the electric current produced in them. The law specifies the magnetic field that is associated with a given current or vice-versa, provided that the electric field doesn't change with time.

question

Ampere's Law can be stated as:

"The magnetic field created by an electric current is proportional to the size of that electric current with a constant of proportionality equal to the permeability of free space."

- Maxwell extended this law with the inclusion of magnetic fields which arise without current by other causes. Thus, Ampere- Maxwell's law is one of the four Maxwell equations.
- The integral form of the law is as below:

  $\oint Bds = \mu_\circ I$

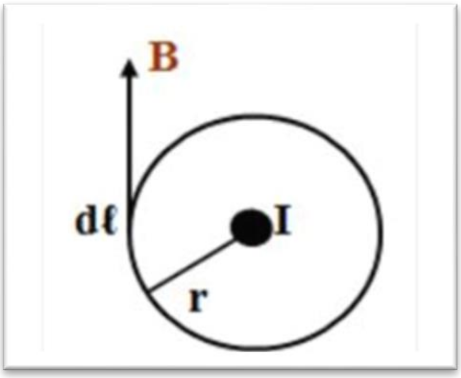
Where :

$\mu_\circ$ is the permeability of free space and 'I' is current.

This law allows us to maintain a proper bridge to fulfil the gap between electricity and magnetism. It also provides the mathematical relation between magnetic fields and electric . Amperes law gives the way to calculate the magnetic field, produced due to the result of an electric current moving through a wire of any shape.

❖ Interpretation of Ampere's Law:

1-If we consider that there is a straight wire through which a current passes
Electricity in a direction and field lines are generated around it
Wire-centered magnetic circles (closed path)
as shape



And the magnetic induction (B) is at a distance (r)
Tangent to one of the closed paths of the field is

$$B = \frac{\mu_\circ I}{2\pi r}$$

Assuming a length of (dℓ) from the closed path at point (P) and since the direction of the inductance is in the same direction as the element (dℓ), so the angle is (θ = 0), and by substituting it in the Ampere equation, it produces:

$$\oint B.cos\theta.dl = \int_0^{2\pi r} \frac{\mu_\circ I}{2\pi r} dl = \frac{\mu_\circ I}{2\pi r}\int_0^{2\pi r} dl = \frac{\mu_\circ I}{2\pi r} \times 2\pi r = \mu_\circ I$$

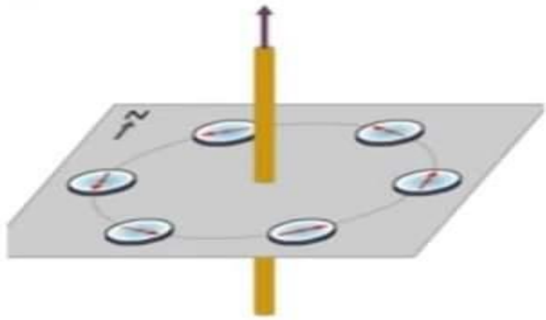
Here is the general form of Ampere's law:

$$\oint B.cos\theta.dl = \mu_\circ I$$

Or :-

2-The circular integration of the magnetic field B with the path ds gives us

$\oint \vec{B}.\overrightarrow{ds}$=B$\oint ds$=$\frac{\mu_\circ I}{2\pi r}(2\pi r)$=$\mu_\circ I$



This relationship is applied to any closed path

that surrounds an electric current I.

We used the right hand rule to find the direction

where the thumb is pointing up and the rest of

the fingers is the direction of the magnetic field generated current inside the wire
And to find the value of the field generated at a point far from the wire

$\oint \vec{B}.\overrightarrow{ds}$=$\mu_\circ I$

The loop on the integral sign means that the scalar (dot) product $\vec{B}.\overrightarrow{ds}$ is to be integrated around a *closed* loop, called an *Amperian loop*.

question

What is Ampere's Circuital Law?

Ampere's circuital law can be written as the line integral of the magnetic field surrounding closed-loop equals the number of times the algebraic sum of currents passing through the loop.

$$\oint H.dL = I_{enc}$$

- The equation's left side describes that if an imaginary path encircles the wire and the magnetic field is added at every point, then it is numerically equal to the current encircled by this route, indicated by I_{enc}.

❖ Determining Magnetic Field by Ampere's Law

Suppose you have a long enough wire that carries a constant current I in amps. How would you determine the magnetic field wrapping the wire at any distance r from the wire؟

In the figure below (Figure 1), a long wire exists that carries current in Amps. We need to find out how much is the magnetic field at a distance r. Therefore, we sketch an imaginary route around the wire indicated by dotted blue toward the right in the figure.

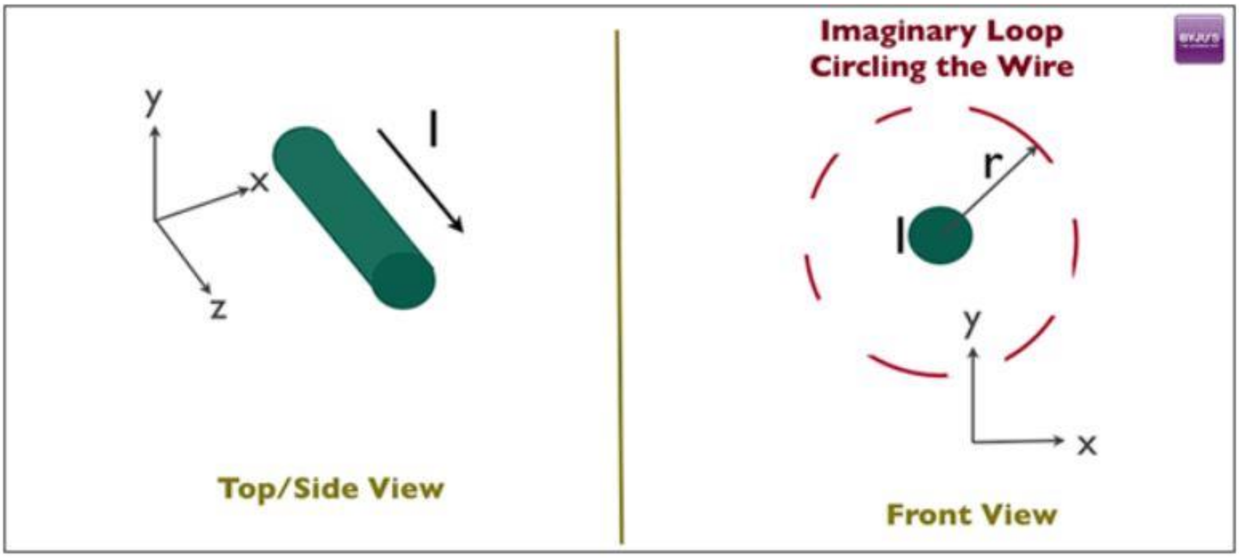


Figure 1

According to the second equation, if the magnetic field is integrated along the blue path, then it has to be equal to the current enclosed, I.

The magnetic field doesn't vary at a distance r due to symmetry. The path length (in blue) in figure 1 is equal to the circumference of a circle,**2πr.**

When a constant value H is added to the magnetic field, the equation's left side looks like this:

$$\oint H.dL = 2\pi r H = I_{enc}$$

$$H = \frac{I_{enc}}{2\pi r}$$

Hence, Ampere's law can be applied to calculate the extent of the magnetic field surrounding the wire. The field H is a vector field that reveals that each region has a direction and a magnitude. The field's direction is tangential at every point to the imaginary loops, as shown in figure 2, and the right-hand rule finds the direction of the magnetic field.

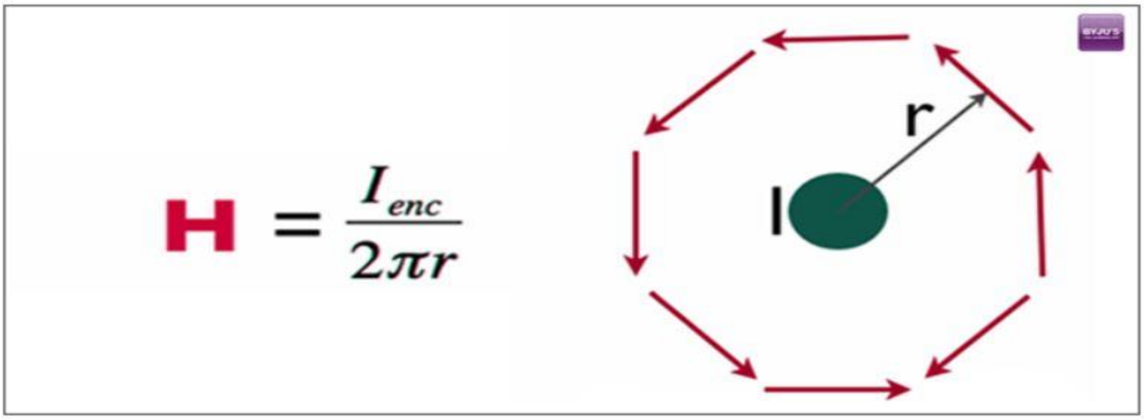


Figure 2

❖ Ampère's law with Maxwell's addition

Ampère's law with Maxwell's addition states that magnetic fields can be generated in two ways: by electric current (this was the original "Ampère's law") and by changing electric fields (this was "Maxwell's addition", which he called displacement current). In integral form, the magnetic field induced around any closed loop is proportional to the electric current plus displacement current (proportional to the rate ofchange of electric flux) through the enclosed surface.

Maxwell's addition to Ampère's law is particularly important: it makes the set of equations mathematically consistent for non static fields, without changing the laws of Ampere and Gauss for static fields. However, as a consequence, it predicts that a changing magnetic field induces an electric field and vice versa. Therefore, these equations allow self-sustaining "electromagnetic waves" to travel through empty space (see electromagnetic wave equation).

The speed calculated for electromagnetic waves, which could be predicted from experiments on charges and currents, matches the speed oflight; indeed, light *is* one form of electromagnetic radiation (as are X- rays, radio waves, and others). Maxwell understood the connection between electromagnetic waves and light in 1861, thereby unifying the theories of electromagnetism and optics.

Formulation in SI units convention

Name	Integral equations	Differential equations
Gauss's law	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega}\rho\, dV$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\iint_{\Sigma}\mathbf{B}\cdot d\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0\left(\iint_{\Sigma}\mathbf{J}\cdot d\mathbf{S} + \varepsilon_0\frac{d}{dt}\iint_{\Sigma}\mathbf{E}\cdot d\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

Formulation in Gaussian units convention

Name	Integral equations	Differential equations
Gauss's law	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{E}\cdot d\mathbf{S} = 4\pi\iiint_{\Omega}\rho\, dV$	$\nabla\cdot\mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{B}\cdot d\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{1}{c}\frac{d}{dt}\iint_{\Sigma}\mathbf{B}\cdot d\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot d\boldsymbol{\ell} = \frac{1}{c}\left(4\pi\iint_{\Sigma}\mathbf{J}\cdot d\mathbf{S} + \frac{d}{dt}\iint_{\Sigma}\mathbf{E}\cdot d\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial\mathbf{E}}{\partial t}\right)$

❖ Applications of Ampere's Law

Ampere's Law is used to :

- Determine the magnetic induction due to a long current-carrying wire.
- Determine the magnetic field inside a toroid.
- Determine the magnetic field created by a long current carrying conducting cylinder.
- Determine the magnetic field inside the conductor.
- Find forces between currents

Questions and Answers

Q\Name the scientist who performed experiments with forces that act on current-carrying wires?

André-Marie Ampère.

Q\State true or false: Ampere's law is used to determine the magnetic field inside a toroid.

True.

Q\State true or false: If the direction of the current is reversed, the direction of the magnetic field reverses.

True.

Q\State Ampere's circuital law.

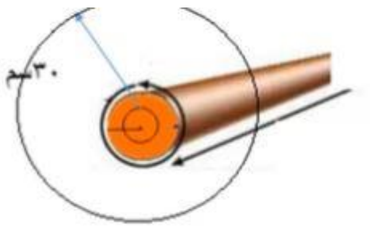
Ampere's circuital law states that "the line integral of the magnetic field surrounding closed-loop equals to the number of times the algebraic sum of currents passing through the loop."

Example :-Calculate the magnetic field 30 cm from the center?(I=50)

$$B = \frac{\mu_\circ I}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times 50}{2\pi(0.3)}$$

$$= 3.33 \times 10^{-5}\ T$$



Example:- A current passes through a thin wire, resulting in a magnetic field of inductance of $10^{-4} T$ at a point 5 cm from the middle of the wire. What is the value of this electric current?

Solution:-

$$B = \frac{\mu_\circ I}{2\pi r}$$

$$I = \frac{2\pi r B}{\mu_\circ}$$

$$I = \frac{2\pi \times 5 \times 10^{-2} \times 10^{-4}}{4\pi \times 10^{-7}}$$

$$I = \frac{10 \times 10^{-6} \times 10^{7}}{4}$$

$I = 25\ A$

Homework:-

1-Calculate the value of the magnetic field strength at a point 100 cm away from a thin conductor carrying a current of 1 ampere ?