

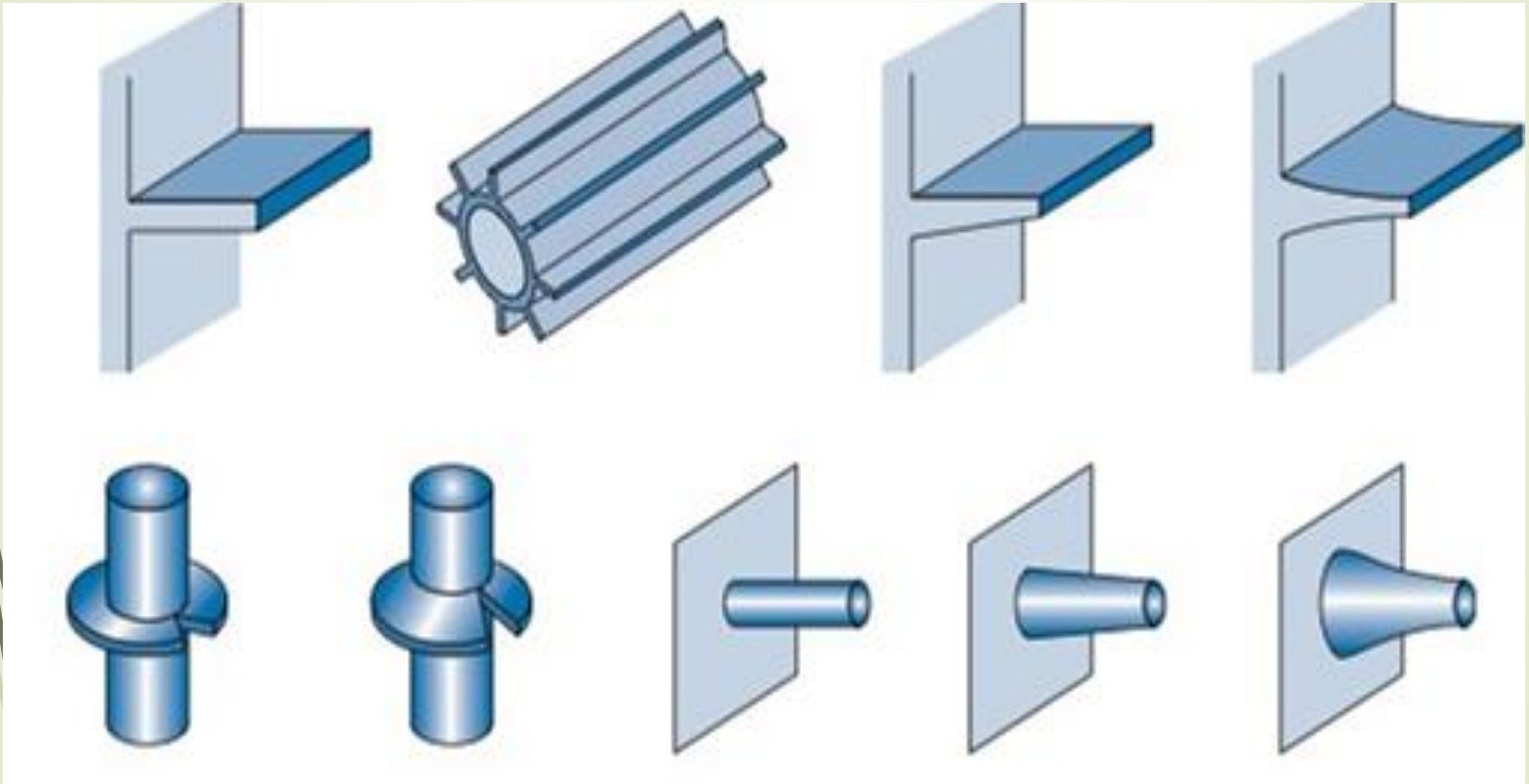
# Heat Transfer by Extended Surfaces (Fins)

Prof. Dr. Majid H. Majeed

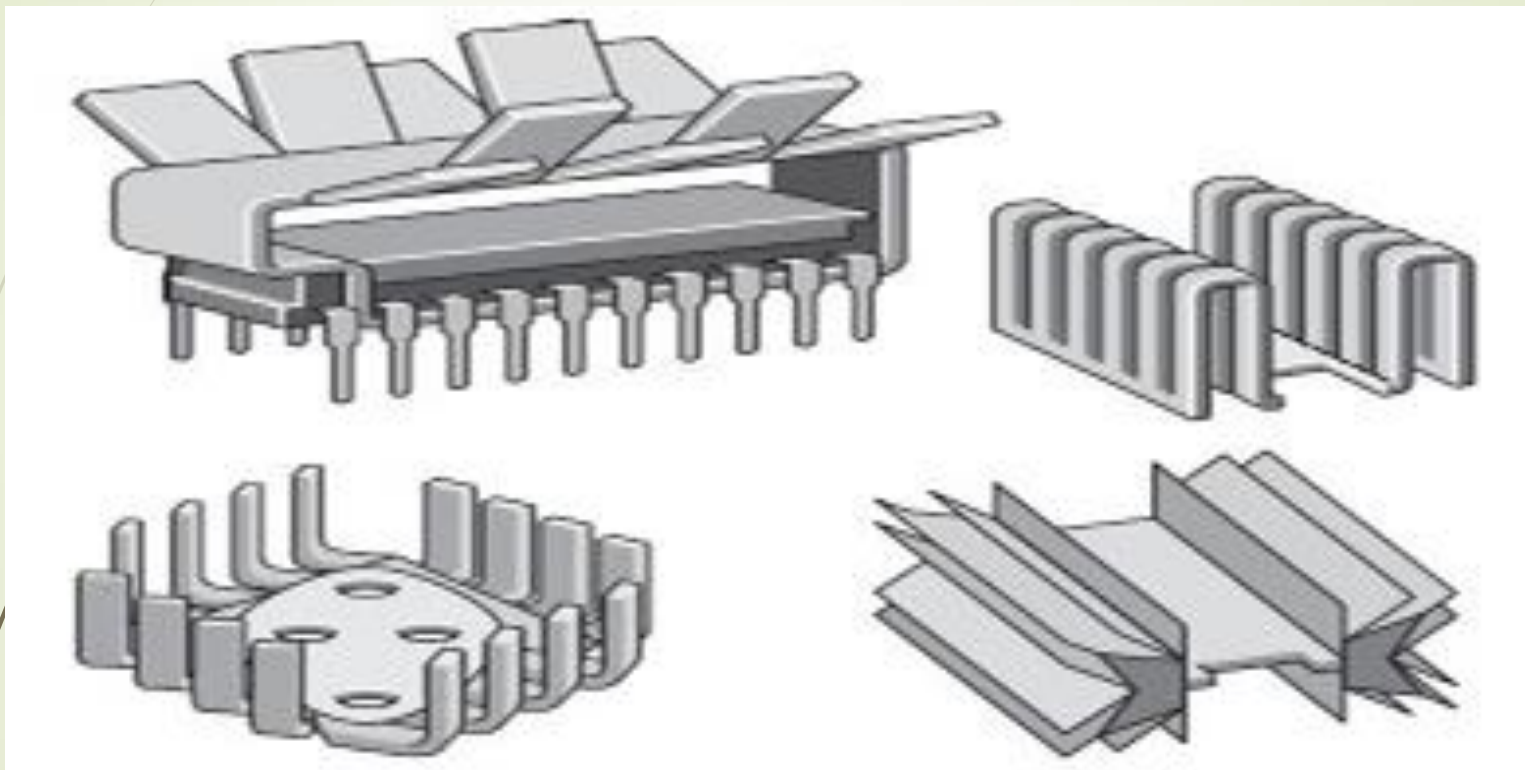
## Heat transfer from Extended Surfaces

The heat transfer by convection is increased by increasing the area that exposed to convection. This led to increase the area of heat transfer in some application that need high heat transfer rate. The increasing of the area of heat transfer is done by extending the surfaces that exposed to convection. This extended surfaces is called fins as shown in Figure

# Fins

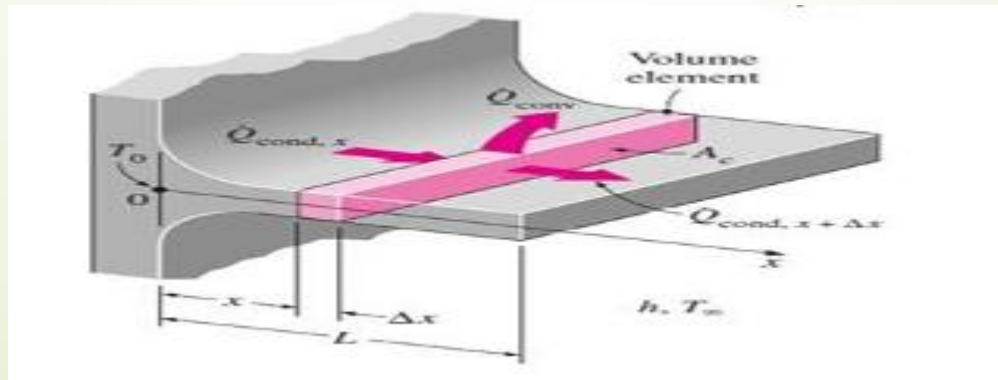


# Fins



# Equation of Fin

- Let us consider an element from a fin at  $x$  location with length  $\Delta x$  and sectional area  $A_c$  and its perimeter of  $p$  as shown in Figure. The equation of energy balance for steady state condition of this element can be expressed as:



$$\left\{ \begin{array}{l} \text{Conduction Heat} \\ \text{Transfer Rate into} \\ \text{The Element at } x \end{array} \right\} - \left\{ \begin{array}{l} \text{Conduction Heat} \\ \text{Transfer Rate from} \\ \text{The Element at } x + \Delta x \end{array} \right\} = \left\{ \begin{array}{l} \text{Convection Heat} \\ \text{Transfer Rate from} \\ \text{The Element} \end{array} \right\}$$

$$\dot{Q}_{\text{cond},x} - \dot{Q}_{\text{cond},x+\Delta x} = \dot{Q}_{\text{conv}}$$

And  $\dot{Q}_{\text{cond},x+\Delta x} = -kA \frac{dT}{dx} + \frac{d}{dx} \left( -kA \frac{dT}{dx} \right) dx$

And  $\dot{Q}_{\text{conv}} = hPdx(T - T_{\infty})$

$$-kA \frac{dT}{dx} - \left[ -kA \frac{dT}{dx} + \frac{d}{dx} \left( -kA \frac{dT}{dx} \right) dx \right]$$

$$= hPdx(T - T_{\infty})$$

7

$$kA \frac{d^2T}{dx^2} dx = hpdx(T - T_\infty)$$

By dividing this equation by  $kA dx$ , we get

$$\frac{d^2T}{dx^2} - \frac{hp}{kA} (T - T_\infty) = 0$$

By assuming  $\theta = T - T_\infty$  then  $\frac{d\theta}{dx} = \frac{dT}{dx}$  and

$$\frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}. \text{ And } \frac{hp}{kA} = m^2$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

The solution of this equation is

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \quad (A)$$

8

- Where  $C_1$  and  $C_2$  are integration constants of arbitrarily whose values are to be determined by the applying of the boundary conditions.
- These boundary conditions are at the base and at the tip of the fin.
- The only two conditions are to be needed to determine  $C_1$  and  $C_2$  which are uniquely.
- B.C.1 is that temperature at the base of fin is known at its value is *at*  $x = 0 \quad T = T_o$
- Then  $\theta = T_o - T_\infty \rightarrow \theta = \theta_o$



➤ By substituting in eq.(A)  $x=0$  and  $\theta = \theta_o$

9  $\theta_o = C_1 + C_2 \quad (1)$

➤ The second boundary depending on the free end of the fin. There are three cases for this end.

➤ Case 1. the fin is very long that at  $x=\infty$   $T = T_\infty$

➤ It means that at  $x=\infty$   $\theta = T_\infty - T_\infty = 0$

➤ Then  $\theta = C_1 e^{-m\infty} + C_2 e^{m\infty} = 0 \quad (2)$

➤ From this equation  $C_1 \neq 0$ , then  $C_2 = 0$

➤ By substituting this in eq.(1) we get

➤  $C_1 = \theta_o$

➤  $\theta = \theta_o e^{-mx}$  (T.D.E) for long fin

10

➤ 
$$\frac{\theta}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = e^{-\sqrt{\frac{hp}{kA}}x} \quad (3)$$

## 1- long fin

➤ Heat transfer from fin is equal to heat flow from its base by conduction

➤ 
$$\begin{aligned} \dot{Q}_{fin} &= -kA \left. \frac{dT}{dx} \right|_{x=0} = -kA \left. \frac{d\theta}{dx} \right|_{x=0} = -kA \frac{d}{dx} \left( \theta_o e^{-\sqrt{\frac{hP}{kA}}x} \right)_{x=0} \\ &= -kA \left( -\theta_o \sqrt{\frac{hP}{kA}} \right) e^{-0} = \sqrt{hPkA} \theta_o \end{aligned}$$

➤ 
$$\dot{Q}_{fin} = \sqrt{hPkA} (T_o - T_\infty)$$

➤ Also heat transfer from fin is equal to heat transfer by convection

➤ from all the fin.

$$\dot{Q}_{fin} = \int_0^{\infty} hP(T - T_{\infty})dx = \int_0^{\infty} hP\theta dx$$

$$\dot{Q}_{fin} = hP \int_0^{\infty} \theta_0 e^{-mx} dx = -hP \frac{\theta_0}{m} e^{-mx} \Big|_0^{\infty}$$

$$\dot{Q}_{fin} = -\frac{hP\theta_0}{\sqrt{\frac{hP}{kA}}} (e^{-\infty} - e^0) = -\sqrt{hPkA}\theta_0(0 - 1)$$

$$\dot{Q}_{fin} = \sqrt{hPkA}\theta_0$$

➤ It equal to that transfer from the base so it is O.K.

## 2- short Insulated end fin

12

in this case the fin is of known length and its end is insulated.

The boundary conditions are

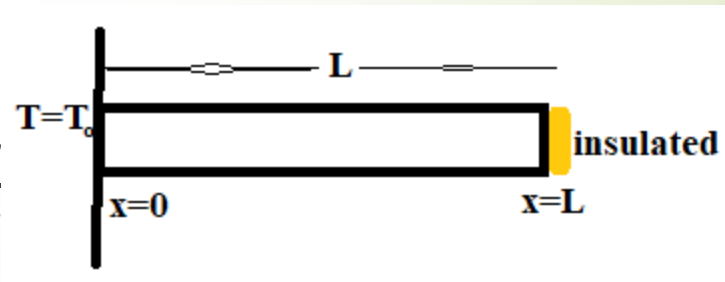
B.C.1 is as in eq.(1)  $T = T_o$  at  $x=0$

B.C.2 is at  $x=L$   $\frac{dT}{dx} = 0$  for insu

The T.D.E is  $\frac{\theta(x)}{\theta_o} = \frac{T(x)-T_\infty}{T_o-T_\infty} = \frac{\cos}{c}$

Heat transfer from the fin is

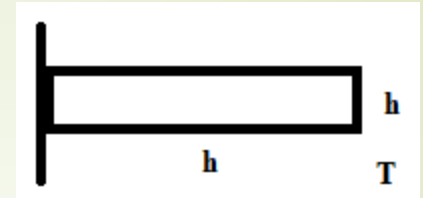
$$\dot{Q}_{fin} = \sqrt{hPkA}\theta_o \tanh(mL)$$



### 3- Short non insulated end fin

13

Fin with heat transfer by convection from its end.



B.C.1 is as in eq.1 at  $x=0$   $T = T_0$  or  $\theta = \theta_0$

B.C.2 at  $x=L$   $-k \frac{d\theta}{dx} \Big|_{x=L} = h\theta_L$

$$\frac{\theta(x)}{\theta_0} = \frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)] + \frac{h}{mk} \sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mk} \sinh(mL)}$$

$$\dot{Q}_{FIN} = \sqrt{hpkA_c} \theta_0 \frac{\sinh(mL) + \frac{h}{km} \cosh(mL)}{\cosh(mL) + \frac{h}{km} \sinh(mL)}$$

## ► Fin efficiency

14

$$\eta_{Fin} = \frac{\left[ \text{Actual Heat Transfer from Fin} \right]}{\left[ \text{Heat Transfer from Fin When The Entire Fin Area at Base Temp.} \right]}$$

The efficiency of long fin is

$$\eta_{Fin} = \frac{\dot{Q}_{Fin}}{hA_{fin}\theta_o}$$

$$\eta_{Fin} = \frac{\dot{Q}_{Fin}}{hA_{fin}\theta_o} = \frac{\sqrt{hpkA_c}\theta_o}{hpL\theta_o} = \sqrt{\frac{kA_c}{hp}} \frac{1}{L} = \frac{1}{mL}$$

Fin effectiveness

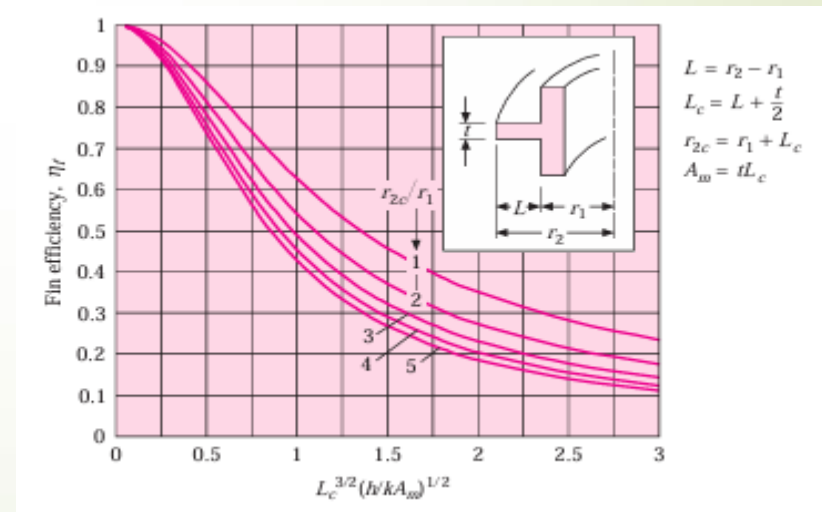
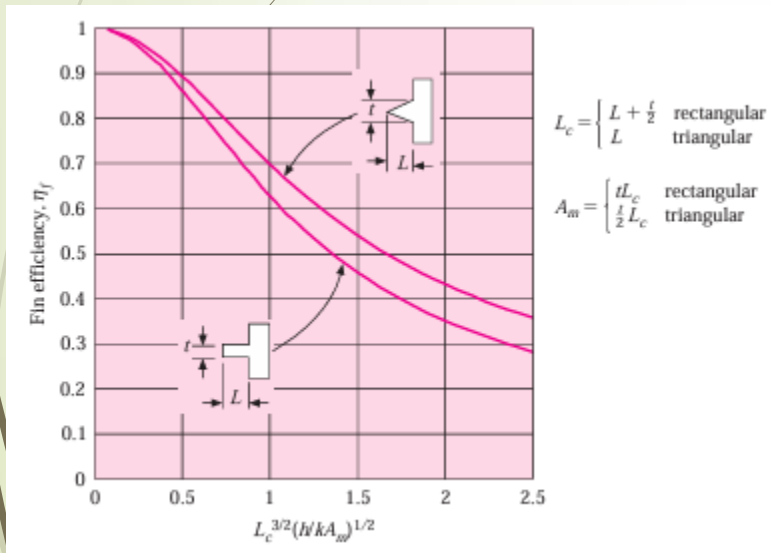
$$\varepsilon_{fin} = \frac{\left[ \text{Heat Transfer Rate From The Fin of Base Area } A_o \right]}{\left[ \text{Heat Transfer Rate From The Surface of Area } A_o \right]} = \frac{\dot{Q}_{Fin}}{\dot{Q}_{no\ Fin}} = \frac{\dot{Q}_{Fin}}{A_o h(T_o - T_\infty)}$$

- And also we can find the flowing charts to find the efficiency of fin.

15

- heat transfer from the fin

- $\dot{Q}_f = \eta_f A_f h (T_o - T_\infty)$



## Long Fin specifications

16

Specifications	Mathematic Relation
<b>Boundary conditions</b>	B.C.1 $x=0$ $T = T_o$ or $\theta = \theta_o = (T_o - T_\infty)$ B.C.2 $x=\infty$ $T = T_\infty$ or $\theta = 0 = (T_\infty - T_\infty) = 0$
<b>T.D.E</b>	$\theta = \theta_o e^{-mx}$ or $\frac{T-T_\infty}{T_o-T_\infty} = \exp\left(-\sqrt{\frac{hP}{kA}}x\right)$
<b>Heat transfer</b>	$\dot{Q}_{fin} = \sqrt{hPkA}\theta_o = \sqrt{hPkA}(T_o - T_\infty)$
<b>Fin efficiency <math>\eta_f</math></b>	$\eta_{fin} = \frac{1}{mL} = \sqrt{\frac{kA}{hP}} \frac{1}{L}$
<b>Fin effectiveness <math>\varepsilon_f</math></b>	$\varepsilon_{fin} = \sqrt{\frac{Pk}{A_o h}}$



## Insulated end fin

17

Specifications	Mathematic Relation
<b>Boundary conditions</b>	B.C.1 $x=0$ $T = T_o$ or $\theta = \theta_o = (T_o - T_\infty)$ B.C.2 $x=L$ $\frac{dT}{dx} = 0$ or $\frac{d\theta}{dx} = 0$
<b>T.D.E</b>	$\frac{\theta}{\theta_o} = \frac{T-T_\infty}{T_o-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$
<b>Heat transfer</b>	$\dot{Q}_{fin} = \sqrt{hPkA}\theta_o \tanh(mL)$ $\dot{Q}_{fin} = \sqrt{hPkA}(T_o - T_\infty)\tanh(mL)$
<b>Fin efficiency</b> $\eta_f$	$\eta_{fin} = \frac{\tanh(mL)}{mL}$
<b>Fin effectiveness</b> $\epsilon_f$	$\epsilon_{fin} = \sqrt{\frac{Pk}{A_o h}} \tanh(mL)$

# Convection from end tip

18

Specifications	Mathematic Relation
<b>Boundary conditions</b>	B.C.1 $x=0$ $T = T_o$ or $\theta = \theta_o = (T_o - T_\infty)$ B.C.2 $x=L$ $\frac{dT}{dx} = -\frac{L}{k}(T_L - T_\infty)$ or $\frac{d\theta}{dx} = -\frac{h}{k}\theta_L$
<b>T.D.E</b>	$\frac{\theta}{\theta_o} = \frac{T-T_\infty}{T_o-T_\infty} = \frac{\cosh(m(L-x)) + \frac{h}{km}\sinh(m(L-x))}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$
<b>Heat transfer</b>	$\dot{Q}_{fin} = \sqrt{hPkA}\theta_o \frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$ $\dot{Q}_{fin} = \sqrt{hPkA}(T_o - T_\infty) \frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$
<b>Fin efficiency <math>\eta_f</math></b>	$\eta_{fin} = \frac{1}{mL} \frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$
<b>Fin effectiveness <math>\varepsilon_f</math></b>	$\varepsilon_{fin} = \sqrt{\frac{Pk}{A_o h}} \frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$

- ➔ Example: A fin of material with thermal conductivity ( $100\text{W/m}\cdot^{\circ}\text{C}$ ). The length of the fin is  $0.5\text{m}$ . The fin of square cross section of side length  $0.1\text{m}$ . The base temperature of the fin is  $100^{\circ}\text{C}$ . The surrounding temperature and convection heat transfer coefficient are  $20^{\circ}\text{C}$  and  $20\text{W}/\text{m}^2\cdot^{\circ}\text{C}$ . Find the temperature Distribution equation and draw that with length of fin. Find the heat transfer and fin efficiency and effectiveness. By 1) assuming the fin of infinite length, 2) insulated end fin, 3) non-insulated fin.

- **Solution:** Fin of length  $L=0.5\text{m}$ , It is of square cross section of side length  $b=0.1\text{m}$  the  $P=4b=0.4\text{m}$  and  $A=b^2 = 0.1 \times 0.1 = 0.01\text{m}^2$ .
- The base temperature  $T_o = 100^\circ\text{C}$ .
- The surrounding temperature  $T_\infty = 20^\circ\text{C}$
- The heat transfer coefficient  $h = 10\text{W}/\text{m}^2.\text{ }^\circ\text{C}$
- Thermal conductivity of fin material  $k=100\text{W}/\text{m}.\text{ }^\circ\text{C}$
- **Properties:** The properties are constant.
- **Assumption:** straight constant cross-sectional area fin with steady state heat conduction.
- **Analysis:** for the fin, we can determined firstly  $m$ .

- $$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10 \times 0.4}{100 \times 0.01}} = 2\text{m}^{-1}$$

➤ 1- Long Fin

The T.D.E  $\frac{T-T_\infty}{T_0-T_\infty} = \exp\left(-\sqrt{\frac{hP}{kA}}x\right) = e^{-mx} = e^{-2x}$

➤  $\frac{T-20}{100-20} = \frac{T-20}{80} = e^{-2x}$

➤ 2- insulated fin

➤  $\frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$

➤  $\frac{T-20}{100-20} = \frac{\cosh[2(0.5-x)]}{\cosh(2 \times 0.5)}$

➤ Convective end tip fin

➤  $\frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh(m(L-x)) + \frac{h}{km} \sinh(r)}{\cosh(mL) + \frac{h}{km} \sinh(n)}$

➤  $\frac{T-20}{100-20} = \frac{\cosh(2(0.5-x)) + \frac{10}{200} \sinh(2(0.5-x))}{\cosh(0.5 \times 2) + \frac{10}{200} \sinh(0.5 \times 2)}$

x	T(1)	T(2)	T(3)
0	100	100	100
0.05	92.39	94.30	93.29
0.1	85.50	89.34	88.23
0.15	79.26	85.07	83.85
0.2	73.62	81.46	80.10
0.25	68.52	78.46	76.96
0.3	63.90	76.05	74.39
0.35	59.72	74.20	72.36
0.4	55.9	72.88	70.46
0.45	52.52	72.10	69.87
0.5	49.43	71.84	69.37

➤ The heat transfer from the fin

➤ 1) long Fin  $\dot{Q}_{fin} = \sqrt{hPkA}\theta_o = \sqrt{hPkA}(T_o - T_\infty)$

➤  $\dot{Q}_{fin} = \sqrt{10 \times 0.4 \times 100 \times 0.01}(100 - 20) = 160W$

➤ 2) insulated end tip fin

➤  $\dot{Q}_{fin} = \sqrt{hPkA}(T_o - T_\infty)\tanh(mL)$

➤  $\dot{Q}_{fin} = \sqrt{10 \times 0.4 \times 100 \times 0.01}(100 - 20) \tanh(2 \times 0.5)$   
 $= 121.855W$

➤ 3) convective end fin

➤  $\dot{Q}_{fin} = \sqrt{hPkA}(T_o - T_\infty) \frac{\sinh(mL) + \frac{h}{km} \cosh(mL)}{\cosh(mL) + \frac{h}{km} \sinh(mL)}$

➤  $\dot{Q}_{fin} = 160 \frac{\sinh(1) + 0.05 \cosh(1)}{\cosh 1 + 0.05 \sinh(1)} = 125.1W$

➤ Fin efficiency

➤ 1) long fin  $\eta_{fin} = \frac{1}{mL} = \sqrt{\frac{kA}{hP}} \frac{1}{L}$

➤  $\eta_{fin} = \frac{1}{mL} = \sqrt{\frac{100 \times 0.01}{10 \times 0.4}} \frac{1}{0.5} = 100\%$

➤ 2) insulated tip fin  $\eta_{fin} = \frac{\tanh(mL)}{mL}$

➤  $\eta_{fin} = \frac{\tanh(2 \times 0.5)}{2 \times 0.5} = 76.16\%$

➤ 3) convective end fin  $\eta_{fin} = \frac{1}{mL} \frac{\sinh(mL) + \frac{h}{km} \cosh(mL)}{\cosh(mL) + \frac{h}{km} \sinh(mL)}$

➤  $\eta_{fin} = \frac{1}{2 \times 0.5} \frac{\sinh(1.0) + \frac{10}{2 \times 100} \cosh(1.0)}{\cosh(1.0) + \frac{10}{2 \times 1000} \sinh(1.0)} = 78.18\%$

► Fin effectiveness

24

► 1) long Fin  $\epsilon_{fin} = \sqrt{\frac{Pk}{A_o h}} = \sqrt{\frac{0.4 \times 100}{0.01 \times 10}} = 20$

► 2) Insulated tip fin  $\epsilon_{fin} = \sqrt{\frac{0.4 \times 100}{0.1 \times 10}} \tanh(2 \times 0.5)$

►  $\epsilon_{fin} = 15.23$

► 3) convective end tip fin

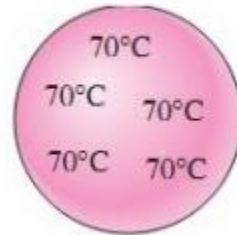
►  $\epsilon_{fin} = \sqrt{\frac{Pk}{A_o h} \frac{\sinh(mL) + \frac{h}{km} \cosh(mL)}{\cosh(mL) + \frac{h}{km} \sinh(mL)}}$

►  $\epsilon_{fin} = 20 \frac{\sinh(1.0) + 0.05 \cosh(1.0)}{\cosh(1.0) + 0.05 \sinh(1.0)} = 15.30$

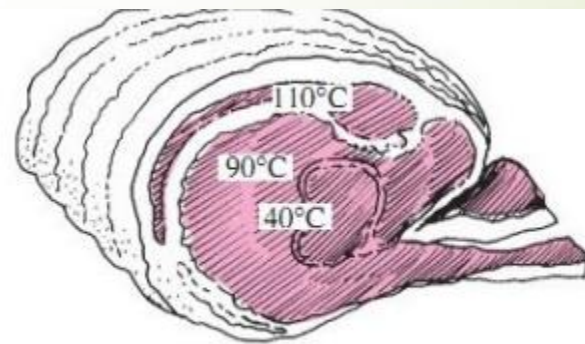


# Transient Heat conduction

- In this case the temperature be function on time as it is function of dimensions.
- $T = T(x, y, z, \tau)$
- **LUMPED SYSTEM CONCEPT**



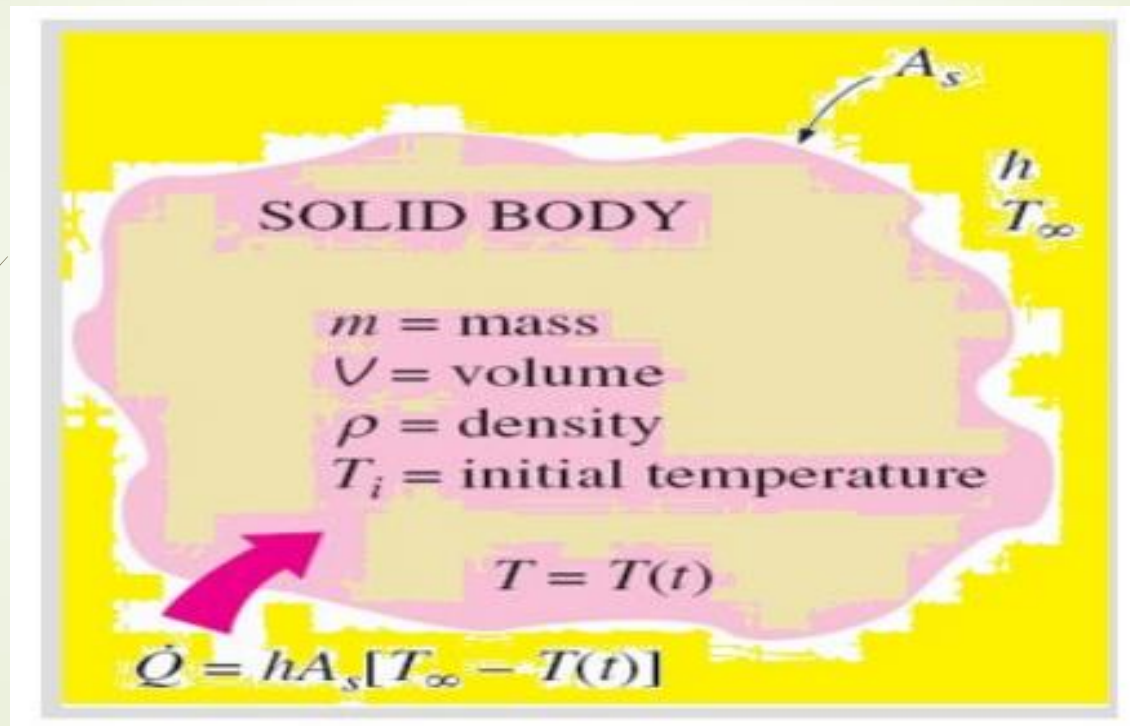
(a) metal with constant Temp. distribution



(b) Roast beef with variation in temp. with location

- Consider a cold solid of arbitrary shape, with mass  $m$ , initially at a uniform temperature  $T_0$ , suddenly immersed into a higher-temperature environment. As heat flows the hot environment into the cold body, the temperature of the solid increases. It is assumed that the lumped system approximation is applicable, namely, that the distribution of temperature within the solid at any instant can be regarded as almost uniform (i.e., the temperature gradients within the solid are neglected).

$$\left\{ \begin{array}{l} \text{Increase of the Internal} \\ \text{Energy of the Solid Over} \\ \text{the Time Interval } d\tau \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat Transfer to the Solid} \\ \text{Through the outer surface} \\ \text{Over the Time Interval } d\tau \end{array} \right\}$$



$$\left\{ \begin{array}{l} \text{Increase of the Internal} \\ \text{Energy of the Solid Over} \\ \text{the Time Interval } d\tau \end{array} \right\} = (\text{mass})CdT = \rho V C dT$$

Where  $\rho$ ,  $C$ , and  $V$  are density, specific heat, and volume of the solid body respectively. The temperature of the solid body  $T$  is a function of time,  $T=T(\tau)$ .

Let  $Q(\tau)$  be the total heat rate following into the body through its boundary surfaces at any instant  $\tau$ .

$$\left\{ \begin{array}{l} \text{Heat Transfer to the Solid} \\ \text{Through the Outer Surface} \\ \text{Over the Time Interval } d\tau \end{array} \right\} = \dot{Q}(\tau)d\tau$$

$$\rho VC dT = \dot{Q}(\tau) d\tau$$
$$\frac{dT(\tau)}{d\tau} = \frac{\dot{Q}(\tau)}{\rho VC}$$

## For Convection Heat Transfer

$$\dot{Q}(\tau) = A_s h [T_\infty - T(\tau)]$$

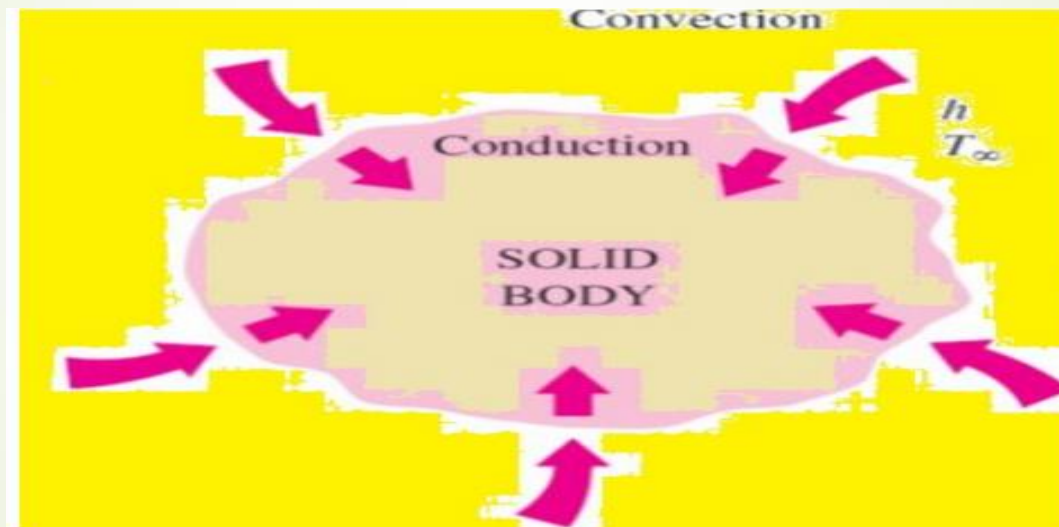
$$\frac{dT(\tau)}{d\tau} = \frac{A_s h}{\rho VC} [T_\infty - T(\tau)]$$

$$\frac{dT(\tau)}{d\tau} = -\frac{A_s h}{\rho VC} [T(\tau) - T_\infty] \quad \text{for } \tau > 0$$

$$T(\tau) = T_i \quad \text{for } \tau = 0$$

- For convenience in the analysis, we measure the temperature in excess of the ambient temperature  $T_\infty$ ; that is, we choose  $T_\infty$  as the reference temperature. Then assume that:

$$\theta(\tau) = T(\tau) - T_\infty$$



$$\theta_i = T_i - T_\infty \quad \& \quad \frac{dT(\tau)}{d\tau} = \frac{d\theta(\tau)}{d\tau}$$

And also a quantity  $m$  is introduced as

$$m = \frac{A_s h}{\rho V C}$$

- Where  $m$  has the dimension of  $(time)^{-1}$ .

31

$$\frac{d\theta(\tau)}{d\tau} + m\theta = 0 \quad \text{for } \tau > 0$$

$$\theta(\tau) = \theta_i \quad \text{for } \tau = 0$$

And by the separating of variables, we get

$$\frac{d\theta(\tau)}{\theta(\tau)} = -m d\tau$$

The integration of this equation will give

$$\ln \theta(\tau) = -m\tau + C$$

Where  $C$  is the integrating constant.

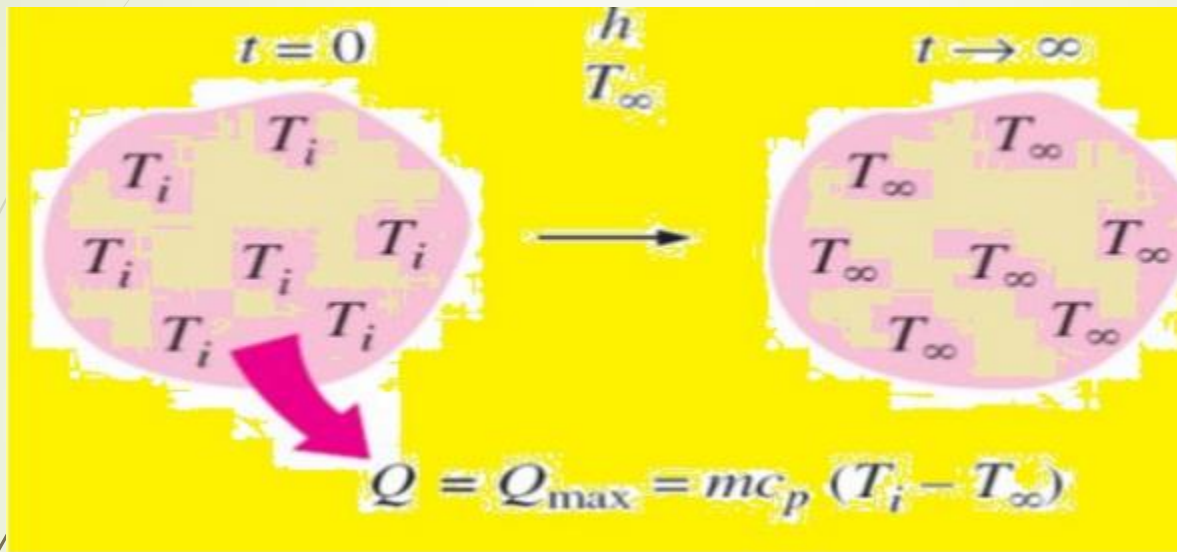
$$\ln \theta_i = -(m)0 + C \rightarrow C = \ln \theta_i$$

$$\ln \theta(\tau) = -m\tau + \ln \theta_i$$

$$\ln \theta(\tau) - \ln \theta_i = -m\tau \rightarrow \ln \left( \frac{\theta(\tau)}{\theta_i} \right) = -m\tau$$

$$\frac{\theta(\tau)}{\theta_i} = e^{-m\tau}$$

$$\frac{T(\tau) - T_{\infty}}{T_i - T_{\infty}} = e^{-\frac{A_s h}{\rho V C} \tau}$$



$$L_c = \frac{V}{A_s}$$

$$m = \frac{h}{\rho C L_c}$$

We can now find the relation of the characteristic length ( $L_c$ ) value for some geometries such as



$$\rightarrow L_C = \frac{V}{A_S}$$

1- A solid sphere

$$L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6}$$

2- A solid cube

$$L_c = \frac{L^3}{6L^2} = \frac{L}{6}$$

3- A solid cylinder of length L

$$L_c = \frac{\pi LR^2}{2\pi RL + 2\pi R^2} = \frac{LR}{2(L + R)}$$

4- A solid long cylinder

$$L_c = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2} = \frac{D}{4}$$

5- A solid large plate with thickness L and one side surface area A\*

$$L_c = \frac{A^* L}{2A^*} = \frac{L}{2}$$

## Criteria For Lumped system Analysis

34

$$Bi = \frac{\text{Convection at The Surface of the Body}}{\text{Conduction Within The Body}} = \frac{h \Delta T}{k / L_c \Delta T} = \frac{hL_c}{k}$$

$$Bi = \frac{\text{Conduction Resistance Within the Body}}{\text{Convection Resistance at The Surface of the Body}} = \frac{L_c / k}{1/h} = \frac{hL_c}{k}$$

A small Biot number means that a small heat conduction resistance and thus a small temperature gradient within the body. In general, the analysis lumped system can be applied

$$Bi \leq 0.1$$

The parameter ( $m\tau$ ) can be modified by the following

$$m\tau = \frac{h}{\rho C L_c} \tau = \frac{h}{\rho C L_c} \frac{k L_c}{k L_c} \tau = \frac{h L_c}{k} \frac{k}{\rho C} \frac{1}{L_c^2} \tau$$

$$m\tau = \frac{h L_c}{k} \frac{\alpha \tau}{L_c^2} = Bi \cdot Fo$$

Where **Fo** is the **Fourier Number**

$$\frac{T(\tau) - T_\infty}{T_i - T_\infty} = e^{-Bi \cdot Fo}$$

➤ **Example** Using lumped system method, determine the time required for a solid steel ball of radius (3cm), thermal conductivity ( $55\text{W/m}\cdot^{\circ}\text{C}$ ), density ( $7830\text{kg/m}^3$ ), and specific heat ( $460\text{J/kg}\cdot^{\circ}\text{C}$ ) to cool from ( $1000^{\circ}\text{C}$ ) to ( $250^{\circ}\text{C}$ ). If the ball is exposed to stream of air at ( $100^{\circ}\text{C}$ ) having a coefficient of heat transfer ( $100\text{W/m}^2\cdot^{\circ}\text{C}$ ).

➤ **Solution:** In this problem the time of cooling steel ball from ( $1000^{\circ}\text{C}$ ) to ( $250^{\circ}\text{C}$ ) by using the lumped capacity method when the ball is facing to convection heat transfer with coefficient of heat transfer of ( $100\text{W/m}^2\cdot^{\circ}\text{C}$ ) and environment temperature of ( $100^{\circ}\text{C}$ ). The radius of the ball  $R=0.03\text{m}$ .

- **Assumption:** The ball material thermal properties and the coefficient of heat transfer are constant. The radiation effect is negligible. There is no temperature gradient through the ball.
- **Properties:** The properties of the ball material are constant and they are  $k=55\text{W/m}\cdot\text{oC}$ ,  $\rho=7830\text{kg/m}^3$ ,  $C=460\text{J/kg}\cdot\text{oC}$ .
- **Analysis:** The characteristic length of the ball with  $R=3\text{cm}$  is:
  - $L_c = \frac{R}{3} = \frac{0.03}{3} = 0.01\text{m}$
  - $m = \frac{h}{\rho C L_c} = \frac{100}{7830 \times 460 \times 0.01} = 0.00278\text{s}^{-1}$
  - Now to find the time spending for cooling is

$$\frac{T(\tau) - T_{\infty}}{T_i - T_{\infty}} = e^{-m\tau} \quad \rightarrow \quad \frac{250 - 100}{1000 - 100} = -0.00277\tau$$

$$\rightarrow \tau = -\frac{1}{0.00277} \ln\left(\frac{150}{900}\right) = 646.8 \text{ sec} = 10.78 \text{ min}$$

➤ The checking for lumped system criteria we can find Biot number

$$\rightarrow Bi = \frac{hL_c}{K} = \frac{100 \times 0.01}{55} = 0.018 > 0.1$$