Heat conduction through wall

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Heat conduction in the wall

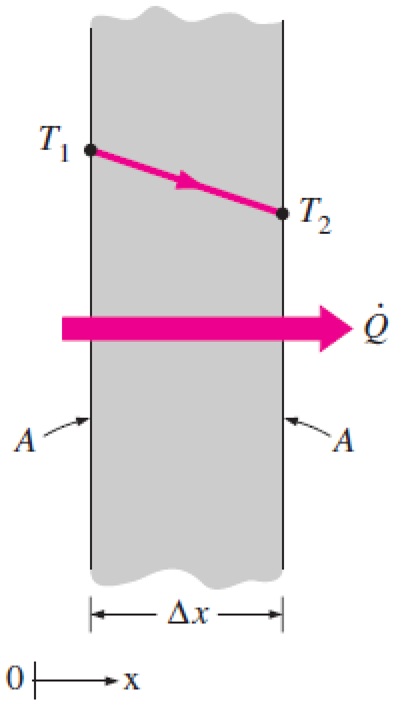
Heat conduction through Plane wall

Fourier Equation is

$$Q = kA\frac{T_1 - T_2}{\Delta x}$$

The heat flux

$$q = \frac{Q}{A} = k\frac{T_1 - T_2}{\Delta x}$$



Heat Conduction in a wall

$$\dot{Q} = \frac{T_1 - T_2}{\frac{\Delta x}{kA}} = \frac{T_1 - T_2}{R_{th}}$$

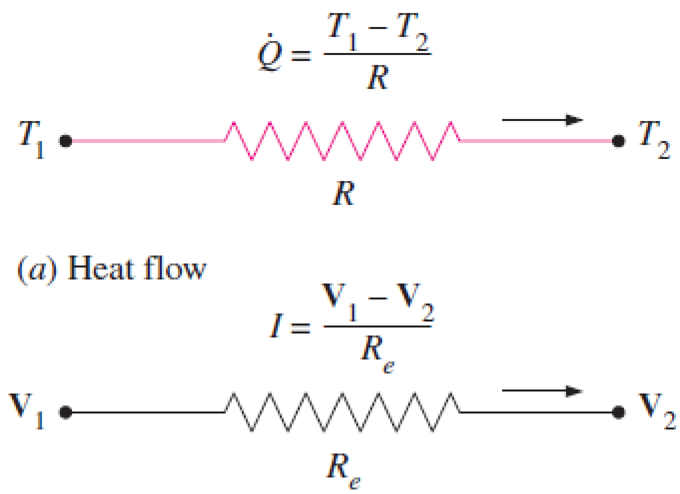
$$R_{th} = \frac{\Delta x}{kA}$$

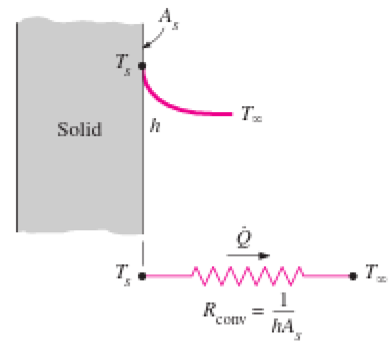
Convection Resistance

$$\dot{Q} = Ah(T_s - T_\infty)$$

$$\dot{Q} = \frac{T_s - T_\infty}{\frac{1}{Ah}} = \frac{(T_s - T_\infty)}{R_{conv}}$$

$$R_{conv} = \frac{1}{Ah}$$





Conduction in a wall

Radiation and convection combination.

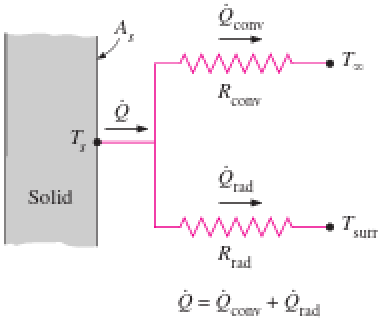
$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

$$Q = Ah_{conv}(T_s - T_\infty) + Ah_{rad}(T_s - T_\infty)$$

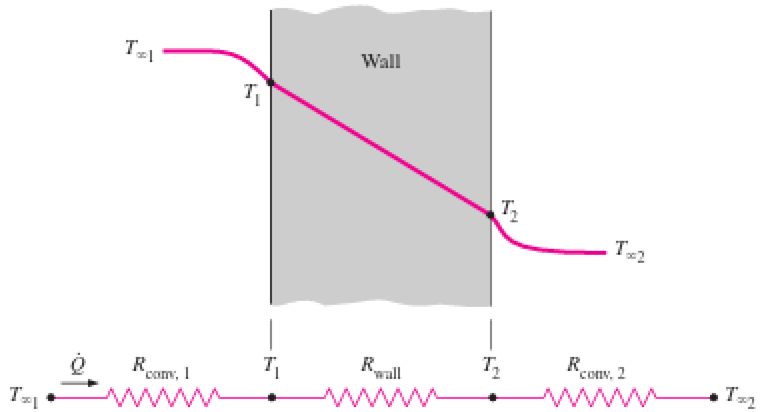
$$\dot{Q} = A(h_{conv} + h_{conv})(T_s - T_\infty)$$

$$\dot{Q} = Ah_{comb}(T_s - T_\infty)$$

$$h_{comb} = h_{conv} + h_{rad}$$



Thermal Resistance Network



$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

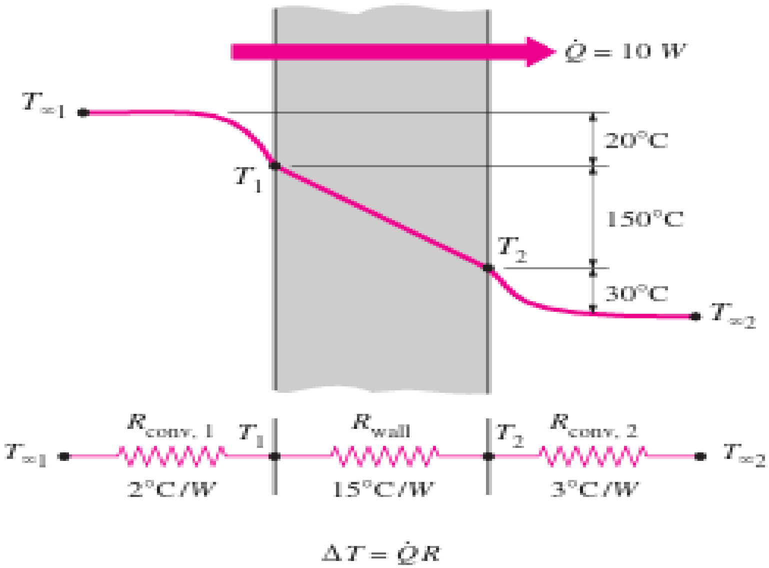
$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

$$\begin{aligned}\dot{Q} &= \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}\\ &= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}\end{aligned}$$

Conduction in a wall

Adding the numerators and denominators yields

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)}$$



where

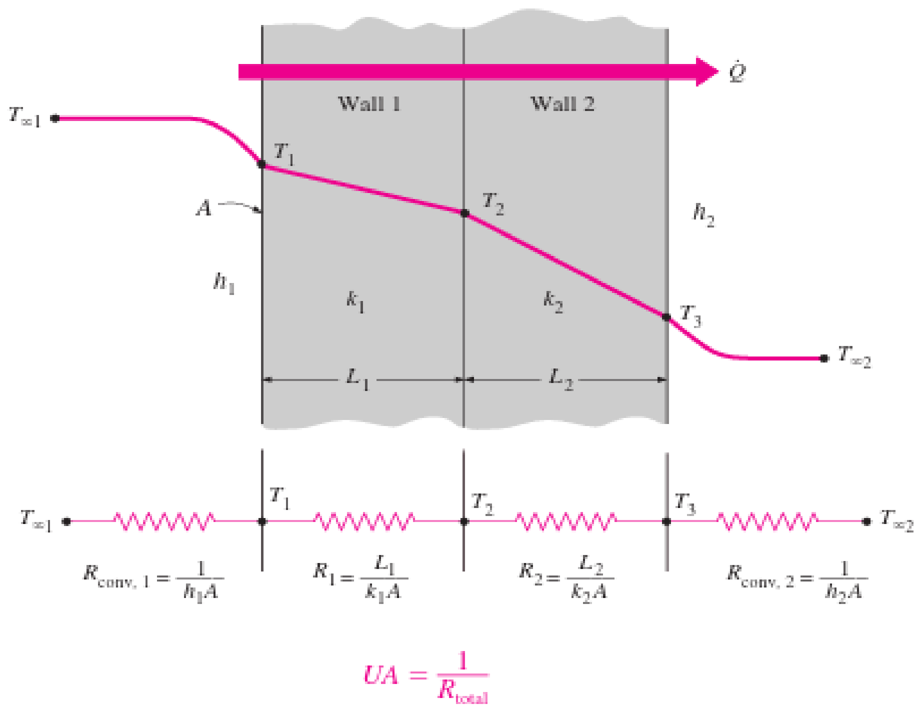
$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad \text{(°C/W)}$$

$$\dot{Q} = UA\,\Delta T \qquad \text{(W)}$$

Conduction in a wall

The over all heat transfer coefficient is U

$$UA = \frac{1}{R_{total}} \quad \rightarrow \quad U = \frac{1}{AR_{total}}$$



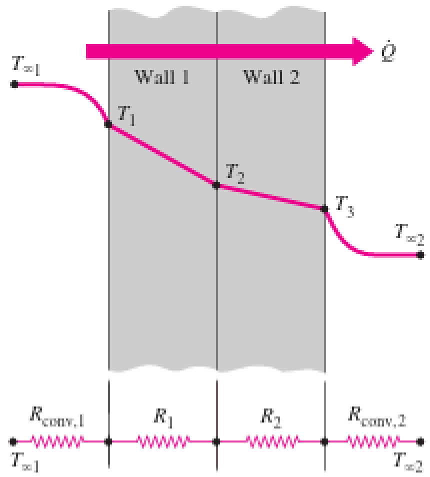
Conduction in a wall

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

Once $\dot{Q}$ is *known*, an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{wall, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

examples

Example.1. Determine the heat transfer across a plane wall of (20cm) thickness and a constant thermal conductivity of (10W/m.K). The temperatures of surface are steady at (120^{o}C) and (20^{o}). The area of the wall is ($2m^2$). Also determine the temperature gradient in the direction of flow and the temperature at the midpoint of the wall.

Examples

Solution: heat transfer by conduction through the wall of thickness Δx=20cm=0.2m and area A=$2m^2$. The surface temperatures are (T_1=120^oC) and (T_2=20^oC)

Property: constant thermal conductivity, k=10W/m.K

Assumption: Steady-state and one dimensional heat conduction

Analysis: The heat transfer through the wall can be determined by Fourier's law of conduction.

Examples

$$\dot{Q} = kA\frac{T_1 - T_2}{\Delta x} = (10W/m.K)(2m^2)\frac{(120-20)^o C}{0.2m} = 10000W = 10kW$$

The temperature gradient is dT/dx, and $\dot{Q} = -kA\frac{dT}{dx}$ then

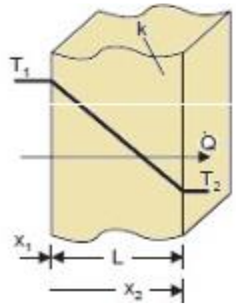
$$\frac{dT}{dx} = -\frac{\dot{Q}}{kA} = -\frac{10000}{10\times 2} = -500^o C/m$$

Or

$$\frac{dT}{dx} = \frac{T_2 - T_1}{\Delta x} = \frac{20-120}{0.2} = -500^o C/m$$

The temperature at the midpoint of the wall can be determined by Fourier's law

$$\dot{Q} = kA\frac{T_1 - T_m}{x_m - x_1} \rightarrow 10000 = (10)(2)\frac{120 - T_m}{0.1}$$



$$T_m = 120 - \frac{10000\times 0.1}{20} = 70^o C$$

Examples

Example 2. The temperature of gases in a furnace is (500K). The temperature of inside surface of the furnace is (400K). The heat transfer between the gases and the surface of the furnace occurs by convection and radiation. The convection coefficient is (20W/$m^2.^o C$). The furnace surface emissivity is (0.9). Find the combined heat transfer coefficient and the heat transfer rate per unit area.

Examples

Solution: heat transfer by convection and radiation between furnace surface and the hot gases T_s=400K, and T_∞=500K

Property: constant property h=20W/ $m^2.^o\, C$ and ε=0.9

Assumption: steady-state radiation and convection

Analysis: the heat transfer coefficient of radiation can be determined by

Examples:

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right) \qquad (W/m^2.K)$$

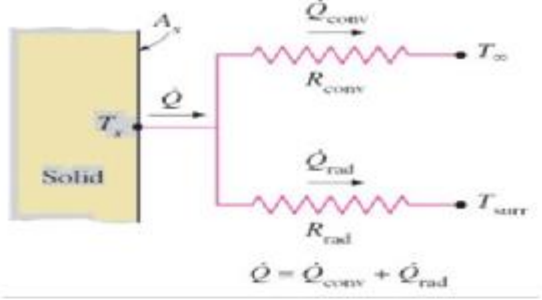
$$h_{rad} = 0.9\left(5.67\times10^{-8}\right)\left(400+500\right)\left(400^2+500^2\right) = 18.83W/m^2K$$

The combined heat transfer coefficient is calculated by

$$h_{combind} = h + h_{rad} = 20 + 18.83 = 38.83W/m^2.^oC$$

The heat transfer rate per area

$$\dot{q} = h_{combined}\left(T_\infty - T_s\right) = 38.83\left(500-400\right) = 3883W/m^2$$



Examples

Example 3 A wall of thickness (3cm) and thermal conductivity of (24W/$m.^{o}$C). The wall is exposed to heat transfer by convection on both sides. The temperature and coefficient of heat transfer on the inner face are (100^{o} C) and (10W/ $m^{2}.^{o}$ C) respectively. The temperature and coefficient of heat transfer on the other surface are (25^{o}C) and (20W/m^{2o}C) respectively. If the wall area is (4m^{2}), find the heat transfer rate. Determine the temperatures at the two sides of the wall.

Examples

Solution: A wall is exposed on its two sides to convection

h_1=10W/m^2.°C, $T_{\infty 1}$=100°C, h_2=20W/m^2.°C, $T_{\infty 2}$=25°C, Δx=3cm=0.03m

Property: constant thermal conductivity k=24W/m.°C

Assumption: Steady state and one-dimensional heat conduction with convection on the two sides of the wall.

Analysis: The overall heat transfer coefficient can be calculated from

$$U = \frac{1}{\left(\frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2}\right)} = \frac{1}{\frac{1}{10} + \frac{0.03}{24} + \frac{1}{20}} = 6.6121 W / m^2.^oC$$

$$\dot{Q} = AU\Delta T = 4m^2 \left(6.612 W / m^2.^oC\right)\left(100 - 25\right) = 1983.6 W$$

Examples

To find the temperature T_1 on the side facing the temperature of fluid (100°C) we can use the following equation

$$\dot{Q} = Ah_1(T_{\infty 1} - T_1) \rightarrow 1983.6 = (4m^2)(10W/m^2.^oC)(100 - T_1)$$

$$T_1 = 50.41^oC$$

And the other side of the wall temperature can be calculated by the following equation

$$\dot{Q} = \frac{(T_{\infty 1} - T_2)}{\left(\frac{1}{Ah_1} + \frac{\Delta x}{kA}\right)} \quad \rightarrow \quad 1983.6 = \frac{100 - T_2}{\frac{1}{4\times 10} + \frac{0.03}{24\times 4}} \quad \rightarrow T_2 = 49.79^oC$$

Or

$$\dot{Q} = Ah_2(T_2 - T_{\infty 2}) \rightarrow 1983.6 = (4m^2)(20W/m^2.^oC)(T_2 - 25)$$

$$T_2 = 49.79^oC$$

Examples

Example 4. A multilayer wall is made of three layers. Layer (1) is of thickness (4cm) with thermal conductivity of (24W/$m.^{o}$C). Layer (2) is of thickness (6cm) with thermal conductivity of (12W/$m.^{o}$C). Layer (3) is of thickness (2cm) and thermal conductivity (0.8W/$m.^{o}$C). The layer (1) and layer (3) are exposed to convection heat transfer at the outer faces. The temperatures and heat transfer coefficients are (120^{o}C) and (20W/$m^{2o}C$) and (10^{o}C) and (60W/$m^{2o}C$). Determine the heat transfer through the wall for area of 4.5m^{2}, the two sides temperatures and the interface temperatures.

Examples

Solution: multilayer plane wall consists of three layers with Δx_1=4cm, k_1 =24W/$m.^o$C, Δx_2=6cm, k2=12W/m^{2o}C, Δx_3=2cm k=0.8W/m^{2o}C, $h_{\infty 1}$=20W/$m^{2o}C$, $h_{\infty 2}$=60W/m^{2o}C, $T_{\infty 1}$=120^oC, T?2=10oC, A=4.5m^2

Property: Constant thermal conductivities Assumption: Steady state heat transfer by conduction through a plane wall

Analysis: firstly we calculate the thermal resistance of every layer and convection resistance on the two sides:

Examples

$$R_{conv,1} = \frac{1}{Ah_1} = \frac{1}{\left(4.5m^2\right)\left(20W/m^{2o}C\right)} = 0.111^oC/W$$

$$R_1 = \frac{\Delta x_1}{Ak_1} = \frac{0.04}{\left(4.5m^2\right)\left(24W/m.^oC\right)} = 0.00037^oC/W$$

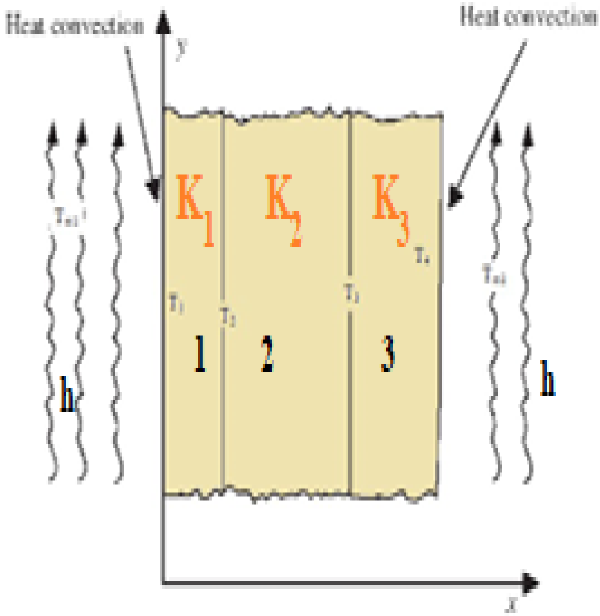
$$R_2 = \frac{\Delta x_2}{Ak_2} = \frac{0.06}{4.5 \times 12} = 0.00111^oC/W,$$

$$R_3 = \frac{\Delta x_3}{Ak_3} = \frac{0.02}{4.5 \times 0.8} = 0.00556^oC/W$$

$$R_{conv,2} = \frac{1}{Ah_2} = \frac{1}{4.5 \times 60} = 0.00370^oC/W$$

$$R_{total} = \sum R = 0.12174^oC/W$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{120-10}{0.12174} = 903.565W$$



Examples

To determine the temperature T_1 $\quad T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 120 - 903.565(0.111) = 19.7^o C$

And T_2 $\quad T_2 = T_{\infty 1} - \dot{Q}\left(R_{conv,1} + R_1\right) = 120 - 903.565(0.111 + 0.00037) = 19.37^o C$

So T_3

$$T_3 = T_{\infty 1} - \dot{Q}\left(R_{conv,1} + R_1 + R_2\right) = 120 - 903.565(0.111 + 0.00037 + 0.00111) = 18.37^o C$$

Also $\quad T_4 = T_{\infty 1} - \dot{Q}\left(R_{conv,1} + R_1 + R_2 + R_3\right)$

$$T_4 = 120 - 903.565(0.111 + 0.00037 + 0.00111 + 0.00556) = 13.343^o C$$

Examples

Example5 Consider a (1m) high and (1.5m) wide double glassing window consisting of two (thick layers (6mm) of glass (k=0.8W/$m.^{o}$C) separated by a stagnant air space (k=0.025W/m^{o}C) of (8mm) wide. Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surfaces for a day during which the room is maintained at (25^{o}C) while the temperature of the outdoor is (45^{o} C). Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be (10W/$m^{2o}C$) and (40W/$m^{2o}C$) which include the effects of radiation.

Examples

Solution: a double pane window of two glass layers separated by stagnant air space, L=1m, w= 1.5m, glass layer Δx_g=6mm, k_g=0.8W/$m.^oC$, stagnant air Δx_a=8mm, k_a=0.025W/$m.^oC$, T_i=25^{o}C, T_o=45^{o}C, h_i=10W/$m^2.^o C$, h_o=40W/$m^2.^o C$.

Property: constant property for air and glass

Assumption: steady state heat transfer and one-dimensional

Analysis: There are five resistance as shown in Fig. We determine these thermal resistance. The area of the window A=Lxw=1.5x 1=1.5m^2.

Examples

$$R_o = \frac{1}{h_o A} = \frac{1}{(40)\times 1.5} = \frac{1}{60} = 0.0167^o C/W$$

$$R_i = \frac{1}{h_i A} = \frac{1}{(10)\times 1.5} = \frac{1}{15} = 0.0667^o C/W$$

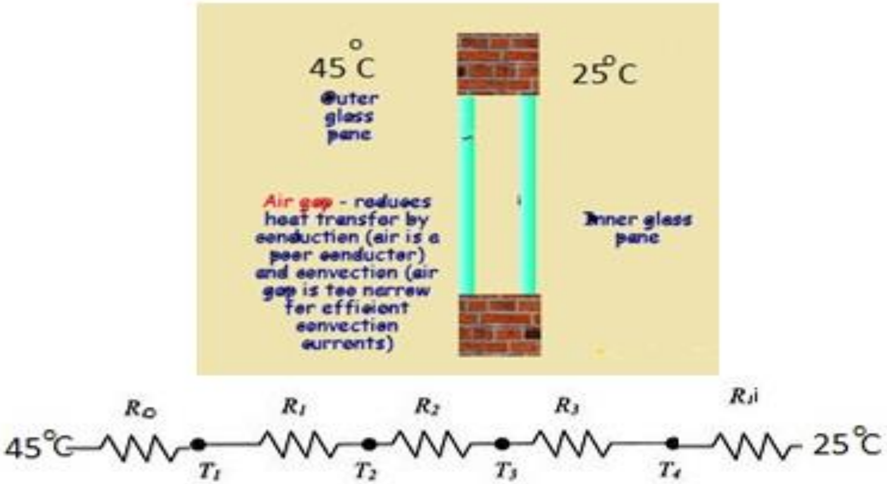
$$R_1 = R_3 = \frac{\Delta x}{A k_g} = \frac{0.006}{0.8\times 1.5} = 0.005^o C/W$$

$$R_2 = \frac{\Delta x_a}{A k_a} = \frac{0.008}{0.025\times 1.5} = 0.2133^o C/W$$

$$R_{total} = \sum R = 0.0167 + 0.0667 + 2\times 0.005 + 0.2133 = 0.3067^o C/W$$

$$\dot{Q} = \frac{T_o - T_i}{R_{total}} = \frac{45-25}{0.3067} = 65.21W$$

And $$\dot{Q} = \frac{T_4 - T_i}{R_i} = \frac{T_4 - 25}{0.0667} = 65.21W \rightarrow T_4 = 29.35^o C$$



Conduction in a wall

Thermal Resistance Networks in Parallel. The concept of thermal resistance or the electric analogy is used to solve problems of heat transfer that involved multilayer or combined parallel and series arrangement. Such problems are often two or three dimensional, but the solution is approximated by assuming the heat transfer in one-dimension using the thermal resistance networks.

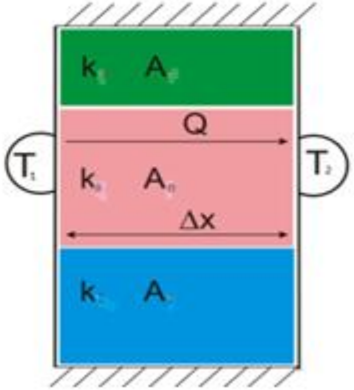
Conduction in a wall

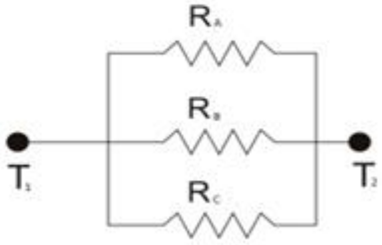
$$\dot{Q} = \dot{Q}_A + \dot{Q}_B + \dot{Q}_C = \frac{T_1 - T_2}{R_A} + \frac{T_1 - T_2}{R_B} + \frac{T_1 - T_2}{R_C} = (T_1 - T_2)\left\{\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right\}$$

Then $$\dot{Q} = \dot{Q}_A + \dot{Q}_B + \dot{Q}_C = (T_1 - T_2)\left\{\frac{1}{R_{total}}\right\}$$

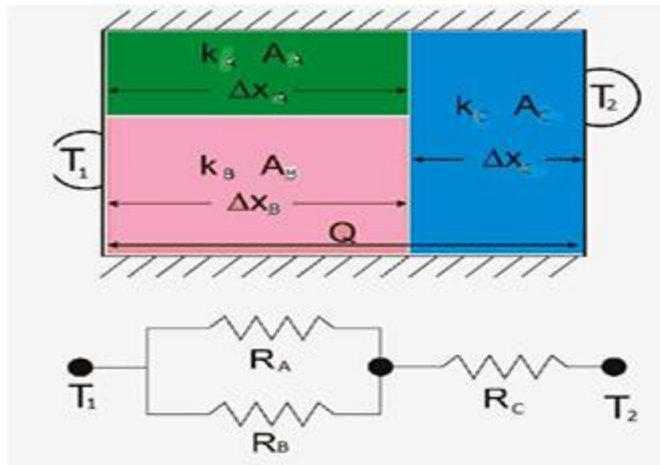
By utilizing the electrical resistance analogy $$\frac{1}{R_{total}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$$

Since the resistance are in parallel.





Conducton in a wall



$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

Where $$R_{total} = R_{A-B} + R_C = \frac{R_A R_B}{R_A + R_B} + R_C$$

And $$R_A = \frac{\Delta x_A}{A_A k_A}, \quad R_B = \frac{\Delta x_B}{A_B k_B}, \quad R_C = \frac{\Delta x_C}{A_C k_C}$$

Conduction in a wall

Example 6 A wall of (3m)high and (4m) wide consists a long (16cmx22cm)cross- section horizontal bricks of thermal conductivity (0.72W/$m.^{o}$C), separated by (3cm) thick plaster layer of thermal conductivity (0.21W/$m.^{o}$C). There are (2.5cm) thick plasters on the brick on each of its sides and (3cm) of rigid foam with thermal conductivity (0.026W/$m.^{o}$C) on the inner wall side as shown in Figure. 3. The temperatures at indoor and outdoor are (22^{o}C) and (47^{o}C), and the coefficients of convection heat transfer on inner and outer sides are (12W/$m^2.^{o}\,C$) and (42W/$m^2.^{o}\,C$). Determine heat transfer rate through the wall neglecting radiation by assuming it one-dimensional heat transfer .

Conduction in a wall

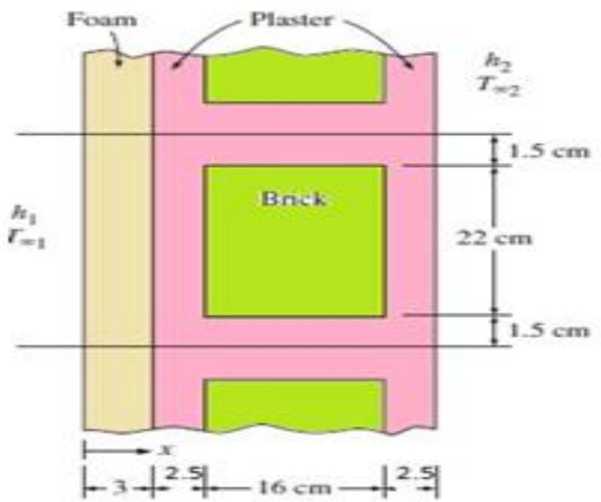
Solution: a wall of brick of cross-section area of (16cmx22cm)with a layer of plaster on each side and rigid foam layer on the inner side the wall is exposed to convection heat transfer on the two sides. From Figure. we can determine the thermal resistance for each component in this composite wall.

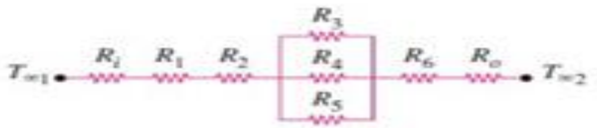
Property: constant thermal conductivity

Assumption: steady state one- dimensional heat transfer, and no effect of radiation

Analysis: firstly we calculate the resistance of a pattern as shown in the Figure.

Conduction in a wall





$$R_i = \frac{1}{h_i A_i} = \frac{1}{12(1\times 0.25)} = 0.333°C/W$$

$$R_1 = \frac{\Delta x_1}{k_1 A_1} = \frac{0.03}{(0.026)(1\times 0.25)} = 4.615°C/W$$

Conduction in a wall

$$R_2 = R_6 = \frac{\Delta x_2}{k_2 A_2} = \frac{0.025}{(0.21)(1\times 0.25)} = 0.4762^o C/W$$

$$R_3 = R_5 = \frac{\Delta x_3}{k_3 A_3} = \frac{0.21}{(0.21)(1\times 0.015)} = 50.793^o C/W$$

$$R_4 = \frac{\Delta x_4}{k_4 A_4} = \frac{0.16}{(0.72)(1\times 0.22)} = 1.0101^o C/W$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{42(1\times 0.25)} = 0.0952^o C/W$$

For parallel resistance the equivalent resistance is

$$\frac{1}{R_{ep}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{50.793} + \frac{1}{1.0101} + \frac{1}{50.793} = \frac{1}{0.9715} \quad R_{ep} = 1.0294^o C/W$$

The total resistance is

$$R_{total} = R_i + R_1 + R_2 + R_{ep} + R_6 + R_o$$

$$= 0.333 + 4.615 + 0.4762 + 1.0294 + 0.4762 + 0.0952 = 7.025$$

The heat transfer to the pattern

$$\dot{Q}_p = \frac{T_o - T_i}{R_{total}} = \frac{47-22}{7.025} = 3.5587W$$

$$\dot{Q} = 4\times(3/0.25)\times 3.5587 = 170.818W$$

Conduction in Cylinders

HEAT CONDUCTION IN CYLINDERS Let us consider a layer of long cylinder (long circular pipe) with inner and outer radii of (r_1 and r_2) respectively. The length is L. Thermal conductivity of material, the cylindrical layer made of is k. The temperature of the two surfaces of the cylindrical layer are T_1 and T_2. By considering that the thermal conductivity is constant and no heat generation, we have T(r) only. The Fourier's law of heat conduction can be expressed for heat flow through the cylindrical layer as expressed for heat flow through the cylindrical layer as

$$\dot{Q} = -kA\frac{dT}{dr}$$

Where the heat transfer area is A=2πrL at a radius r.

A is depending on r and thus it varies as r is changed in the heat transfer direction. By the variables separating in this eq. and integrating it from r=r_1 to r=r_2

where T(r_1)=T_1 and T(r_2)=T_2, gives

Conduction in cylindrical wall

$$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{A} dr = -\int_{T_1}^{T_2} k dT$$

Substituting A=2πrL, then $$\int_{r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{2\pi L} \frac{dr}{r} = -\int_{T_1}^{T_2} k dT$$

Since $\dot{Q}_{cond,cyl}$ is constant the integration becomes

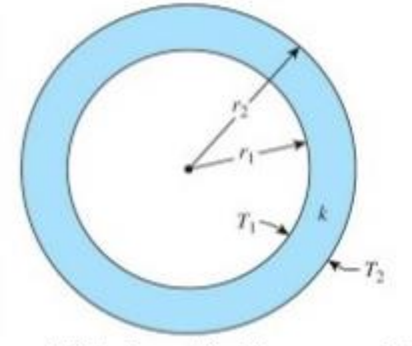
$$\frac{\dot{Q}_{cond,cyl}}{2\pi L} \ln\frac{r_2}{r_1} = k(T_1 - T_2)$$

By rearranging this equation, we get

$$\dot{Q}_{cond,cyl} = \frac{(T_1 - T_2)}{\frac{1}{2\pi L k}\ln\frac{r_2}{r_1}}$$

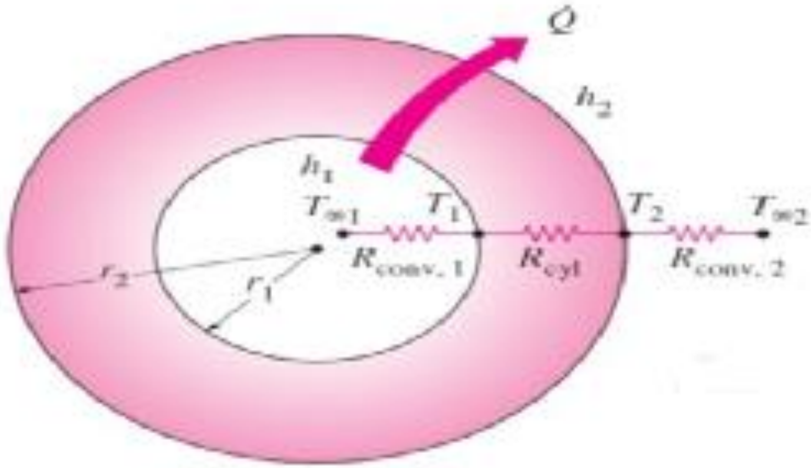
This equation can be rearranged to $$\dot{Q}_{cond,cyl} = \frac{(T_1 - T_2)}{R_{cyl}}$$

Where $$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi L k} = \frac{\ln(outer\ radius / inner\ radius)}{2\pi \times (length) \times (thermal\ conductivity)}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

Conduction in cylindrical wall



$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$$

The Resistance Network for Cylindrical shell With Convection

$$R_{total} = R_{conv.1} + R_{cyl} + R_{conv.2}$$

$$R_{total} = \frac{1}{2\pi L r_1 h_1} + \frac{1}{2\pi L k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi L r_2 h_2}$$

Conduction in a cylindrical wall

Example 7 Cylindrical shell of inner diameter (0.2m) and outer diameter (0.4m). The thermal conductivity of shell material is (60W/$m.^{o}$C). The shell is exposed to heat convection on its both sides. The temperature inside and outside the shell are (80^{o} C) and (25^{o} C)respectively. The heat transfer coefficient on inside and outside the shell are (40W/$m^2.^{o}\,C$) and (10W/$m^2.^{o}\,C$). Determine the heat transfer through the shell for (1m) long.

Examples

Solution: cylindrical shell exposed to convection on its two surfaces. D_1 =0.2m and D_2=0.4m the temperature of fluids in and out the shell are T_1=80^oC, T_2=25^oC

Property: constant thermal conductivity k=60W/m.^{o}C, constant heat transfer coefficient on the two sides of the shell are h_1=40W/$m^2.^oC$, and h_2=10 W/$m^2.^oC$.

Assumption: steady one dimensional heat transfer through the shell.

Analysis: to calculate the heat transfer through the cylindrical shell, we can find the thermal resistance as following.

Convection resistance inside the shell

examples

$$R_{con.1} = \frac{1}{2\pi r_1 L h_1} = \frac{1}{\pi D_1 L h_1} = \frac{1}{\pi (0.2m)(1m)(40W/m^2.^oC)} = 0.0398^o C/W$$

Conduction resistance of the shell

$$R_{cond} = \frac{1}{2\pi L k}\ln\left(\frac{r_2}{r_1}\right) = \frac{1}{2\pi L k}\ln\left(\frac{D_2}{D_1}\right) = \frac{1}{2\pi(1m)(60W/m.^oC)}\ln\left(\frac{0.4}{0.2}\right) = 0.00184^o C/W$$

Convection resistance outside the shell

$$R_{con.2} = \frac{1}{2\pi r_2 L h_2} = \frac{1}{\pi D_2 L h_2} = \frac{1}{\pi (0.4m)(1m)(10W/m^2.^oC)} = 0.0796^o C/W$$

The total thermal resistance of heat transfer is

$$R_{total} = R_{conv.1} + R_{cyl} + R_{conv.2}$$

$$R_{total} = (0.0398 + 0.00184 + 0.0796)^o C/W = 0.12124^o C/W$$

The heat transfer per unit length will be

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-25)^o C}{0.12124^o C/W} = 453.65W$$

Conduction in a cylindrical wall

The heat transfer by steady state through shell of multi-cylinders can be treated just like multilayer discussed before. Simply make summation for the resistances in series for all layers. An example is steady state heat transfer through the composite cylinder of two layers of length L shown in Fig. 3.16 with convection affects on each sides can be represented as.

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{total}}$$

Where the total thermal resistance is R_{total} , represented as

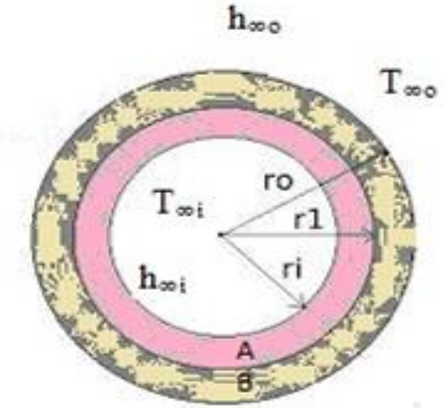
$$R_{total} = R_{conv,i} + R_{cyl,1} + R_{cyl,2} + R_{conv,o}$$

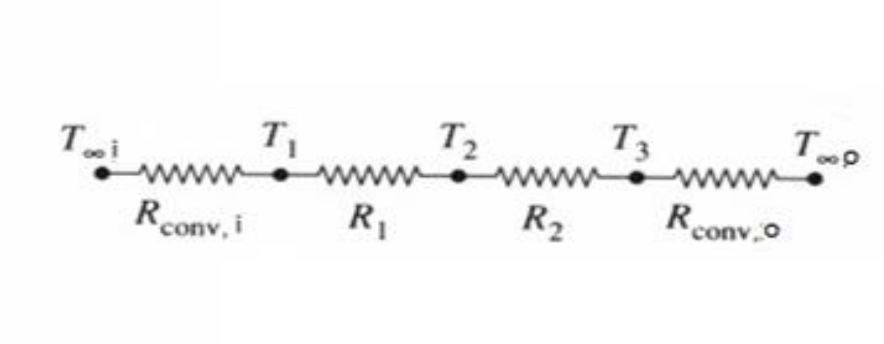
$$R_{total} = \frac{1}{A_i h_{\infty i}} + \frac{1}{2\pi L k_A} \ln\left(\frac{r_1}{r_i}\right) + \frac{1}{2\pi L k_B} \ln\left(\frac{r_o}{r_1}\right) + \frac{1}{A_o h_{\infty o}}$$

Where $A_i=2\pi r_i L h_{\infty i}$ and $A_o=2\pi r_o L h_{\infty o}$

As $\dot{Q}$ is known, the temperature can be calculated by applying the relation $\dot{Q} = \frac{T_{\infty i} - T_j}{R_{total,i-j}}$ across a layer or layers that involves a known temperature T_i on one side and the unknown temperature T_j at the other side. As in Fig. $\dot{Q}$ is calculated firstly, and then the interface temperature T_j at any location between two layers or on the surfaces. They are

Conduction in a cylindrical wall





$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{total}}$$

$$\dot{Q} = \frac{T_{\infty i} - T_i}{R_{conv,1}} = \frac{T_i - T_{\infty o}}{R_{cyl,1} + R_{cyl,2} + R_{conv,o}}$$

$$\dot{Q} = \frac{T_{\infty i} - T_1}{R_{conv,1} + R_{cyl,1}} = \frac{T_1 - T_{\infty o}}{R_{cyl,2} + R_{conv,o}}$$

$$\dot{Q} = \frac{T_{\infty i} - T_o}{R_{conv,i} + R_{cyl,1} + R_{cyl,2}} = \frac{T_o - T_{\infty o}}{R_{conv,o}}$$

Examples

Example .8 Steam at (300^oC) flows in a cast iron pipe (k=75W/$m.^oC$). The inner and outer diameters of the pipe are (5cm) and (5.5cm)respectively. It is covered with glass wool insulation of (4cm) thick and thermal conductivity (0.05W/$m.^oC$). Heat is transfer to the surrounding at (25^oC) by free convection and radiation, with a coefficient of combined heat transfer to be (20W/$m^2.^oC$). Taking the heat transfer coefficient inside the pipe to be (65W/$m^2.^oC$). Determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature at interface between the pipe and insulation and the inner and outer surfaces temperature.

Examples

Solution: A steam pipe of cast iron covered with glass wool insulation is subjected with convection on inner surface and combined convection and radiation on the outer surface.

D_1=5cm, D_2=5.5cm, Insulation thickness t=4cm. D_3=5.5+8=13.5cm

Property: the thermal conductivities of materials in this problem are constant and they are: k_A=75W/$m.^o C$, and k_B=0.05W/$m.^o C$. The coefficient of heat transfer inside the tube is: $h_{\infty i}$=65W/$m^2.^o C$,

outside the insulation is: $h_{\infty o}$=20W/$m^2.^o C$. $T_{\infty i}$=$300^o C$, $T_{\infty o}$=$25^o C$

Assumption: The heat transfer is steady state and one –dimensional

Analysis: from the data of the example, we can fined

Examples

Inner surface area $A_i = 2\pi r_1 L = \pi D_1 L = \pi(0.05m)(1m) = 0.157m^2$

Outer surface of insulation $A_o = 2\pi r_3 L = \pi D_3 L = \pi(0.135m)(1m) = 0.424m^2$

Thermal resistance of inner convection $R_{conv,i} = \dfrac{1}{A_i h_{\infty i}} = \dfrac{1}{(0.157)(65)} = 0.098^o C/W$

Thermal resistance of the pipe material

$$R_A = \frac{1}{2\pi L k_A}\ln\left(\frac{r_2}{r_1}\right) = \frac{1}{2\pi(1m)(75)}\ln\left(\frac{5.5}{5}\right) = 0.000202^o C/W$$

Thermal resistance of insulation

$$R_B = \frac{1}{2\pi L k_B}\ln\left(\frac{r_3}{r_2}\right) = \frac{1}{2\pi(1m)(0.05)}\ln\left(\frac{13.5}{5.5}\right) = 2.858^o C/W$$

Thermal resistance of outer combined convection and radiation

Examples

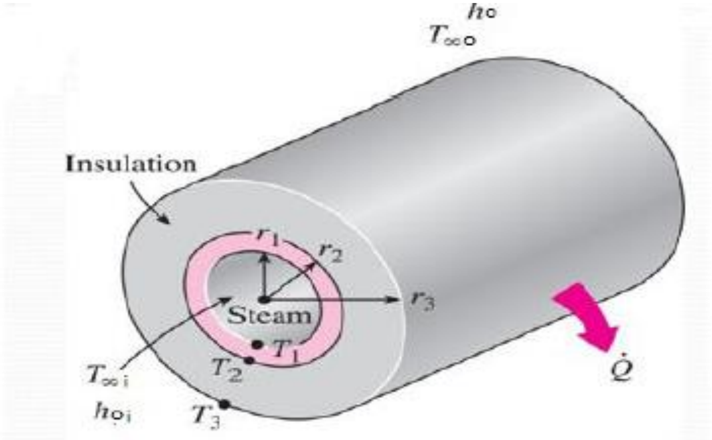


Fig. 3.17 Schematic For Example 3.8

$$R_{conv,o} = \frac{1}{A_o h_{\infty o}} = \frac{1}{(0.424)(20)} = 0.118°C/W$$

The total resistance $R_{total} = \sum R = 0.098 + 0.000202 + 2.858 + 0.118 = 3.0742°C/W$

The heat lost from steam is equal to heat transfer from the pipe

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{total}} = \frac{300 - 25}{3.0742} = 89.454W$$

The temperature of the inner surface is calculated by

$$\dot{Q} = \frac{T_{\infty i} - T_1}{R_{conv,i}} = \frac{300 - T_1}{0.098} = 89.454W \rightarrow T_1 = 291.233°C$$

Examples

The temperature of the inner surface is calculated by

$$\dot{Q} = \frac{T_{\infty i} - T_1}{R_{conv,i}} = \frac{300 - T_1}{0.098} = 89.454W \rightarrow T_1 = 291.233^oC$$

The temperature of the interface between pipe and insulation

$$\dot{Q} = \frac{T_{\infty i} - T_2}{R_{conv,i} + R_A} = \frac{300 - T_2}{0.098 + 0.000202} = 89.454W \rightarrow T_2 = 291.215^oC$$

The temperature of outer surface

$$\dot{Q} = \frac{T_{\infty i} - T_3}{R_{conv,i} + R_A + R_B} = \frac{300 - T_3}{0.098 + 0.000202 + 2.858} = 89.454W \rightarrow T_2 = 35.556^oC$$

Or $$\dot{Q} = \frac{T_3 - T_{\infty o}}{R_{conv,o}} = \frac{T_3 - 25}{0.118} = 89.454W \rightarrow T_2 = 35.556^oC$$

Overall Heat Transfer Coefficient

Overall Heat Transfer Coefficient For Cylindrical Wall

The heat transfer equation for cylindrical wall can rewrite in the form of the overall heat

transfer coefficient as follows

$$\dot{Q} = AU\Delta T = \frac{\Delta T}{R_{total}}$$

Or

$$AU = \frac{1}{R_{total}} = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{2\pi L k_1}\ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{1}{h_o A_o}}$$

Where r_i is the inner radius and it is equal to r_1. And r_o is outer radius and it is equal the radius of the outer cover on the cylinder. $A_i = 2\pi L r_i$, and $A_o = 2\pi L r_o$. We can assume that

Over all Heat Transfer Coefficient

.

$$AU = A_i U_i = A_o U_o$$

U_i, U_o are overall heat transfer coefficient based on inner surface and outer surface respectively. The Eq.(3.41) can be rewritten as:

$$\frac{1}{AU} = \frac{1}{A_i U_i} = \frac{1}{A_o U_o} = \frac{1}{h_i A_i} + \frac{1}{2\pi L k_1} \ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{1}{h_o A_o}$$

The overall heat transfer coefficient based on inner surface is calculated from the following equation:

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{r_i}{h_o r_o}$$

The overall heat transfer coefficient based on outer surface is calculated from the following equation.

Overall Heat Transfer Coefficient

The overall heat transfer coefficient based on inner surface is calculated from the following equation:

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{r_i}{h_o r_o}$$

The overall heat transfer coefficient based on outer surface is calculated from the following equation.

$$\frac{1}{U_o} = \frac{r_o}{h_i r_i} + \frac{r_o}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{1}{h_o}$$

Examples

Example 9 Determine Uo , the overall heat transfer coefficient based on the outer surface for steel pipe of inner diameter (3.0cm) and outer diameter (3.7cm). thermal conductivity of steel that the pipe made of is (54.0W/ $m.^{o}C$). The inside and out side heat transfer coefficient of the pipe are (1000W/ $m^2.^{o}$ C) and (2000W/$m^2.^{o}C$). if the temperature of inner fluid and outer fluid are ($500^{o}C$) and ($200^{o}C$), calculate the heat transfer along one meter length and the temperature of inner and outer surface.

Examples

Solution: steel pipe is exposed to convection from its both sides. The U_o, rate of heat transfer from the pipe per unit meter length and the temperature of inner and outer surfaces of the pipe are to be calculated. Dimensions of the pipe D_i=3cm, D_o=3.7cm. then r_i=1.5cm=0.015m, r_o=1.85cm=0.0185m $T_i=500^oC$, $T_o=200^oC$

Property: The thermal conductivity of steel $k=54 W/m.^oC$, and $h_i=1000 W/m^2.^oC$, $h_o=2000 W/m^2.^oC$

Assumption: One dimensional heat transfer through a cylindrical wall and steady state.

Analysis: we can utilize the eq. to find the overall heat transfer coefficient based on outer surface for a pipe of single layer,

$$\frac{1}{U_o} = \frac{r_o}{h_i r_i} + \frac{r_o}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \cdots + \frac{1}{h_o}$$

Examples

And $$\frac{1}{U_o} = \frac{1.85}{1.5(1000)} + \frac{0.0185}{54}\ln\left(\frac{1.85}{1.5}\right) + \frac{1}{2000} \quad , U_o = 554 W/m^2.^oC$$

To determine the heat transfer rate from eq.(3.40)

$$\dot{Q} = A_o U_o \Delta T$$

And $A_o = \pi D_o L = \pi(0.037\text{m})\text{x}1.0\text{m} = 0.11624\text{m}^2$, $A_i = 0.09425\text{m}^2$ then

$$\dot{Q} = A_o U_o \Delta T = (0.11624)(554)(500 - 200) = 19319.1W$$

Examples

To fined the temperature of the inner surface

$$(T_i - T_1) = \dot{Q} / (h_i A_i)$$

$$(500 - T_1) = (19319)/(0.09425)(1000) = 205^{\circ}C \rightarrow T_1 = 295^{\circ}C$$

The temperature of the outer surface

$$(T_2 - T_o) = \dot{Q} / (h_o A_o)$$

$$(T_2 - 200) = (19319)/(0.037\pi)(2000) = 83.0^{\circ}C \rightarrow T_1 = 283^{\circ}C$$

To check the solution we cal calculate the heat transfer by conduction through the wall only from Fourier equation through cylinder wall

$$\dot{Q} = 2\pi Lk(T_1 - T_2) / \ln\left(\frac{r_2}{r_1}\right)$$

$$= 2\pi(1.0m)(54W / m.^{\circ}C)(295 - 283) / \ln\left(\frac{1.85}{1.5}\right) = 19319W$$

Conduction in Sphere

HEAT CONDUCTION IN SPHERE

Let us consider a spherical layer of inner and outer radii r_1 and r_2 respectively as shown Fig. 3.18. The thermal conductivity is of constant value k W/m.°C. The temperature of the two surfaces are T_1 and T_2. There is no heat generation. Then T is function of r. Fourier's law of heat transfer by conduction through spherical layer can represented by

$$\dot{Q} = -kA\frac{dT}{dr}$$

Where $A=4\pi r^2$ is the surface area of heat transfer at location r. A is depending on r, and so it varies in direction of heat transfer. By using the principle of variable separation in eq.(3.46) as $T(r_1)=T_1$ and $T(r_2)=T_2$

$$\int_{r=r_1}^{r=r_2} \frac{\dot{Q}_{cond,sph}}{A}\,dr = -\int_{T=T_1}^{T=T_2} k\,dT$$

Substituting $A=4\pi r^2$, then

$$\int_{r=r_1}^{r=r_2} \frac{\dot{Q}_{cond,sph}}{2\pi r^2}\,dr = -\int_{T=T_1}^{T=T_2} k\,dT$$

Since the heat transfer by conduction through the shell is constant ($\dot{Q}_{cond,sph} = const.$)

Conduction in Sphere

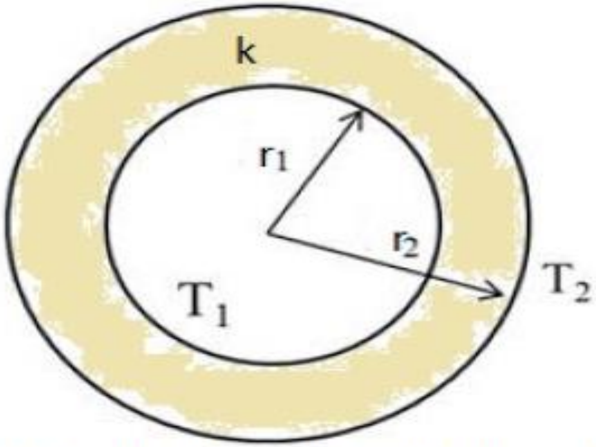


Fig. 3.18. Conduction Through Spherical Layer

$$\frac{\dot{Q}_{cond.sph}}{4\pi}\left\{\frac{1}{r_1}-\frac{1}{r_2}\right\}=k(T_1-T_2)$$

By rearranging this equation, we get that

$$\dot{Q}_{cond.sph}=4\pi k\frac{(T_1-T_2)}{\left\{\frac{1}{r_1}-\frac{1}{r_2}\right\}}$$

This equation can be rearranged as

$$\dot{Q}_{cond.sph}=\frac{(T_1-T_2)}{R_{sph}}$$

Conduction in Spheres

$$\dot{Q}_{cond.sph} = \frac{(T_1 - T_2)}{R_{sph}}$$

Where $R_{sph} = \dfrac{r_2 - r_1}{4\pi k r_1 r_2} = \dfrac{(outer\ radius - inner\ radius)}{4\pi \times (Thermal\ conductivity)(inner\ radius)(outer\ radius)}$

is the resistance of thermal conductance of the spherical layer or the spherical layer conduction resistance.

Now let us consider heat transfer in one-dimensional direction through a spherical layer that exposed to heat transfer by convection on the two sides with temperatures $T_{\infty i}$ and $T_{\infty o}$ and coefficients of heat transfer h_i and h_o respectively. Fig. shows schematic of the model. In this case the thermal resistance consists of one conduction resistance and two convection resistances in series, like that one of the plane wall. The heat flow rate of under condition of steady can be represented as following.

$$\dot{Q}_{sph} = \frac{T_{\infty i} - T_{\infty o}}{R_{total}}$$

Where $$R_{total} = R_{conv,i} + R_{cond} + R_{conv,o}$$

Conduction in Spheres

$$R_{total} = \frac{1}{A_i h_i} + \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{1}{A_o h_o}$$

$$\dot{Q}_{sph} = AU\left(T_i - T_o\right)$$

And $$AU = A_i U_i = A_o U_o$$

$A_o = 4\pi r_o^2$ and $A_i = 4\pi r_i^2$ U_i, U_o are as defined before, so that

$$A_i U_i = A_o U_o = \frac{1}{R_{total}} = \frac{1}{\frac{1}{A_i h_i} + \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{1}{A_o h_o}}$$

Then $$U_i = \frac{1}{\frac{A_i}{A_i h_i} + \frac{A_i}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{A_i}{A_o h_o}}$$

Conduction in Spheres

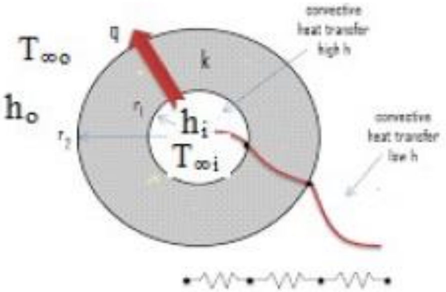
From the value of A_i and A_o we obtain:

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i^2}{k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{r_i^2}{r_o^2 h_o}}$$

This is called the overall heat transfer coefficient based on inner area, and

$$U_o = \frac{1}{\frac{r_o^2}{r_i^2 h_i} + \frac{r_o^2}{k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{1}{h_o}}$$

And this is called overall heat transfer coefficient based on outer area



Schematic for Overall heat Transfer coefficient of Sphere Layer.

Examples

Example 10. A spherical tank is of a internal and external diameter (4m) and (4.04m)respectively. The tank material is stainless steel of thermal conductivity (16W/m.^{o}C). The tank contains iced water at ($0^{o}C$) and located in a room of temperature ($23^{o}C$). The heat transfer coefficient in and out the sphere are (90W/$m^{2}.^{o}C$) and (8W/$m^{2}.^{o}C$)respectively. Determine the heat transfer rate to the iced water in the tank and the ice mass that melts at ($0^{o}C$). the latent heat for ice water melting is (333.7kJ/kg)

Examples

Solution: the heat transfer through spherical shell exposed to convection on both sides. The inner diameter D_i=4m and outer diameter D_o=4.04m then the radii are r_i=2.0m and r_o=2.02m. the temperatures inside and outside the shell are $T_{\infty i}=0^oC$, and $T_{\infty o}=23^oC$.
Property: Constant property k=16W/m.°C, latent heat h_{fg}=333.7 kJ/kg and $h_i=90W/m^2.^oC$, and $h_o=8W/m^2.^oC$
Assumption: one-dimensional heat conduction with steady state
Analysis: determine the heat transfer through the shell, we must calculate the overall heat transfer coefficient based on the inner or outer area. The coefficient of heat transfer based on the inner surface area is

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i^2}{k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{r_i^2}{r_o^2 h_o}} = \frac{1}{\frac{1}{90} + \frac{2^2}{16}\left(\frac{1}{2} - \frac{1}{2.02}\right) + \frac{2^2}{2.02^2(8)}} = 7.414W/m^2.^oC$$

Examples

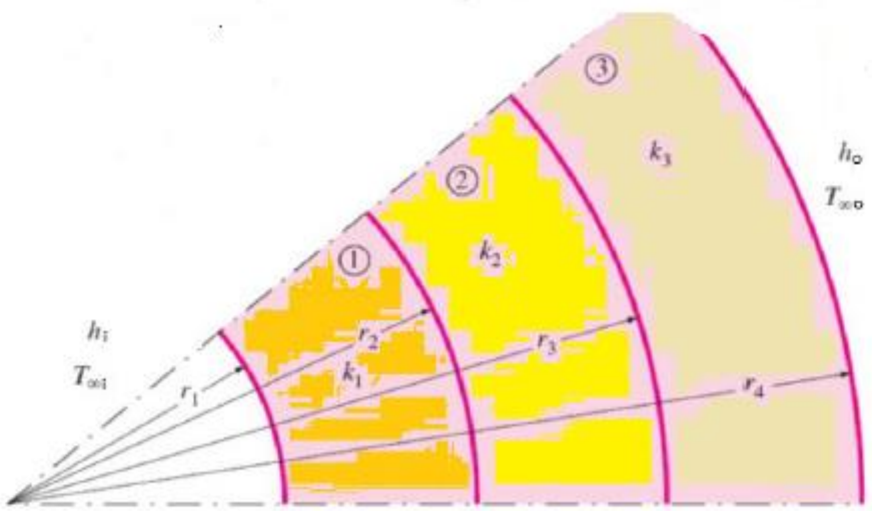
To calculate the amount of iced water to melted during (24hr). By using the following equation: the heat transfer rate multiplying by the time dividing by the latent heat

$$m = \frac{\dot{Q} \times time}{h_{fg}} = \frac{8.571(24 \times 3600)}{333.7} = 2219.16kg$$

Conduction in Multi Sphere

For multilayer spheres the total thermal resistance can be determined like in that of multilayer plane wall as noted in Fig. 3.20. from the basic equation of heat transfer

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{total}}$$



Conduction in Multi Sphere

Where $$R_{total} = R_{conv,i} + R_{cond.1} + R_{cond,2} + R_{cond.3} + R_{conv,o}$$

By substituting this in equation of heat transfer as

$$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{R_{conv,i} + R_{cond.1} + R_{cond,2} + R_{cond.3} + R_{conv,o}}$$

Then $$\dot{Q} = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{4\pi r_1^2 h_i} + \frac{1}{4k_1}\left(\frac{r_2 - r_1}{r_1 r_2}\right) + \frac{1}{4k_2}\left(\frac{r_3 - r_2}{r_2 r_3}\right) + \frac{1}{4k_3}\left(\frac{r_4 - r_3}{r_3 r_4}\right) + \frac{1}{4\pi r_4^2 h_o}}$$

Once $\dot{Q}$ is known, we can calculate the interface temperature T_j between any two layers or on inner and outer surface just like that of the plane wall.

Examples

Example 11. A sphere carrying steam at (240^oC)has an internal diameter of (2m). The spherical shell thickness is (0.075m). The thermal conductivity of the sphere material is (50W/$m.^o$C). The convective heat transfer coefficient on the inside is (1.1W/$m^2.^o C$). The sphere is covered by two insulation layers, one of (5cm)thickness of thermal conductivity of (0.15W/$m.^o C$) and the another (5cm) thickness and thermal conductivity of (0.475W/$m.^o C$). The outside is exposed to air of temperature (40^oC) with heat transfer coefficient of (18W/$m^2.^o C$). Determine (a)The overall heat transfer coefficient based on the outer and inner area, (b) Heat transfer rate from the spherical wall, (c) also determine the temperature of the interface between the shell and insulation and between the two insulation layers.

Examples

Solution: A sphere contains steam is insulated by two layers of insulation with following data D_i=2m, $r_1=r_i$=1m, r_2=1+0.075=1.075m, r_3=1.075+0.05=1.125m and $r_o=r_4$=1.25+0.05=1.175m, $h_{\infty i}$ =1.1W/m^2.°C, $T_{\infty i}$ =240°C, $h_{\infty o}$=18W/m^2.°C, $T_{\infty o}$=40°C. the schematic diagram is shown in Fig. 3.21

Property: the thermal conductivity of materials is constant k_A=50W/m.°C, k_B=0.15W/m.°C, k_C=0.475W/m.°C.

Assumption: one-dimensional heat conduction

Solution; Firstly we can find the inner and outer surface area.

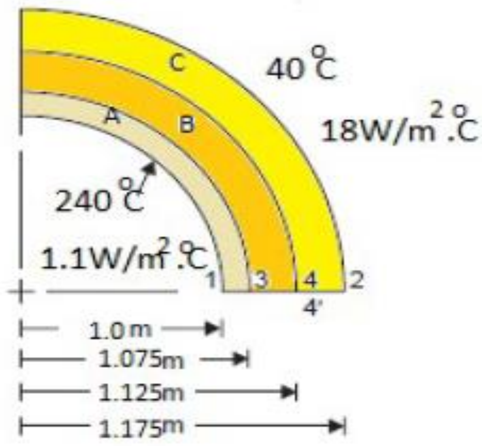
$$A_i = 4\pi r_i^2 = 4\pi(1)^2 = 12.566m^2 \quad A_o = 4\pi r_o^2 = 4\pi(1.175)^2 = 17.349m^2$$

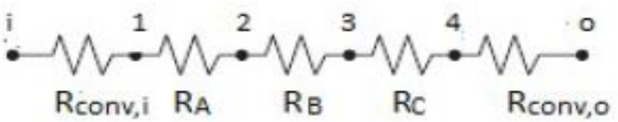
The overall heat transfer coefficient

$$U_i = \frac{1}{4\pi r_i^2\left\{\frac{1}{4\pi r_1^2 h_i} + \frac{1}{4\pi k_A}\left(\frac{r_2 - r_1}{r_1 r_2}\right) + \frac{1}{4\pi k_B}\left(\frac{r_3 - r_2}{r_2 r_3}\right) + \frac{1}{4\pi k_C}\left(\frac{r_4 - r_3}{r_3 r_4}\right) + \frac{1}{4\pi r_4^2 h_o}\right\}}$$

$$U_i = \frac{1}{\left\{\frac{1}{h_i} + \frac{r_1^2}{k_A}\left(\frac{r_2 - r_1}{r_1 r_2}\right) + \frac{r_1^2}{k_B}\left(\frac{r_3 - r_2}{r_2 r_3}\right) + \frac{r_1^2}{k_C}\left(\frac{r_4 - r_3}{r_3 r_4}\right) + \frac{r_1^2}{r_4^2 h_o}\right\}}$$

Examples





Schematic Diagram for Example 3.11

a) overall coefficient of heat transfer based on inner and outer area

$$U_i = \frac{1}{\left\{\frac{1}{1.1} + \frac{1^2}{50}\left(\frac{0.075}{1 \times 1.075}\right) + \frac{1^2}{0.15}\left(\frac{0.05}{1.075 \times 1.125}\right) + \frac{1^2}{0.48}\left(\frac{0.05}{1.125 \times 1.175}\right) + \frac{1^2}{(1.175)^2(18)}\right\}}$$

$$U_i = 0.7662 W / m^2.^\circ C$$

from eq. . we can find U_o

$$U_o = \frac{A_i}{A_o} U_i = \frac{12.566}{17.349}(0.766) = 0.555 W / m^2.^\circ C$$

Examples

b) the heat transfer rate

$$\dot{Q} = A_i U_i (T_{\infty i} - T_{\infty o}) = 12.566(0.766)(240-40) = 1925W$$

c) the temperature at the point of interface between the wall and the first layer of insulation T_2. In this case we can use the equation of heat transfer as below

$$\dot{Q} = \frac{T_{\infty i} - T_2}{R_{conv,i} + R_A} = \frac{T_{\infty i} - T_2}{\frac{1}{A_i h_i} + \frac{1}{4\pi k_A}\left(\frac{r_2 - r_1}{r_1 r_2}\right)}$$

$$T_2 = T_{\infty i} - \dot{Q}\left[\frac{1}{A_i h_i} + \frac{1}{4\pi k_B}\left(\frac{r_2 - r_1}{r_1 r_2}\right)\right]$$

Then $$T_2 = 240 - 1925\left[\frac{1}{(12.566)(1.1)} + \frac{1}{4\pi(50)}\left(\frac{1.075-1}{1.075\times 1}\right)\right] = 100.52^{\circ}C$$

The temperature at the interface between the two insulations T_3

Examples

$$\dot{Q} = \frac{T_{\infty i} - T_3}{R_{conv,i} + R_A + R_B} = \frac{T_{\infty i} - T_3}{\frac{1}{A_i h_i} + \frac{1}{4\pi k_A}\left(\frac{r_2 - r_1}{r_1 r_2}\right) + \frac{1}{4\pi k_B}\left(\frac{r_3 - r_2}{r_2 r_3}\right)}$$

And now $$T_3 = T_{\infty i} - \dot{Q}\left[\frac{1}{A_i h_i} + \frac{1}{4\pi k_A}\left(\frac{r_2 - r_1}{r_1 r_2}\right) + \frac{1}{4\pi k_B}\left(\frac{r_3 - r_2}{r_2 r_3}\right)\right]$$

$$T_3 = 240 - 1925\left[\frac{1}{(12.566)(1.1)} + \frac{1}{4\pi(50)}\left(\frac{1.075 - 1.0}{(1.075)(1.0)}\right) + \frac{1}{4\pi(0.15)}\left(\frac{1.125 - 1.075}{(1.125)(1.075)}\right)\right] = 62.84^o C$$

Or it can be calculated from the other side

$$\dot{Q} = \frac{T_3 - T_{\infty o}}{R_C + R_{conv,o}} = \frac{T_3 - T_{\infty o}}{\frac{1}{4\pi k_C}\left(\frac{r_4 - r_3}{r_3 r_4}\right) + \frac{1}{A_o h_o}}$$

$$T_3 = T_{\infty o} + \dot{Q}\left[\frac{1}{4\pi k_C}\left(\frac{r_4 - r_3}{r_3 r_4}\right) + \frac{1}{A_o h_o}\right]$$

And now $$T_3 = 40 + 1925\left[\frac{1}{4\pi(0.48)}\left(\frac{1.175 - 1.125}{(1.175)(1.125)}\right) + \frac{1}{(17.349)(18)}\right] = 62.84^o C$$

Critical Thickness of Insulation

CRITICAL RADIUS OF INSULATION

It is known that more insulation thickness to a plane wall decreases heat transfer. The lower rate of heat transfer is because of the thicker insulation. This resulted, since the area of heat transfer is still constant. The addition of insulation is always increasing wall thermal resistance without convection resistance affecting.

The adding of insulation to a cylindrical or spherical shells, however, has a different effect. The additional increasing in resistance of the conduction of the insulation layer but decreases the resistance of convection because of the increasing of the area of outer surface of convection heat transfer. The heat transfer rate from the pipe may be decreases or increases, depending on which effect is dominating.

Now let use consider a shell of cylindrical shape with outer radius r_1 and temperature T_1 which is constant. This shell is now insulated with a insulation material of thermal conductivity k and thickness that give outer radius r_2. The heat lost from the cylinder to the surrounding of temperature T_∞ with a coefficient of heat transfer h. The rate of heat transfer from the insulated shell to the out side surrounding medium can be represented as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2 / r_1)}{2\pi Lk} + \frac{1}{(2\pi L r_2)h}}$$

Critical thickness of insulation

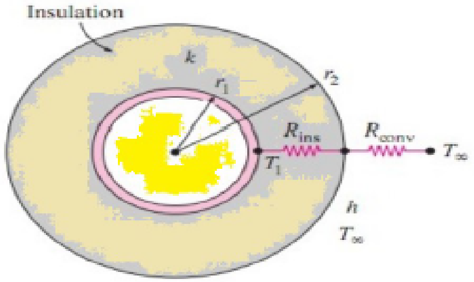


Fig. 3.22 The Critical Radius Effect on cylinder

The differentiation of $\dot{Q}$ to the outer radius of insulated material r_2 will be equal zero or optimum value (minimum heat transfer). It means that $\left(d\dot{Q}/dr_2\right)=0$ as shown in Fig.

$$\frac{d\dot{Q}}{dr_2}=\frac{(T_1-T_\infty)\left[\frac{r_1}{r_2}\frac{1}{r_1}\frac{1}{2\pi Lk}-\frac{1}{r_2^2}\frac{1}{2\pi Lh}\right]}{\left\{\frac{\ln(r_2/r_1)}{2\pi Lk}+\frac{1}{(2\pi Lr_2)h}\right\}^2}=0$$

By simplifying this equation we get

$$\frac{1}{k}-\frac{1}{r_2h}=0$$

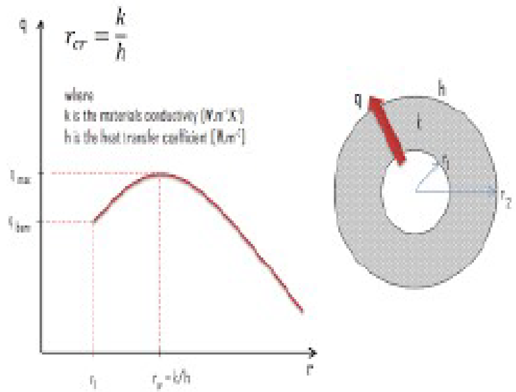
Or $$r_2=\frac{k}{h}\quad (m)\ \textit{for cylinder}$$

Critical Thickness of Insulation

And we can drive a relation for critical radius of insulation of spherical shell as follows

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{\left(4\pi r_2^2\right)h}}$$

The differentiation of eq.



The Effect of Insulation on Heat Transfer rate

Critical thickness of insulation

$$\frac{d\dot{Q}}{dr_2} = \frac{(T_1 - T_\infty)\left[\frac{1}{4\pi k}\frac{1}{r_2^2} - \frac{1}{4\pi h}\frac{2}{r_2^3}\right]}{\left[\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{(4\pi r_2^2)h}\right]^2} = \mathbf{0}$$

And simplifying the eq. will give

$$\frac{1}{k} - \frac{2}{r_2 h} = 0$$

Or $$r_2 = \frac{2k}{h} \quad (m) \quad \text{for Sphere}$$

Examples

Example 12. Copper pipe carrying refrigerant at (-25oC)of (0.01m)radius is exposed to convection at (30W/$m^2.^o C$) and air temperature of ($25^o C$). An insulation of thermal conductivity (0.6W/m.$^o C$). Find the critical radius of the insulation. Find also the heat transfer rate at different thickness of the insulation from (0.0cm) to (2.0cm) by (0.2cm) step. Draw the result of the relation between heat transfer and the insulation radius.

Example

Solution: copper pipe of r_1=0.01m the outside radius, T_1=-25°C, the convection temperature T_∞=25°C and h=30W/m^2.°C. Insulation thermal conductivity k=0.6W/m.°C.

Property: Thermal conductivity and heat transfer coefficient are constants

Assumption: one-dimensional steady state heat conduction

Analysis: the critical radius of the insulation $r_{cr} = \frac{k}{h} = \frac{0.6}{30} = 0.02m$

The heat transfer rate through the wall and insulation is calculated from the following relation.

$$\dot{Q} = \frac{T_{\infty 0} - T_1}{R_{ins,c} + R_{conv}} = \frac{T_{\infty 0} - T_1}{\frac{\ln(r_2 / r_1)}{2\pi kL} + \frac{2}{2\pi r_2 Lh}}$$

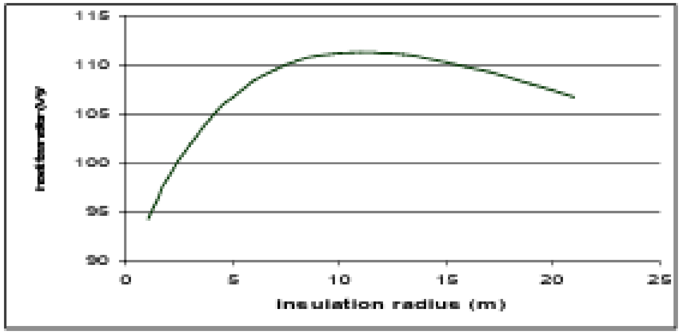
Examples

For different values of insulation thickness (r_2 value) from r_2=r_1 (no insulation) passing through the critical value of insulation radius (r_2=r_{cr})until r_2=2r_{cr}. Table 3-1 shows the result of heat transfer with radius

heat transfer through cylinder with variation of insulation radius (example 3.12)

r_2	0.01	0.012	0.014	0.016	0.018	0.020
Q	94.286	101.986	106.837	109.634	110.996	111.373
r_2	0.022	0.024	0.026	0.028	0.03	
Q	111.084	110.353	109.333	108.132	106.823	

also the effect of the insulation radius on the heat transfer through the cylinder.



The Insulation Effect on Heat Transfer Through Cylinder Wall