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Lecture Two

Basic Laws

2.1 Introduction

Lecture 1 introduced basic concept of circuits. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as **Ohm's law** and **Kirchhoff's laws**, form the foundation upon which electric circuit analysis is built. In addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis.

2.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1. Network: Any arrangement of the various, electrical energy source along with the different circuit elements is called an electrical network. Such a network is shown in the **Fig. 2.1**.

2. Network Element: Any individual circuit element with two terminals which can be connected to other circuit element is called a network element. Network elements can be either active elements or passive elements.

3. Branch: A part of the network which connects the various points of the network with one another is called a branch. In the **Fig. 2.1**, **AB, BC, CD, DA, DE, CF** and **EF** are the various branches. The branch may consist of more than one element.

4. Junction Point: A point where three or more branches meet is called a junction point. Points D and C are the junction points in the network shown in the **Fig. 2.1**.

5. Node: A point at which two or more elements are joined together is called node. The junction points are also the nodes of the network. In the network shown in the **Fig. 2.1**, **A, B, C, D, E** and **F** are the nodes of the network.

6. Mesh (or Loop): Mesh (or Loops) is a set of branches forming a closed path in a network in such way that if one branch is removed then remaining branches do not form a closed path. In the **Fig. 2.1** paths **A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D** etc are the loops of the network.

In this lecture, the analysis of d.c. circuits consisting of pure resistors and d.c. sources is included.

2.3 Classification of Electric Networks

The behavior of the entire network depends on the behavior and characteristics of its elements. Based on such characteristics electrical network can be classified as below,

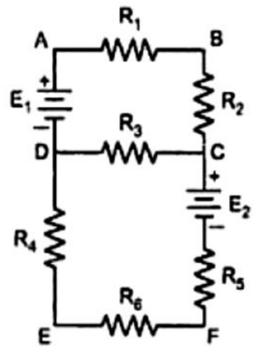


Figure 2.1 An electrical network.

i) Linear Network: A circuit or network whose parameter i.e. elements are always constant irrespective of the change in time, voltage, temperature etc. is known as linear network.

ii) Nonlinear Network: A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as nonlinear network.

iii) Bilateral Network: A circuit whose characteristics, behavior is same irrespective of the direction of current through various elements of it is called bilateral network.

iv) Unilateral Network: A circuit whose, operation, behavior is dependent on the direction of the current through various elements is called unilateral network.

v) Active Network: A circuit whose contain at least one source of energy is called active. An energy source may be a voltage or current source.

vi) Passive Network: A circuit which contains no energy source is called passive circuit. This is shown in the **Fig 2.2**.

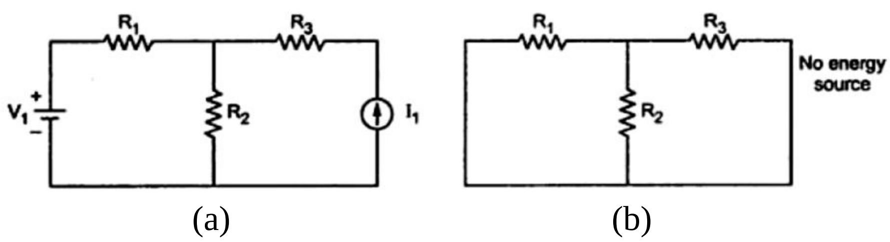


Figure 2.2 (a) Active network, (b) Passive network

vii) Lumped Network: A network in which all the network elements are physically separable is known as **lumped network**. Most of the electric networks are lumped in nature, which consists of element like R, L, C, and voltage source etc.

viii) Distributed Network: A network in which the network elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called distributed network. The best example of such a network is a transmission line, where resistances, inductance and. capacitance of a transmission line are distributed all along its length and cannot be shown as separate elements, anywhere in the circuit.

2.4 OHM'S LAW

As shown in lecture 1, the materials in general have a characteristic behavior of resisting the flow of electric charge. The resistance *R* of any material with a uniform cross-sectional area *A* depends on *A* and its length ***l***.

The circuit element used to model the current-resisting behavior of a material is the resistor. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit symbol for the resistor is shown in **Fig. 2.3**, where *R* stands for the resistance of the resistor. The resistor is the simplest passive element. Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

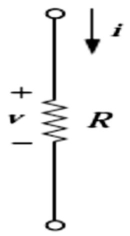


Figure 2.3 Circuit symbol for resistance.

> Key Point: *Ohm's law states that the voltage* **v** *across a resistor is directly proportional to the current I flowing through the resistor.*

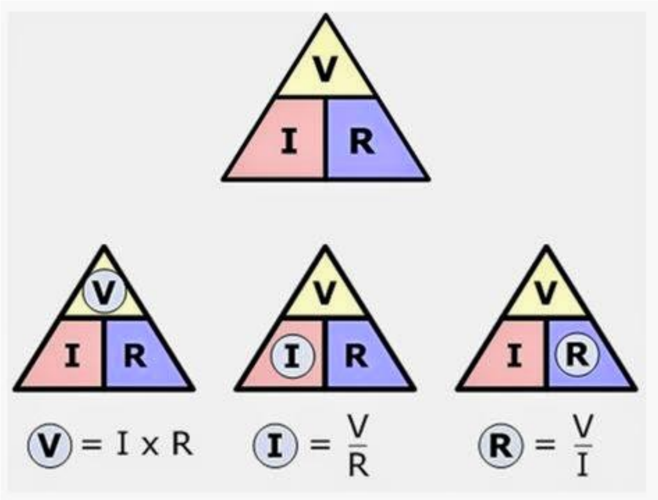
Ohm defined the constant of proportionality for a resistor to be the resistance; *R*. (The resistance is material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus,

$$V = IR \quad (2.1)$$

The resistance *R* of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

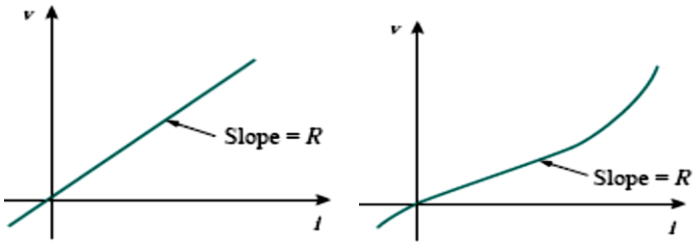
Then $$R = V/I \quad (2.2)$$

so that **1 Ω= 1 V/A**



It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a linear resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in **Fig. 2.4(a)**. A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in **Fig. 2.4 (b)**. Examples of devices with nonlinear resistance are the light bulb and the diode. A useful quantity in circuit analysis is the reciprocal of resistance *R*, known as conductance and denoted by *G*:

$$\mathbf{G = 1/R = I/V} \tag{2.3}$$



(a) **(b)**

Figure 2.4 The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mhos, in this lectures we prefer to use the Siemens (S), the SI unit of conductance:

1 S = 1 ℧ = 1 A/V

Thus,

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or Siemens (S).

From **Eq. (2.3)**, we may write

$$\mathbf{I = GV} \qquad (2.4)$$

The power dissipated by a resistor can be expressed in terms of **R**. Using **Eqs. (1.23)** and **(2.1)**,

$$\mathbf{P = VI = I^2R = V^2/R} \qquad (2.5)$$

The power dissipated by a resistor may also be expressed in terms of **G** as

$$\mathbf{P = VI = V^2G = I^2/G} \qquad (2.6)$$

We should note two things from **Eqs. (2.5)** and **(2.6)**:

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit.

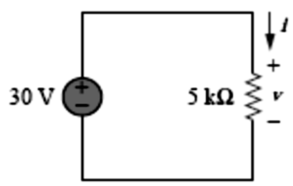
2.4.1 Limitations of Ohm's Law

The Limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diode, zener diode, voltage regulators.

2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by, $V = kI^m$ where **k, m** are constants.

Example 2.1: In the circuit shown below, calculate the current i, the conductance G, and the power **P**.



Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$\mathbf{i = v / R = 30 \times 5 \times 10^3 = 6\ mA}$$

The conductance is $\mathbf{G = 1/ R = 1/ (5 \times 10^3) = 0.2\ mS}$

We can calculate the power in various ways using either **Eqs. (1.29)**, **(2.5)**, or **(2.6)**.

$$\mathbf{p = vi = 30 \times (6 \times 10^{-3}) = 180\ mW}$$

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2.5 SERIES RESISTORS

A series circuit is one in which several resistances are connected one after the other. There is only one path for the flow of current. Consider the resistances shown in the **Fig. 2.5**. The resistance $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, said to be in series.

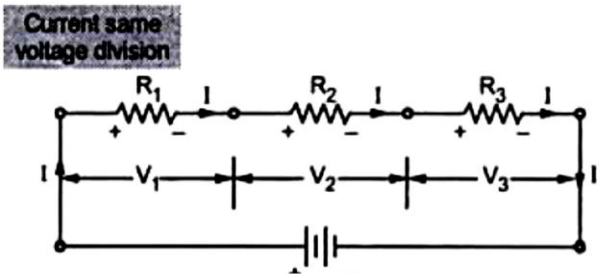


Fig. 2.5 series circuit

Req= Equivalent resistance of the circuit.

$$\mathbf{R_{eq} = R_1 + R_2 + R_3}$$

i.e. total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For **N** resistances in series, $\mathbf{R = R_1 + R_2 + R_3 + \ldots + R_N}$ (2.7)

If $\mathbf{R_1 = R_2 = \cdot\ \cdot\ \cdot = R_N = R}$, then

$$\mathbf{R_{eq} = N \times R} \qquad (2.8)$$

2.5.1 Characteristics of Series Circuits

1) The same current flows through each resistance.

2) The supply voltage **V** is the sum of the individual voltage drops across the resistances.

$$\mathbf{V = V_1 + V_2 + V_3 + \ldots + V_N} \qquad (2.9)$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e. $\mathbf{R > R_1, R > R_2, \ldots R > R_N}$

2.6 PARALLEL RESISTORS

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the **Fig. 2.6**.

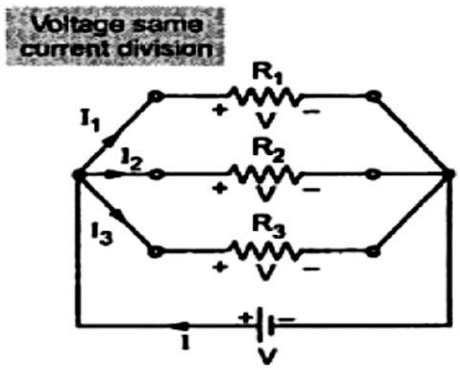


Fig. 2.6 A parallel circuit.

R_{eq}= Total or equivalent resistance of the circuit,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if '**N**' resistances are in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \quad (2.10)$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdot\ \cdot\ \cdot = R_N = R$, then

$$R_{eq} = R/N \quad (2.11)$$

Conductance (G):

It b known that, **1/R = G** (conductance) hence,

$$G = G_1 + G_2 + G_3 + \ldots + G_N \quad (2.12)$$

Important result:

Now If **N** = 2, two resistance are in parallel then,.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R = \frac{R_1 R_2}{R_1 + R_2} \quad (2.13)$$

2.6.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of the individual currents.
3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances $R < R_1, R < R_2, R < R_N$.
5) The equivalent conductance is the arithmetic addition of the individual conductance's.

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single equivalent resistance **Req**.

Example 2.2: Find R_{eq} for the circuit shown in **Fig. 1**.

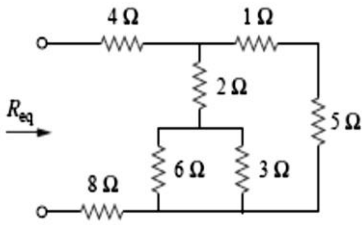


Figure 1

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\ \Omega = 6 \times 3/(6+3) = 2\ \Omega$$

(The symbol || is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$\mathbf{1\ \Omega + 5\ \Omega = 6\ \Omega}$$

Thus the circuit in **Fig. 1** is reduced to that in **Fig. 2(a)**. **In Fig. 2(a)**, we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$\mathbf{2\ \Omega + 2\ \Omega = 4\ \Omega}$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in **Fig. 2 (a)**; their equivalent resistance is

$$\mathbf{4\ \Omega \parallel 6\ \Omega = 4 \times 6/\ (4 + 6) = 2.4\ \Omega}$$

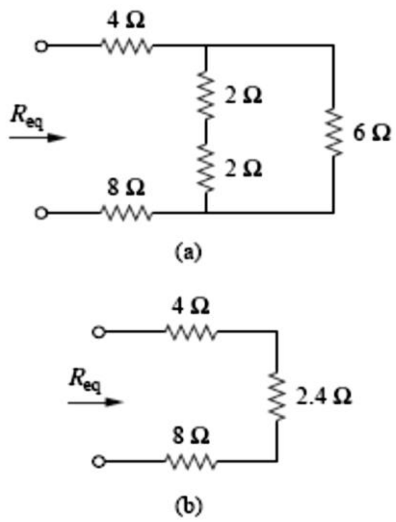


Figure 2

The circuit in **Fig. 2 (a)** is now replaced with that in **Fig. 2 (b)**. In **Fig. 2 (b)**, the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$\mathbf{R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega}$$

2.7 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role. Since the value of *R* can range from zero to infinity, it is important that we consider the two extreme possible values of *R*.

2.7.1 Short Circuit

When any two points in a network are joined directly to each other with a thick meta1ic conducting wire the two points are said to be short circuited. The resistance of such short circuit is zero.

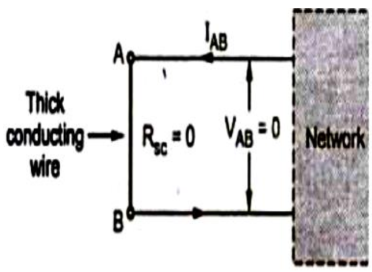


Figure 2.7 Short circuit (R_{sc} = 0)

The part of the network, which is short circuited, is shown in the **Fig. 2.7**. The points A and B are short circuited. The resistance of the branch **AB** is **Rsc**=0. The Current $\mathbf{I_{AB}}$ is flowing through the short circuited path. According to Ohm's law,

$$\mathbf{V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V}$$

Key Point: ***The voltage across short circuit is always zero though current flows through the short circuited path.***

2.7.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

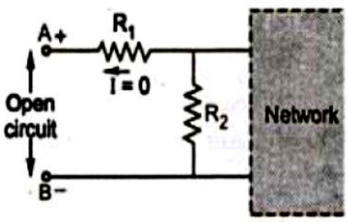


Figure 2.8 Open circuit (R_{OC} =∞).

As there is no direct connection in an open circuit, the resistance of the open circuit is ∞. The part of the network which is open circuited is shown in the **Fig. 2.8**. The points A and B are said to be open circuited. The resistance of the branch AB is $\mathbf{R_{OC}} = \infty\ \Omega$.

According to Ohm's law,

$$I_{OC} = V_{AB}/R_{OC} = V_{AB}/\infty = 0\text{ A}$$

Key Point: ***The current through open circuit is always zero though there exist voltage across open circuited terminals.***

2.8 The voltage-divider and current-divider circuits

2.8.1 The voltage-divider circuit

Voltage-divider circuit, shown in **Fig.2.9**. We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis we introduce the current ***i*** as shown in **Fig.2.9 (b)**. From Kirchhoff's current law $\mathbf{R_1}$ and $\mathbf{R_2}$, carry the same current. Applying **Kirchhoff's voltage law** around the closed loop yields

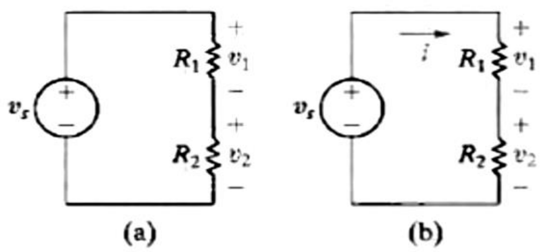


Figure 2.9 (a) A voltage-divider circuit and (b) The voltage-divider circuit with current i indicated

$$v_s = i\,R_1 + i\,R_2\,,$$

Now we can use Ohm's law to calculate v_1 and, v_2:

$$v_1 = \frac{R_1\,v_s}{R_1 + R_2},\quad v_2 = \frac{R_2\,v_s}{R_1 + R_2} \tag{2.14}$$

In general, if a voltage divider has **N** resistors ($\mathbf{R_1, R_2, \ldots, R_N}$) in series with the source voltage v_s, the Nth resistor ($\mathbf{R_N}$) will have a voltage drop of

$$v_N = \frac{R_N\,v_s}{R_1 + R_2 + \cdots + R_N} = \frac{R_N\,v_s}{R_{eq}} \tag{2.15}$$

2.8.2 The current-divider circuit

The **current-divider circuit** shown in **Fig. 2.10.** The current divider is designed to divide the current i_s between R_1 and R_2. We find the relationship between the current i_s, and the current in each resistor (that is, i_1 and i_2) by directly applying **Ohm's law and Kirchhoff's current law**. The voltage across the parallel resistors is

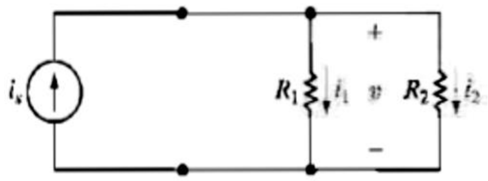


Figure2.10 the current-divider circuit.

$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s$$

$$i_1 = \frac{R_2\, i_s}{R_1 + R_2}, \quad i_2 = \frac{R_1\, i_s}{R_1 + R_2} \tag{2.16}$$

If we divide both the numerator and denominator by R_1R_2, **Eq. (2.16)** become

$$i_1 = \frac{G_1\, i_s}{G_1 + G_2}, \quad i_2 = \frac{G_2\, i_s}{G_1 + G_2} \tag{2.17}$$

Thus, in general, if a current divider has **N** conductors (**G_1, G2, . . . , G_N**) in parallel with the source current i, the nth conductor (**G_N**) will have current

$$i_N = \frac{G_N\, i_s}{G_1 + G_2 + \cdots + G_N} = \frac{R_{eq}\, i_s}{R_N} \tag{2.18}$$

Example 2.3: Find i_o and v_o in the circuit shown in **Fig. 1(a)**. Calculate the power dissipated in the 3-Ω resistor.

Solution: The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

6 Ω || 3 Ω = 6 × 3/ (6 + 3) = 2 Ω

By apply voltage division, since the 12 V in **Fig. 1(b)** is divided between the 4-Ω and 2-Ω resistors. Hence,

v_o =2(12 V)/(2 + 4) = 4 V

Apply current division to the circuit in **Fig. 1(a)** now that we know i, by writing

i = 12/ 4 + 2 = 2 A
i_o = 6 i /(6 + 3) = 4 /3 A

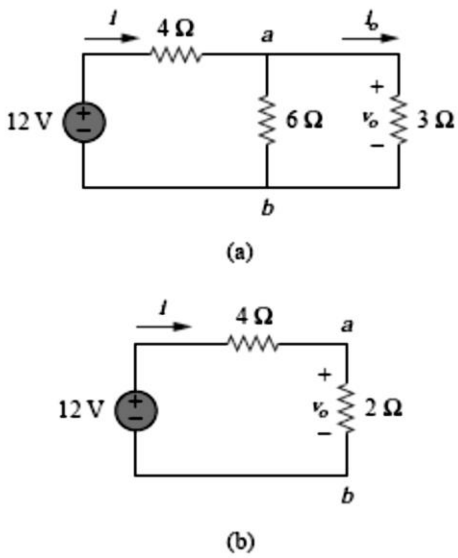


Figure 1(a) Original circuit, (b) Its equivalent circuit.

The power dissipated in the 3-Ω resistor is

$$p_o = v_o\, i_o = 4(4/3) = 5.333\ W$$

2.9 WYE-DELTA TRANSFORMATIONS

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in **Fig. 2.11**. How do we combine resistors **R_1** through **R_6** when the resistors are neither in series nor in parallel? Many circuits of the type shown in **Fig. 2.11** can be simplified by using three-terminal equivalent networks. These are the wye (**Y**) or tee (**T**) network shown in **Fig. 2.12** and the delta (**Δ**) or pi (**π**) network shown in **Fig. 2.13**.

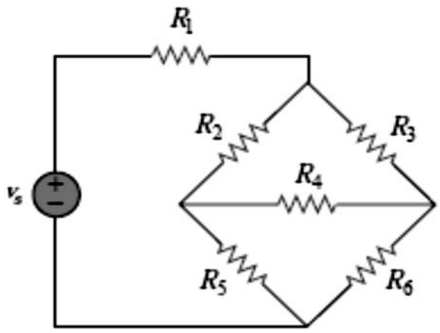


Figure 2.11The bridge network.

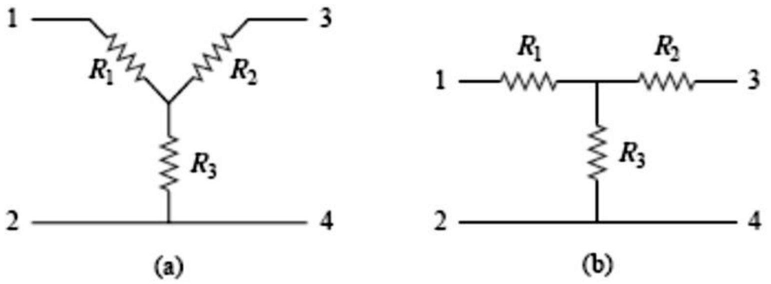


Figure 2.12 Two forms of the same network: (a) Y, (b) T.

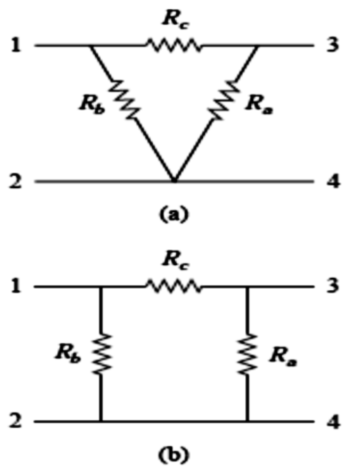


Figure 2.13 Two forms of the same network: (a) Δ, (b) π.

Delta to Wye Conversion

Suppose it is more convenient to work with a **wye** network in a place where the circuit contains a delta configuration. We superimpose a **wye** network on the existing **delta** network and find the equivalent resistances in the **wye** network. For terminals 1 and 2 in **Figs. 2.12** and **2.13**, for example, $\mathbf{R_{12}(Y) = R1 + R3}$, $\mathbf{R_{12}\ (\Delta) = R_b \parallel (R_a + R_c)}$ (2.19)

Setting $\mathbf{R_{12}(Y) = R_{12}\ (\Delta)}$ gives

$$\mathbf{R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}}$$

$$\mathbf{R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}} \qquad \mathbf{R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}} \tag{2.20}$$

By solving previous equations, we get

$$\mathbf{R_1 = \frac{R_b\, R_c}{R_a + R_b + R_c}} \tag{2.21}$$

$$\mathbf{R_2 = \frac{R_c\, R_a}{R_a + R_b + R_c}} \tag{2.22}$$

$$\mathbf{R_3 = \frac{R_a\, R_b}{R_a + R_b + R_c}} \tag{2.23}$$

Wye to Delta Conversion

Reversing the **Δ-to-Y** transformation also is possible. That is, we can start with the **Y** structure and replace it with an equivalent **Δ** structure. The expressions for the three **Δ**-connected resistors as functions of the three **Y**-connected resistors are

$$\mathbf{Ra = \frac{R1R2 + R2R3 + R3R1}{R1}} \tag{2.24}$$

$$\mathbf{Rb = \frac{R1R2 + R2R3 + R3R1}{R2}} \tag{2.25}$$

$$\mathbf{Rc = \frac{R1R2 + R2R3 + R3R1}{R3}} \tag{2.26}$$

The Y and Δ networks are said to be balanced when

$$\mathbf{R_1 = R_2 = R_3 = R_Y,\ R_a = R_b = R_c = R_\Delta} \tag{2.27}$$

Under these conditions, conversion formulas become

$$\mathbf{R_Y = R_\Delta / 3 \text{ or } R_\Delta = 3R_Y} \tag{2.28}$$

Example 2.4: Obtain the equivalent resistance $\mathbf{R_{ab}}$ for the circuit in **Fig. 1** and use it to find current **i**.

Solution:

In this circuit, there are two **Y**-networks and one **Δ**-network. Transforming just one of these will simplify the circuit. If we convert the **Y**-network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$\mathbf{R_1 = 10\ \Omega,\quad R_2 = 20\ \Omega,\quad R_3 = 5\ \Omega}$

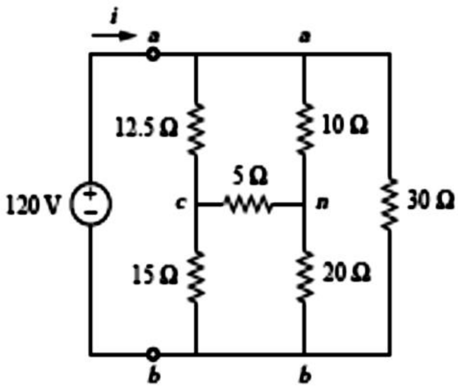


Figure 1.

Thus, from Eqs. (2.24) to (2.26) we have

$$\mathbf{Ra = \frac{R1R2 + R2R3 + R3R1}{R1} = \frac{10\times20+20\times5+5\times10}{10}} = \frac{350}{10} = 35\ \Omega$$

$$\mathbf{Rb = \frac{R1R2 + R2R3 + R3R1}{R2}} = \frac{350}{20} = 17.5\Omega$$

$$\mathbf{Rc = \frac{R1R2 + R2R3 + R3R1}{R3}} = \frac{350}{5} = 70\Omega$$

With the **Y** converted to **Δ**, the equivalent circuit (with the voltage source removed for now) is shown in **Fig. 2 (a)**. Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = 70 \times 30/(70+30) = 21\ \Omega$$

$$12.5 \parallel 17.5 = 12.5 \times 17.5/(12.5+17.5) = 7.2917\ \Omega$$

$$15 \parallel 35 = 15 \times 35/(15+35) = 10.5\ \Omega$$

so that the equivalent circuit is shown in **Fig. 2 (b)**. Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = 17.792 \times 21/(17.792+21) = 9.632\ \Omega$$

Then

$$i = vs/\ Rab = 120/\ 9.632 = 12.458\ A$$

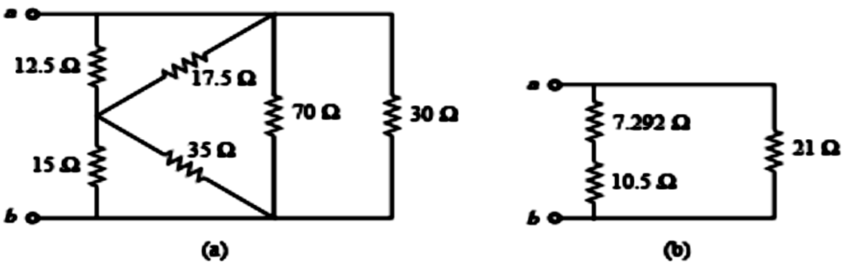


Figure 2 Equivalent circuits to Fig. 1, with the voltage removed.

2.10 Energy Sources

There are basically two types of energy sources; voltage source and current source. These sources are classified as i) Ideal source and ii) Practical source. Let us see the difference between Ideal and practical sources.

2.10.1 Voltage Source

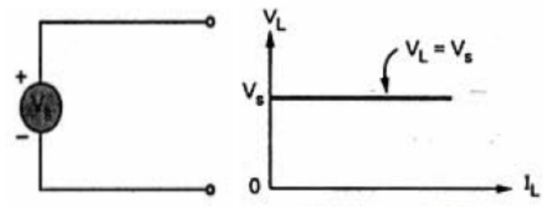
***Ideal voltage source:**

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. This is indicated by **V- I** characteristics shown in the **Fig. 2.14 (b)**.

***Practical voltage source:**

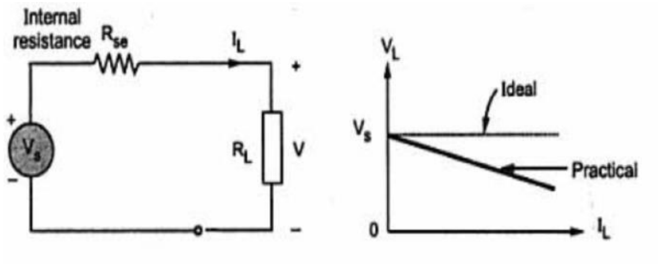
But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by $\mathbf{R_{se}}$ as shown in the **Fig. 2.15**. Because of the $\mathbf{R_{se}}$, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = V_S - I_L R_L$$



(a) symbol (b) characteristics

Figure 2.14 Ideal voltage source.



(a) circuit (b) characteristics

Figure 2.15 Practical voltage source.

Voltage sources are further classified as follows,

i) Time invariant Sources:

The sources in which voltage is not varying with time are known as time invariant voltage source or D.C. sources. These are denoted by capital letters. Such a source is represented in the **Fig. 2.16 (a)**.

ii) **Time Variant Source**:

The sources in which voltage is varying with time are known as time variant voltage sources or A.C. sources. These are denoted by small letters. This is shown in the **Fig. 2.16 (b)**.

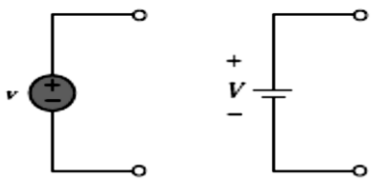


Figure 2.16 (a) D.C. sources.

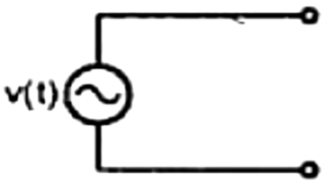


Figure 2.16(b) A.C. source.

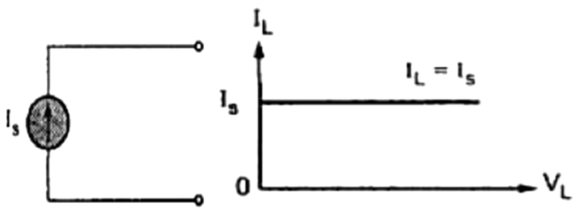
2.10.2 Current Source

***Ideal current source:**

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminal. This is explained by **V-I** characteristics shown in the **Fig. 2.17 (b)**.

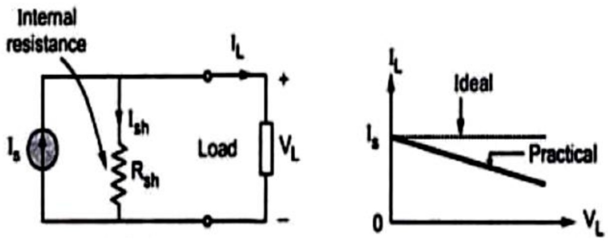
***Practical current source:**

But practically, every current source has high internal resistance, shown in parallel with current source and It is represented by $\mathbf{R_{sh}}$. This is shown in the **Fig. 2.18**. Because of $\mathbf{R_{sh}}$, current through its terminals decreases slightly with voltage at its terminals.



(a) symbol (b) characteristics

Figure 2.17 ideal current source.



(a) circuit (b) characteristics

Figure 2.18 ideal current source.

Similar to voltage sources, current sources are classified as follows,

i) Time Invariant Sources:

The sources in which current is not varying with time are known as time invariant current sources or D.C. sources. These are denoted by capital letters. Such a current source is represented in the **Fig. 2.19 (a)**.

ii) Time Variant Sources:

The sources in which current is varying with time are known as time variant current sources or A.C. sources. These are denoted by small letters. Such source is represented in the **Fig. 2.19 (b)**.

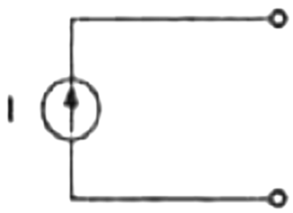


Figure 2.19 (a) D.C. source.

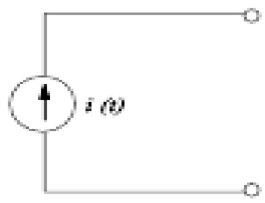


Fig. 2.19 (b) A.C. source.

The sources, which are discussed above are independent sources because these sources does not depend on other voltage or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside

2.10.3 Dependent Sources

Dependent source are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the **Fig. 2.20** and further classified as,

i) **Voltage-Controlled Voltage Source (VCVS)**: It produces a voltage as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20 (a)**. The controlling voltage is named v_x the equation that determines the supplied voltage v_s is

$v_s = \mu v_x$, and the reference polarity for v_s is as indicated. Note that μ is a multiplying constant that is dimensionless.

ii) **Current-Controlled Voltage Source (CCVS)**: It produces voltage as a function of current elsewhere in the given circuit. It is shown In the **Fig. 2.20(b)**. the controlling current is i_x the equation for the supplied voltage v_s is $v_s = \rho\, i_x$,

the reference polarity is as shown and the multiplying constant **ρ** has the dimension volts per ampere

iii) **Voltage-Controlled Current Source (VCCS)**: It produces current as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20(c)**. The controlling voltage is v_x, the equation for the supplied current i_s is $i_s = \alpha\, v_x$,

the reference direction is as shown and the multiplying constant **α** has the dimension amperes per volt

iv) **Current-Controlled Current Source (CCCS)**: It produces current as a function of current elsewhere in the given circuit. It is shown in the **Fig. 2.20 (d)**. the controlling current is i_x the equation for the supplied current i_s is $i_s = \beta\, i_x$,

the reference direction is as shown, and the multiplying constant β is dimensionless.

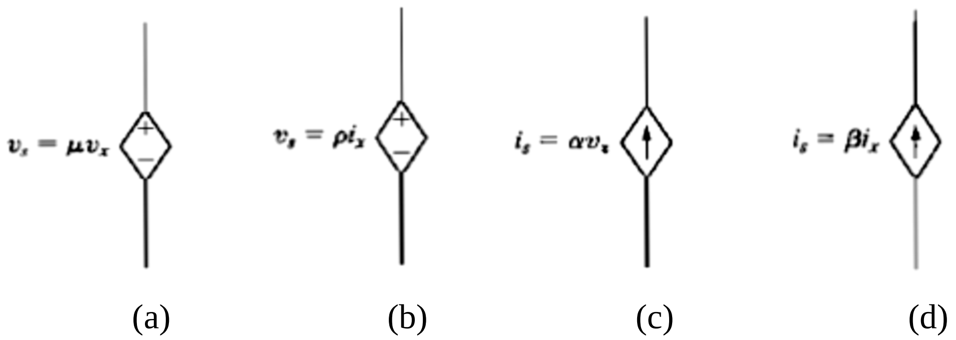


Figure 2.20 The circuit symbols a) an ideal dependent voltage-controlled voltage source, (b) an ideal dependent current-controlled voltages source, (c) an ideal dependent voltage-controlled current source (d) an ideal dependent current-controlled current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits. An example of a current controlled voltage source is shown on the right-hand side of **Fig. 2.21**, where the voltage 10i of the voltage source depends on the current ***i*** through element C.

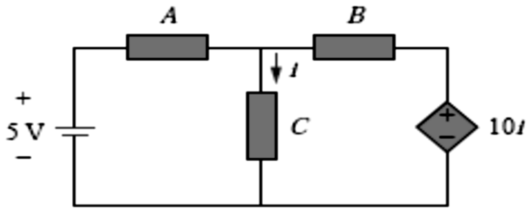


Figure 2.21 the source on the right-hand side is a current-controlled voltage source.

2.11 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much easier. Let us consider such series and parallel combinations of energy sources.

2.11.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources. Consider the two sources as shown in the **Fig. 2.22**.

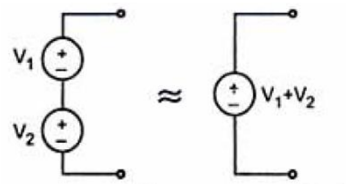


Figure 2.22

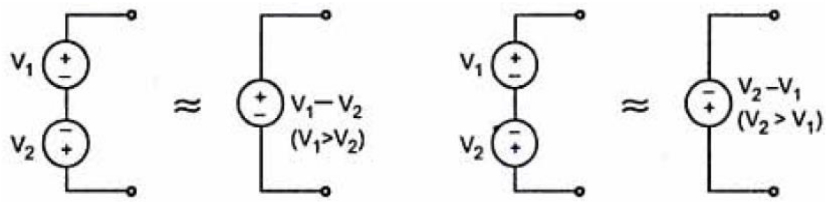


Figure 2.23

If the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the **Fig. 2.23**. If the polarities of the two sources are different then the equivalent single source is the difference between the two voltage sources. The polarity of such source is same as that of the greater of the two sources.

> **Key Point:** *the voltage sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.2 Voltage Sources in Parallel

Consider the two voltage source in parallel as shown in the **Fig. 2.24**. The equivalent single source has a value same as $\mathbf{V_1}$ and $\mathbf{V_2}$. It must be noted that all the open circuit voltage provided by each source must be equal as the sources are in parallel.

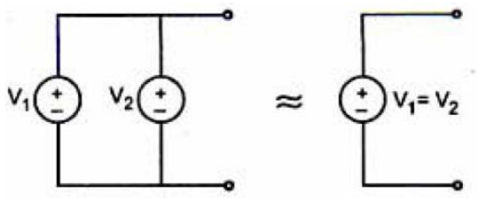


Figure 2.24

Key Point: *the voltage sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

2.11.3 Current Sources in Series

Consider the two current sources in series is shown in the **Fig. 2.25**, the equivalent single source has a value same as $\mathbf{I_1}$ and $\mathbf{I_2}$.

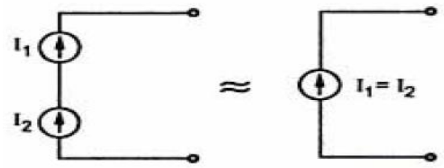
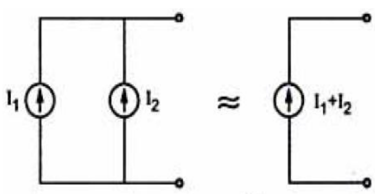


Figure 2.25

Key Point: *the current sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the **Fig. 2.26**.



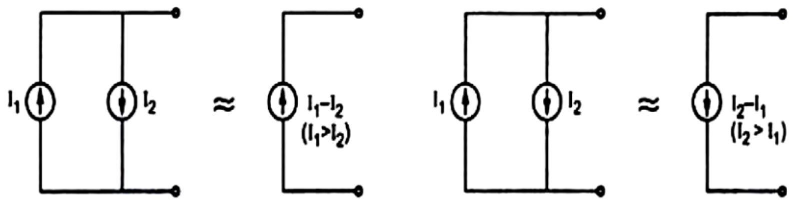


Figure 2.26 **Figure 2.27**

if the directions of the currents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources. Consider the two current sources with opposite directions connected in parallel as shown in the **Fig. 2.27**. If the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with value equal to the difference between the two voltage sources.

Key Point: *the current sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

q/

2.12 Notation

Notation will play an increasingly important role in the analysis to follow.

i) Double-Subscript Notation

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In **Fig. 2.28(a)**, the two points that define the voltage across the resistor **R** are denoted by **a** and **b**. Since a is the first subscript for $\mathbf{V_{ab}}$, point a must have a higher potential than point **b** if $\mathbf{V_{ab}}$ is to have a positive value. If, in fact, point b is at a higher potential than point a, $\mathbf{V_{ab}}$ will have a negative value, as indicated in **Fig. 2.28(b)**.

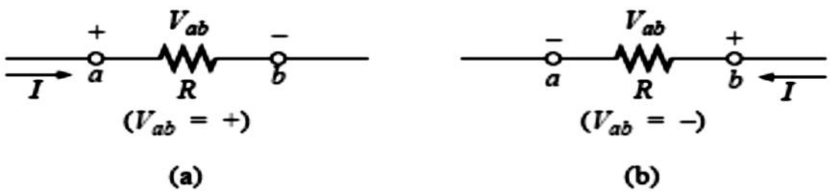


Figure 2.28 defining the sign for double-subscript notation.

In summary:

The voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

ii) Single-Subscript Notation

If point **b** of the notation $\mathbf{V_{ab}}$ is specified as ground potential (zero volts), then a single subscript notation can be employed that provides the voltage at a point with respect to ground.

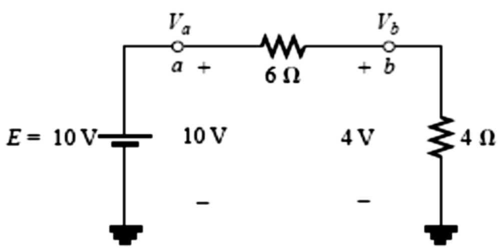


Figure 2.29 defining the use of single-subscript notation for voltage levels.

In **Fig. 2.29**, $\mathbf{V_a}$ is the voltage from point a to ground. In this case it is obviously **10 V** since it is right across the source voltage **E**. The voltage $\mathbf{V_b}$ is the voltage from point **b** to ground. Because it is directly across the **4-Ω** resistor, $\mathbf{V_b}$ = 4 V.

In summary:

The single-subscript notation Va specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of Va .

General Comments

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$V_{ab} = V_a - V_b \tag{2.29}$$

In other words, if the voltage at points **a** and **b** is known with respect to ground, then the voltage V_{ab} can be determined using **Eq. (2.29)**. In **Fig. 2.29**, for example,

$$V_{ab} = V_a - V_b = 10\ \text{V} - 4\ \text{V} = 6\ \text{V}$$

2.13 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

2.13.1 Kirchhoff's current law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero or the sum of the currents entering a node is equal to the sum of the currents leaving the node.

Mathematically, **KCL** implies that

$$\sum_{n=1}^{N} i_n = 0 \tag{2.30}$$

where **N** is the number of branches connected to the node and in is the nth current entering (or leaving) the node.

Consider the node in **Fig. 2.30**. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \tag{2.31}$$

since currents i_1, i_3, and i_4 are entering the node, while currents i_2 and i_5 are leaving it. By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5 \tag{2.32}$$

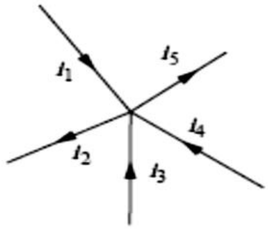


Figure 2.30 Currents at a node illustrating KCL.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in **Fig. 2.31(a)** can be combined as in **Fig. 2.31(b)**. The combined or equivalent current source can be found by applying KCL to node **a**.

$$\mathbf{I_T + I_2 = I_1 + I_3}$$

or

$$\mathbf{I_T = I_1 - I_2 + I_3}$$

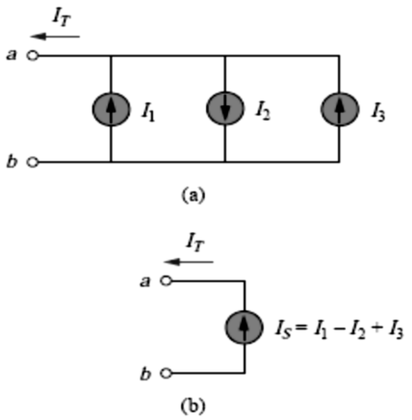


Figure 2.31 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

A circuit cannot contain two different currents, $\mathbf{I_1}$ and $\mathbf{I_2}$, in series, unless $\mathbf{I_1} = \mathbf{I_2}$; otherwise **KCL** will be violated.

2.13.2 Kirchhoff's voltage law

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0 \qquad (2.33)$$

Where **M** is the number of voltages in the loop (or the number of branches in the loop) and $\boldsymbol{v_m}$ is the mth voltage.

To illustrate KVL, consider the circuit in **Fig. 2.32**. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $\mathbf{-v_1}$, $\mathbf{+v_2}$, $\mathbf{+v_3}$, $\mathbf{-v_4}$, and

+v_5, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \tag{2.34}$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.35}$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises (2.36)

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been **$+v_1$, $-v_5$, $+v_4$, $-v_3$,** and **$-v_2$**, which is the same as before, except that the signs are reversed. Hence, **Eqs. (2.34) and (2.35)** remain the same.

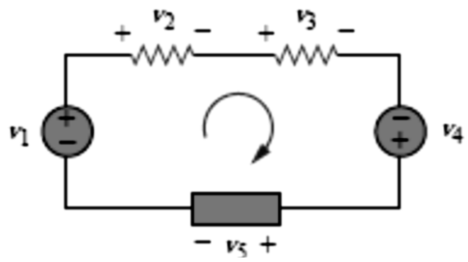


Figure 2.32 A single-loop circuit illustrating KVL.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources.

2.13.3 Steps to Apply Kirchhoff. Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1: Draw the circuit diagram from the given information and insert all the value of sources with appropriate polarities and all the resistances.

Step 2: Mark all the branch currents with assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents as minimum as far as possible to limit the mathematical calculations required to solve them later on. Assumed directions may be wrong; in such case answer of such current will be mathematically negative which indicates the correct direction of the current.

Step 3: Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistance of the network. This is necessary for application of KVL to various closed loops.

Step 4: Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any preview equation.

2.14 Solving Simultaneous Equations and Cramer's Rule

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy. Let us assume that set of simultaneous equations obtained is, as follows,

$$\begin{aligned} &a_{11} x_1 + a_{12} x_2 + \dots\dots\dots\dots + a_{1n} x_n = C_1 \\ &a_{21} x_1 + a_{22} x_2 + \dots\dots\dots\dots + a_{2n} x_n = C_2 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad . \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad . \\ &a_{n1} x_1 + a_{n2} x2 + \dots\dots\dots\dots + a_{nn} x_n = C_n \end{aligned}$$

where $\mathbf{C_1, C_2, \dots\dots\dots, C_n}$ constants. Then Cramer's rule says that form a system determinant $\mathbf{\Delta}$ or **D** as,

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = D$$

Then obtain the subdeterminant **Dj** by replacing $\mathbf{j^{th}}$ column of $\mathbf{\Delta}$ by the column of constants existing on right hand side of equations i.e. $\mathbf{C_1, C_2, \dots. C_n}$;

$$D_1 = \begin{bmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_n & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \qquad D_2 = \begin{bmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{bmatrix}$$

and

$$D_n = \begin{bmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & C_n \end{bmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \quad X_2 = \frac{D_2}{D}, \cdots, X_n = \frac{D_n}{D}$$

Where $\mathbf{D_1}$, $\mathbf{D_2}$, …, $\mathbf{D_n}$ and **D** are values of the respective determents

Example 2.5: Apply Kirchhoff's laws to the circuit shown in figure 1 below Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative?

If yes, explain the significance of the negative sign.

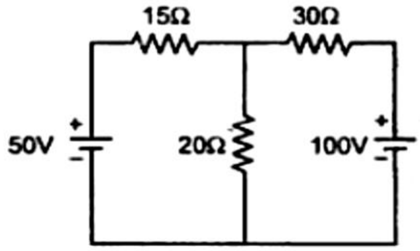


Figure 1

Solution: Application Kirchhoff's laws:

Step 1and 2: Draw the circuit with all the values which are same as the given network.

Mark all the branch currents starting from **+ve** of any of the source, say +ve of 50 V source

Step 3: Mark all the polarities for different voltages across the resistance. This is combined with step 2 shown in the network below in Fig. 1 (a).

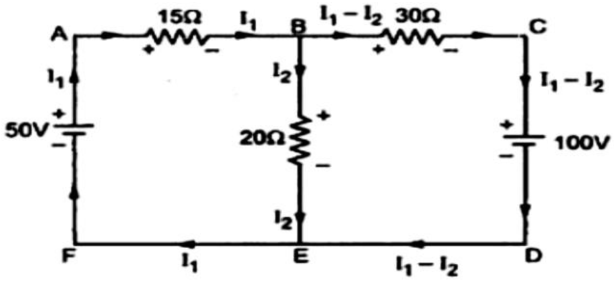


Figure 1 (a)

Step 4: Apply **KVL** to different loops.

Loop 1: **A-B-E-F-A**, $\quad \mathbf{-15\, I_1 - 20\, I_2 + 50 = 0}$

Loop 2: **B-C-D-E-D**, $\quad \mathbf{-30\,(I_1 - I_2) - 100 + 20\, I_2 = 0}$

Rewriting all the equations, taking constants on one side,

$$\mathbf{15\, I_1 + 20\, I_2 = 50}, \qquad \mathbf{-30\, I_1 + 50\, I_2 = 100}$$

Apply Cramer's rule, $\quad D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$

Calculating $\mathbf{D_1}$, $$\boldsymbol{D_1} = \begin{vmatrix} \mathbf{50} & \mathbf{20} \\ \mathbf{100} & \mathbf{50} \end{vmatrix} = \mathbf{500}$$

$$\boldsymbol{I_1} = \frac{\boldsymbol{D_1}}{\boldsymbol{D}} = \frac{\mathbf{500}}{\mathbf{1350}} = \mathbf{0.37}\ \boldsymbol{A}$$

Calculating $\mathbf{D_2}$, $$\boldsymbol{D_2} = \begin{vmatrix} \mathbf{15} & \mathbf{50} \\ \mathbf{-30} & \mathbf{100} \end{vmatrix} = \mathbf{3000}$$

$$\boldsymbol{I_2} = \frac{\boldsymbol{D_2}}{\boldsymbol{D}} = \frac{\mathbf{3000}}{\mathbf{1350}} = \mathbf{2.22}\ \boldsymbol{A}$$

For $\mathbf{I_1}$ and $\mathbf{I_2}$ as answer is positive, assumed direction is correct.

For $\mathbf{I_1}$ answer is **0.37** A. For $\mathbf{I_2}$ answer is **2.22** A

$$\mathbf{I_1 - I_2 = 0.37 - 2.22 = -1.85\ A}$$

Negative sign indicates assumed direction is wrong.

i.e. $\mathbf{I_1 - I_2}$ = 1.85 A flowing in opposite direction to that of the assumed direction.