Thermal Stresses:

The change in temperature causes bodies to expand or contract, the amount of linear deformation (Δth) is expressed as follows:

$\Delta th = \alpha \times L \times \Delta T$

where:

α -------The coefficient of linear deformation in unit of (m/m.C°)

L-------Length of the body (m)

ΔT------Temperature change (C°).

A general procedure for computing the loads and stresses caused when thermal deformation happened as result for temperature changing is outlines in steps:

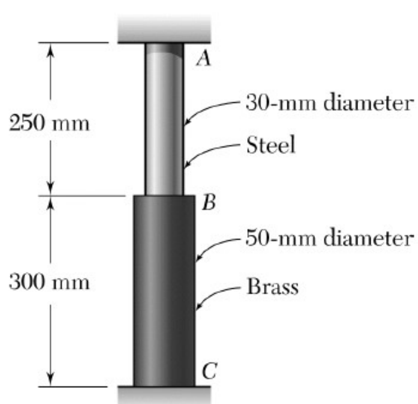
1-Assume that the body is free from all applied loads and constraints so that thermal deformations can occur freely.

2- Apply sufficient load to the body to restore it to the original condition

3- Solve to find unknowns, using equations of equilibrium and equations which are obtained from geometric relations between the temperature and load deformation.

Example:

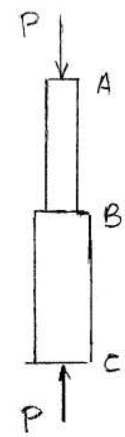
A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°C$) and portion *BC* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°C$). Knowing that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of 50 °C.



SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{mm}^2 = 1.9635 \times 10^{-3} \text{m}^2$$



Free thermal expansion:

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (0.250)(11.7 \times 10^{-6})(50) + (0.300)(20.9 \times 10^{-6})(50)$$
$$= 459.75 \times 10^{-6} \text{m}$$

Shortening due to induced compressive force P:

$$\delta_P = \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}$$
$$= \frac{0.250P}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{0.300P}{(105 \times 10^9)(1.9635 \times 10^{-3})}$$
$$= 3.2235 \times 10^{-9} P$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235 \times 10^{-9} P = 459.75 \times 10^{-6}$$
$$P = 142.62 \times 10^3 \text{N}$$

$P = 142.6$ kN ◀

Example:

A steel railroad track ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}$/°C) was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7 \times 10^{-6})(48 - 6)(10) = 4.914 \times 10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200 \times 10^9} = 50 \times 10^{-12}\sigma$$

$$\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 0$$

$$\sigma = -98.3 \times 10^6 \text{ Pa}$$

(*b*) $\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 3 \times 10^{-3}$

$$\sigma = \frac{3 \times 10^{-3} - 4.914 \times 10^{-3}}{50 \times 10^{-12}}$$
$$= -38.3 \times 10^6 \text{ Pa}$$