## Terzaghi Bearing-Capacity Equation

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) as shown bellow:-


Terzaqhi Equation (1843)

$$
\begin{aligned}
& q_{\mathrm{ult}}=c N_{c} s_{c}+\bar{q} N_{q}+0.5 \gamma B N_{\gamma} s_{\gamma} \quad N_{q}=\frac{a^{2}}{a \cos ^{2}(45+\phi / 2)} \\
& a=e^{(0.75 \pi-\phi / 2) \tan \phi} \\
& N_{c}=\left(N_{q}-1\right) \cot \phi \\
& N_{\gamma}=\frac{\tan \phi}{2}\left(\frac{K_{p \gamma}}{\cos ^{2} \phi}-1\right)
\end{aligned}
$$

For: strip round square

$$
\begin{array}{lll}
s_{c}=1.0 & 1.3 & 1.3 \\
s_{\gamma}=1.0 & 0.6 & 0.8
\end{array}
$$

Where $N_{C}, N_{q}$, and $N_{\gamma}$ are Terzaghi factors, $S_{C}$ and $S_{\gamma}$ shape factors, B and D are the width and depth of foundation respectively. $\boldsymbol{q}^{-}$is an overburden pressure at the base of footing.

By substituting $S_{\boldsymbol{C}}$ and $S_{\boldsymbol{\gamma}}$ in the above equation, we obtain:-
$q_{u}=C N_{C}+q^{-} N_{q}+0.5 \gamma B N_{\gamma}$
( for strip footing)
$q_{u}=1.3 C N_{C}+q^{-} N_{q}+0.3 \gamma B N_{\gamma}$
$q_{u}=1.3 C N_{C}+q^{-} N_{q}+0.4 \gamma B N_{\gamma}$
(round footing)
(Square footing)

Table 3.1 Terzaghi's Bearing Capacity Factors-Eqs. (3.4), (3.5), and (3.6) a From
Kumbhojkar (1993)

| $\boldsymbol{\phi}^{\prime}$ | $\boldsymbol{N}_{\boldsymbol{c}}$ | $\boldsymbol{N}_{\boldsymbol{q}}$ | $\boldsymbol{N}_{\boldsymbol{\gamma}}{ }^{\mathrm{a}}$ | $\boldsymbol{\phi}^{\prime}$ | $\boldsymbol{N}_{\boldsymbol{c}}$ | $\boldsymbol{N}_{\boldsymbol{q}}$ | $\boldsymbol{N}_{\boldsymbol{\gamma}}{ }^{a}$ |
| ---: | ---: | ---: | :--- | :--- | :--- | :--- | ---: |
| 0 | 5.70 | 1.00 | 0.00 | 26 | 27.09 | 14.21 | 9.84 |
| 1 | 6.00 | 1.10 | 0.01 | 27 | 29.24 | 15.90 | 11.60 |
| 2 | 6.30 | 1.22 | 0.04 | 28 | 31.61 | 17.81 | 13.70 |
| 3 | 6.62 | 1.35 | 0.06 | 29 | 34.24 | 19.98 | 16.18 |
| 4 | 6.97 | 1.49 | 0.10 | 30 | 37.16 | 22.46 | 19.13 |
| 5 | 7.34 | 1.64 | 0.14 | 31 | 40.41 | 25.28 | 22.65 |
| 6 | 7.73 | 1.81 | 0.20 | 32 | 44.04 | 28.52 | 26.87 |
| 7 | 8.15 | 2.00 | 0.27 | 33 | 48.09 | 32.23 | 31.94 |
| 8 | 8.60 | 2.21 | 0.35 | 34 | 52.64 | 36.50 | 38.04 |
| 9 | 9.09 | 2.44 | 0.44 | 35 | 57.75 | 41.44 | 45.41 |
| 10 | 9.61 | 2.69 | 0.56 | 36 | 63.53 | 47.16 | 54.36 |
| 11 | 10.16 | 2.98 | 0.69 | 37 | 70.01 | 53.80 | 65.27 |
| 12 | 10.76 | 3.29 | 0.85 | 38 | 77.50 | 61.55 | 78.61 |
| 13 | 11.41 | 3.63 | 1.04 | 39 | 85.97 | 70.61 | 95.03 |
| 14 | 12.11 | 4.02 | 1.26 | 40 | 95.66 | 81.27 | 115.31 |
| 15 | 12.86 | 4.45 | 1.52 | 41 | 106.81 | 93.85 | 140.51 |
| 16 | 13.68 | 4.92 | 1.82 | 42 | 119.67 | 108.75 | 171.99 |
| 17 | 14.60 | 5.45 | 2.18 | 43 | 134.58 | 126.50 | 211.56 |
| 18 | 15.12 | 6.04 | 2.59 | 44 | 151.95 | 147.74 | 261.60 |
| 19 | 16.56 | 6.70 | 3.07 | 45 | 172.28 | 173.28 | 325.34 |
| 20 | 17.69 | 7.44 | 3.64 | 46 | 196.22 | 204.19 | 407.11 |
| 21 | 18.92 | 8.26 | 4.31 | 47 | 224.55 | 241.80 | 512.84 |
| 22 | 20.27 | 9.19 | 5.09 | 48 | 258.28 | 287.85 | 650.67 |
| 23 | 21.75 | 10.23 | 6.00 | 49 | 298.71 | 344.63 | 831.99 |
| 24 | 23.36 | 11.40 | 7.08 | 50 | 347.50 | 415.14 | 1072.80 |
| 25 | 25.13 | 12.72 | 8.34 |  |  |  |  |

${ }^{a}$ From Kumbhojkar (1993)
Example 1. A square foundation is $2^{m *} 2^{m}$ in plan. The soil supporting the foundation has a friction angle $\emptyset=25^{\circ}$ of and $C=20 \mathrm{KN} / \mathrm{m} 2$ The unit weight of soil, is $\gamma=16.5 \mathrm{KN} / \mathrm{m3}$. Determine the allowable gross load on the foundation with a factor of safety (FS) of 3. Assume that the depth of the foundation is 1.5 m and that general shear failure occurs in the soil.
Solution:
Ultimate bearing capacity may be obtained by general Terzaqhi's Equation for square footing:-

$$
q_{U}=1.3 C N_{C}+q N_{q}+0.4 \gamma B N_{\gamma}
$$

For $\varnothing=25^{\circ}$

$$
N_{C}=25.13, \quad N_{q}=12.72, \quad N_{\gamma}=8.34
$$

$$
q_{U}=1.3 * 20 * 25.13+1.5 * 16.5 * 12.72+0.4 * 16.5 * 2 * 8.34=1,078 K N / m 2
$$

$q_{\text {all }}=\frac{q_{U}}{F}=\frac{1078}{3}=359.4 \mathrm{KN} / \mathrm{m} 2$

Thus the total allowable gross load, $Q=359.4 * 2^{2}=1437 K N$

Example 2. Compute the allowable bearing pressure using the Terzaghi equation for the footing and soil parameters shown in the figure. Use a safety factor of 3 to obtain $q_{a}$. Compare this with the value obtained from using Eq. (/). The soil data are obtained from a series of undrained $U$ triaxial tests. Is the soil saturated?


## Solution:

1. The soil is not saturated, since a $U$ test gives a $\emptyset$ angle.
2. Find the bearing capacity. Note that this value is usually what a geotechnical consultant would have to recommend (B not known but D is).

$$
\begin{aligned}
& N_{c}=17.7 \quad N_{q}=7.4 \quad N_{\gamma}=5.0 \\
& s_{c}=1.3 \quad s_{\gamma}=0.8
\end{aligned}
$$

$q_{u}=1.3 C N_{C}+q N_{q}+0.4 \gamma B N_{\gamma}$
(Square footing)
$q_{u}=1.3 * 20 * 17.7+1.2 * 17.3 * 7.4+0.4 * 17.3 * B * 5$
$=613.8+34.6 B$
$q_{a}=\frac{613.8+34.6 B}{3}=205+11.5 B \quad \mathrm{KN} / \mathrm{m} 2$
Assume B=1.5, $q_{a}=222 \mathrm{KN} / \mathrm{m} 2$

## Modification of Bearing Capacity Equations for Water Table

Terzaghi's Equations give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure below).


Case I. If the water table is located so that $\mathbf{0} \leq \boldsymbol{D}_{\boldsymbol{1}} \leq \boldsymbol{D}_{\boldsymbol{f}}$ the factor $q$ in the bearing capacity equations takes the form:

$$
\boldsymbol{q}=\gamma \boldsymbol{D}_{1}+\left(\gamma_{s a t}-\gamma_{w}\right) D_{2}
$$

Where
$\gamma_{\text {sat }}$ : saturated density of soil , $\quad \gamma_{w}$ :density of water
Also, the value of in the last term of the equations has to be replaced by
Case II. For a water table located so that

$$
\begin{gathered}
\gamma^{-}=\gamma_{\text {sat }}-\gamma_{w} \\
0 \leq \mathrm{d} \leq \mathrm{B}, \\
\boldsymbol{q}=\gamma \boldsymbol{D}_{f}
\end{gathered}
$$

In this case, the factor in the last term of the bearing capacity equations must be replaced by the factor:-

$$
\bar{\gamma}=\gamma^{\prime}+\frac{d}{B}\left(\gamma-\gamma^{\prime}\right)
$$

Case III. When the water table is located so that $d>B$ the water will have no effect on the ultimate bearing capacity.

## Example:

A square foundation $(B \times B)$ has to be constructed as shown in Figure 3.7. Assume that $\gamma=16.5 \mathrm{kN} / \mathrm{m}^{3}, \gamma_{\mathrm{sat}}=18.55 \mathrm{kN} / \mathrm{m}^{3}, \phi^{\prime}=34^{\circ}, D_{f}=1.22 \mathrm{~m}$, and $D_{1}=0.61 \mathrm{~m}$. The gross allowable load, $Q_{\text {all }}$, with FS $=3$ is 667.2 kN . Determine the size of the Footing.

Answer: $B=1.3 \mathrm{~m}$

