## Lecture One

## Basic Concepts and Units

### 1.1 Introduction

The study of an electrical engineering involves the analysis of the energy transfer from one point to another. So before beginning the actual study of an electrical engineering, it is necessary to understand some important fundamentals about the basic elements of an electrical engineering such as electromotive force, current, voltage, resistance etc. Practically, electrical system is related with number of other types of systems such as mechanical, thermal etc. Therefore, in this lecture we are going to study the basic concepts and review the International System (SI units) of measurement of different quantities such as work, power, energy etc. in various systems together with the definition of related terms.

### 1.2 Structure of an Atom

The structure of matter plays an important role in the understanding of fundamentals of electricity. Basically, an atom represents the basic element in the structure of the matter. The atom is composed of three fundamental particles: neutron, proton and the electron.

| Fundamental <br> particles of matter | Symbol | Nature of charge <br> possessed | Mass in Kg. |
| :---: | :---: | :---: | :---: |
| Neutron | n | $\mathbf{0}$ | $\mathbf{1 . 6 7 5 \times 1 0 ^ { - 2 7 }}$ |
| proton | $\mathrm{p}+$ | + | $1.675 \times \mathbf{1 0}^{-27}$ |
| electron | $\mathrm{e}^{-}$ | - | $\mathbf{9 . 1 0 7 \times 1 0 ^ { - 3 1 }}$ |

All of the protons and neutrons are bound together into a compact nucleus. Nucleus may be thought of as a central sun, about which electrons revolve in a particular fashion. The electrons are arranged in different orbits as shown in Fig. 1.1.


Fig. 1.1: Shells and subshells of the atomic structure.

### 1.3 Concept and Unit of Charge

Key Point: Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs ( $C$ ).

Table 1.2 below shows the different particles and charge possessed by them.
Table 1.2: Charge for different particles.

| Particle | Charge possessed <br> (in Coulomb) | Nature |
| :---: | :---: | :---: |
| Neutron | 0 | Neutral |
| Proton | $1.602 \times 10^{-19}$ | Positive |
| Electron | $1.602 \times \mathbf{1 0}^{-19}$ | Negative |

It can be seen from the table above that the charge possessed by the electron is very small hence it is not convenient to take it as the unit of charge. The unit of the measurement of the charge is Coulomb, so one coulomb charge is defined as

$$
1 \text { coulomb }=\text { charge on } 6.24 \times 10^{18} \text { electrons }
$$

The charge associated with one electron can then be determined as

$$
\text { Charge/electron }=\mathrm{e}=\frac{1 \mathrm{C}}{6.24 \times 10^{18}}=1.602 \times 10^{-19} \mathrm{C}
$$

### 1.4 Concept of Electromotive Force and Current

- The free electrons are responsible for the flow of electric current.
- A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions.
- The small electrical effort, externally applied to such conductor makes all the free electrons to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such physical phenomenon is represented in Fig.1.2.

Key Point: Electromotive force (e.m.f.): The force that establishes the flow of charge (or current) in a system due to the application of a difference in potential. This term is not applied that often in today's literature.


Fig. 1.2: The flow of current.

- The free electrons as are negatively charged get attracted by positive of the cell connected.
- This movement of electron is called an electric current.
- The flow of electrons is always from negative to positive while flow of current is always assumed as from positive to negative. This called direction of conventional current.


### 1.5 Relation between Charge and Current

The current is flow of electrons. Thus current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second.

Key Point: Electric current is the time rate of change of charge, measured in amperes (A).
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Mathematically we can write the relation between the charge $(\mathbf{Q})$ and the electric current $(\mathbf{I})$ as,

$$
\begin{equation*}
i=\frac{d q}{d t} \tag{1.1}
\end{equation*}
$$

Where current is measured in amperes (A), and

$$
1 \text { ampere }=1 \text { coulomb } / \text { second }
$$

The charge transferred between time $\mathbf{t}_{\mathbf{0}}$ and $\mathbf{t}$ is obtained by integrating both sides of Eq. (1.1). We obtain

$$
\begin{gather*}
q=\int_{t_{0}}^{t} i d t  \tag{1.2}\\
I=\frac{Q}{t} \quad \text { Ampere } \tag{1.3}
\end{gather*}
$$

Where $\mathbf{I}$ : is the average current flowing; $\mathbf{Q}$ : is the total charge transferred; and $\mathbf{t}$ : is the time required for transfer of charge.
Definition of 1 Ampere: A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.

1 Ampere current $=$ Flow of $6.24 \times 10^{18}$ electrons per second
Example 1.1: Determine the time required for $4 \times 10^{16}$ electrons to pass through a surface if the current is 5 mA .

## Solution: Determine t?

$$
\begin{aligned}
& Q=\text { No. of electrons } \times \mathrm{e} \\
& Q=4 \times 10^{16} \times 1.602 \times 10^{-19}=0.641 \times 10^{-2} \mathrm{C}=6.41 \mathrm{mC} \\
& t=\frac{Q}{I}=\frac{6.41 \times 10^{-3} \mathrm{C}}{5 \times 10^{-3} \mathrm{~A}}=1.282 \mathrm{~s}
\end{aligned}
$$

### 1.5.1 Electron Drift Velocity and Velocity of Charge

A speed with which charge drifts in a conductor is called the velocity of charge. Since current is the rate of flow of charge, it is given as

$$
\begin{equation*}
i=\frac{d q}{d t}=\frac{\mathrm{nAe} v d t}{d t} \quad \therefore \quad I=\mathbf{n A e v} \tag{1.4}
\end{equation*}
$$

Where $\mathbf{n}$ : is the number of free electrons available per $\mathrm{m}^{3}$ of the conductor material; $\boldsymbol{v}$ : is the drift velocity $\mathrm{m} / \mathrm{s} ; \mathbf{A}$ : is the cross-section area of the conductor; $\mathbf{e}$ : is the electron's charge.

The speed with which charge drifts in a conductor is called the velocity of charge.
Let the Current density $\mathbf{J}$ (current per unit area), then $\mathbf{J}=\mathbf{I} / \mathbf{A}=\mathbf{n e v} \mathrm{A} / \mathrm{m}^{2}$

Example 1.2: A conductor material has a free-electron density of $10^{24}$ electrons $/ \mathrm{m}^{3}$. When a voltage is applied, a constant drift velocity of $1.5 \times 10^{-2} \mathrm{~m} / \mathrm{s}$ is attained by the electrons. If the cross-sectional area of the material is $1 \mathrm{~cm}^{2}$, calculate the magnitude of the current.

Solution: The magnitude of the current is $\mathbf{I}=\mathbf{n A e v}$
Here, $\mathrm{n}=10^{\mathbf{2 4}}$ electrons $/ \mathrm{m}^{\mathbf{3}} ; \mathrm{A}=\mathbf{1} \mathrm{cm}^{2}=\mathbf{1 0}^{-4} \mathrm{~m}^{\mathbf{2}} ; \boldsymbol{v}=\mathbf{1 . 5} \times \mathbf{1 0}^{-2} \mathrm{~m} / \mathrm{s} ; \mathrm{e}=1.6 \times \mathbf{1 0}^{-19} \mathrm{C}$;
$\therefore \mathrm{I}=10^{24} \times 10^{-4} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-2}=0.24 \mathrm{~A}$

Example 1.3: Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section $1 \mathrm{~cm}^{2}$ and length 10 km . Free electron density of copper $=8.5 \times 10^{28}$ per $\mathrm{m}^{3}$. How many years the electric charge will take to travel from one end of the conductor to the other?

Solution: $I=n e A v$ or $v=I / n e A$ $\therefore \quad \mathrm{v}=1 /\left(\mathbf{8 . 5} \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}\right)=7.35 \times 10^{-7} \mathrm{~m} / \mathrm{s}=\mathbf{0 . 7 3 5} \boldsymbol{\mu \mathrm { m }} / \mathrm{s}$

Time taken by the charge to travel conductor length of $10 \mathbf{~ k m}$ is

$$
t=\text { distance } / \text { velocity }=10 \times 10^{3} / 7.35 \times 10^{-7}=1.36 \times 10^{10} \mathrm{~s}
$$

Now, 1 year $=365 \times 24 \times 3600=31,536,000 \mathrm{~s}$

$$
t=1.36 \times 10^{10} / 31,536,000=431 \text { years }
$$

### 1.6 Concept of Electric Potential and Potential Difference

Key Point: potential is the energy required to move a unit charge through an element, measured in volts (V).

The electric potential at point due to a charge is one volt if one joule of work is done in moving a unit positive charge. Mathematically it is expressed as

$$
\begin{equation*}
\text { Eledrical Potential }=\text { work done } / \text { charge }=\mathrm{dw} / \mathrm{dq}=\mathrm{W} / \mathrm{Q} \tag{1.5}
\end{equation*}
$$

$$
1 \text { volt }=1 \text { joule } / \text { coulomb }=1 \text { newton } \cdot \text { meter } / \text { coulomb }
$$

In electric circuit, flow of current is always from higher electric potential to lower electric potential. So we can define potential difference as below:

Key Point: the difference between the electric potential at any two given points in a circuit is known as potential difference (p.d.) and measured in volts (V).

Thus, when two points have different potential, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. No current can flow if the potential difference between the two points is zero.

### 1.7 Resistance

When the electrons begins flow in the metal. The ions get formed which are charged particles as discussed earlier. Now free electrons are moving in specific direction when connected to external source of e.m.f. So such ions always become obstruction for the flowing electrons. So there is collision between ions and free flowing electrons. This not only reduces the speed of electrons but also produced the heat (increases the metal temperature) and reduces the flow of current. Thus the material opposes the flow of current.

Key Point: This property of an electric current circuit tending to prevent the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.

The resistance is denoted by the symbol ' $\boldsymbol{R}$ ' and is measured in ohm symbolically represented as $\Omega$. We can define unit ohm as below.

Key Point: 1 Ohm: Is the resistance of a circuit, when a current of 1 Ampere generates the heat at the rate of one joules per second.

### 1.7.1 Factors Affecting the Resistance ( $R$ )

1. Length of the material: It varies directly as its length, $\boldsymbol{l}$.
2. Cross-section area: It varies inversely as the cross-section area of the conductor, $\boldsymbol{A}$.
3. The type and nature of the material.
4. Temperature: The temperature of the material affects the value of the resistance.

So for a certain material at a certain temperature we can write a mathematical expression as,

$$
\begin{equation*}
R=\frac{\rho l}{A} \tag{1.6}
\end{equation*}
$$

Where $\boldsymbol{l}$ : is the length in metres; $\boldsymbol{A}$ : is the cross-sectional area in square meters; $\boldsymbol{\rho}$ : is the resistivity in $\Omega \cdot \mathrm{m}$; and $\boldsymbol{R}$ : is the resistance in ohms.

### 1.8 Resistivity and Conductivity

The resistivity or specific resistance of a material depends on nature of material and denoted by $\boldsymbol{\rho}$ (rho). From the eq. (1.6) of resistance it can be expressed as,

$$
\begin{equation*}
\rho=\frac{R A}{l} \quad \text { i.e. } \frac{\Omega \cdot m^{2}}{m}=\Omega \cdot m \tag{1.7}
\end{equation*}
$$

Definition: The resistance of the material having unit length and unit cross-sectional area is known as its specific resistance or resistivity.

Table 1.3 gives the values of resistivity of few common materials.
Table 1.3 Resistivities and temperature coefficients.

| Material | Resistivity in $\boldsymbol{\Omega} \cdot \mathrm{m}$ at $20^{0}\left(\times 10^{-8}\right)$ | Temperature coefficient at $20^{0}\left(\times 10^{-4}\right)$ |
| :---: | :---: | :---: |
| Aluminum, commercial | 2.8 | 40.3 |
| Brass | 6-8 | 20 |
| Carbon | 3000-7000 | -5 |
| Lead | 22 |  |
| Copper (annealed) | 1.73 | 39.3 |
| German silver ( $\mathbf{8 4 \%} \mathbf{C u} \mathbf{1 2 \%} \mathbf{~ N i} \mathbf{4 \%} \mathbf{~ Z n}$ ) | 20.2 | 2.7 |
| Gold | 2.44 | 36.5 |
| Iron | 9.8 | 65 |
| $\begin{gathered} \text { Manganin } \\ (\mathbf{8 4 \%} \mathbf{C u} ; \mathbf{1 2 \%} \mathbf{M n} ; \mathbf{4 \%} \mathbf{N i}) \end{gathered}$ | 44-48 | 0.15 |
| Mercury | 95.8 | 8.9 |
| Nichrome $(\mathbf{6 0 \%} \mathbf{\mathrm { Cu }} ; \mathbf{2 5 \% ~ \mathrm { Fe } ; 1 5 \% \mathrm { Cr } )}$ | 108.5 | 1.5 |
| nickel | 7.8 | 54 |
| platinum | 9-15.5 | 36.7 |
| silver | 1.64 | 38 |
| tungsten | 5.5 | 47 |
| Amber | $5 \times 10^{14}$ |  |
| bakelite | $10^{10}$ |  |
| Glass | $10^{10}-10^{12}$ |  |
| Mica | $10^{15}$ |  |
| Rubber | $10^{16}$ |  |

### 1.8.1 Conductance (G)

The conductance of any material is reciprocal of its resistance and ill denoted as $\mathbf{G}$. It is the indication of ease with which current can flow through the material. It is measured in Siemens (S).

So

$$
\begin{equation*}
G=\frac{1}{R}=\frac{A}{\rho l}=\frac{1}{\rho}\left(\frac{A}{l}\right)=\sigma\left(\frac{A}{l}\right) \tag{1.8}
\end{equation*}
$$

### 1.8.2 Conductivity

The quantity ( $\mathbf{1} / \boldsymbol{\rho}$ ) is called conductivity denoted as $\boldsymbol{\sigma}$ (sigma). Thus the conductivity is the reciprocal of resistivity. It is measured in Siemens $/ \mathbf{m}(\mathbf{S} / \mathbf{m})$.

Key Point: A material having highest value of conductivity is the best conductor while with poorest value of conductivity is the best insulator.

Example 1.4: The resistance of copper wire 25 m long is found to be $50 \Omega$. If its diameter is 1 mm , calculate the resistivity of copper

Solution:

$$
\begin{aligned}
& l=25 \mathrm{~m}, \quad \mathrm{~d}=1 \mathrm{~mm}, \\
& A=(\pi / 4) \times \mathrm{d}^{2}=(\pi / 4) \times\left(1 \times 10^{-3}\right)^{2}=0.7853 \mathrm{~mm}^{2} \\
& \rho=\frac{R A}{l}=\frac{50 \times 0.7853 \times 10^{-6}}{25}=1.57 \mu \Omega \cdot \mathrm{~m}
\end{aligned}
$$

$$
\mathrm{R}=50 \Omega
$$

Example 1.5: A silver wire has resistance of $2.5 \Omega$. What will be the resistance of a manganin wire having a diameter, half of the silver wire and length one third? The specific resistance of manganin is 30 times that of silver.
Solution: $\quad R_{s}=$ silver resistance $=2.5 \Omega, \quad d_{m}=$ manganin diameter $=d_{s} / 2$

$$
l_{\mathrm{m}}=\text { manganin length }=l_{\mathrm{s}} / 3, \quad \rho_{\mathrm{m}}=\text { manganin specific resistance }=\mathbf{3 0} \rho_{\mathrm{s}}
$$

Now the area of cross section for silver $A_{s}=(\pi / 4) \times d_{s}{ }^{2}$

$$
\begin{aligned}
& R_{s}=\frac{\rho_{s} l_{s}}{A_{s}}=\frac{\rho_{s} l_{s}}{\frac{\pi}{4} \times\left(d_{s}\right)^{2}}=2.5 \\
& \begin{aligned}
R_{m}=\frac{\rho_{m} l_{m}}{A_{m}} & =\frac{30 \rho_{s} \times\left(\frac{l_{s}}{3}\right)}{\frac{\pi}{4}\left(d_{m}\right)^{2}}=\frac{10 \rho_{s} l_{s}}{\frac{\pi}{4}\left(\frac{d_{s}}{2}\right)^{2}} \\
& =40 \frac{\rho_{s} l_{s}}{\frac{\pi}{4}\left(d_{s}\right)^{2}}=40 R_{s}=100 \Omega \quad \text { Resistance of manganin }
\end{aligned}
\end{aligned}
$$

### 1.9 Effect of Temperature on Resistance

The resistance of the material affected as temperature of a material change. As example, Atomic structure theory says that under normal temperature when the metal is subjected to
potential difference, ions i.e. unmovable charged particles get formed inside the metal. The electrons which are moving randomly get aligned in a particular direction as shown in the Fig. 1.3. If temperature increases, the ions gain energy and start oscillating about their mean position


Fig. 1.3: Vibrating ions in conductor.
and cause collision and obstruction to the flowing electrons. Due to collision and obstruction due to higher amplitude of oscillations of ions, the resistance of material increases as temperature increases. But this is not true for all materials. In some cases the resistance decreases as temperature increase.

### 1.9.1 Effect of Temperature on Metals

The resistance of all the pure metals like copper, iron, tungsten etc. increases linearly with temperature. This is shown in the Fig. 1.4.

For good conductors, an increase in temperature will result in an increase in the resistance level.


Fig. 1.4: Effect of temperature on metals. Consequently, conductors have a positive temperature coefficient.

### 1.9.2 Effect of Temperature on Carbon and Insulator

The effect of temperature on carbon and insulators is exactly opposite to that of pure metals. Resistance of carbon and insulators decreases as the temperature increase. The result is a negative temperature coefficient.

### 1.9.3 Effect of Temperature on Alloys

The resistance of alloys increase as the temperature increase but rate of increase is not significant. In fact, some of alloys show almost no change in resistance for considerable change in the temperature like Manganin (alloy of copper, manganese and nickel), Eureka (alloy of copper and nickel) etc. Due to this property alloys are used to manufacture the resistance boxes. Fig.1.5 shows the effect of temperature on metals,


Fig. 1.5: Effect of temperature on resistance. insulating materials and alloys.

### 1.9.4 Effect of Temperature on Semiconductors



Fig. 1.6: Effect of temperature on semiconductor. temperature will result in a decrease in the resistance

## level. Consequently, semiconductors have negative temperature coefficients.

The thermistor and photoconductive cell are excellent examples of semiconductor devices with negative temperature coefficients.

### 1.10 Resistance Temperature Coefficient (R.T.C.)

From the discussion up till now we can conclude that the change in resistance is,

1) Directly proportional to the initial resistance.
2) Directly proportional to the change in temperature.
3) Depends on the nature of the material whether it is a conductor, alloy or insulator.


Fig. 1.7: Resistance vs. temperature.

Let us consider a conductor, the resistance of which increases with temperature linearly.
Let $\quad \mathbf{R}_{\mathbf{0}}=$ Initial resistance at $0 \mathrm{C}^{0}, \mathbf{R}_{\mathbf{1}}=$ Resistance at $\mathrm{t}_{1} \mathrm{C}^{\mathrm{o}}, \quad \mathbf{R}_{\mathbf{2}}=$ Resistance at $\mathrm{t}_{2} \mathrm{C}^{\mathrm{o}}$
As shown in the Fig. 1.7. $\quad \mathbf{R}_{\mathbf{2}}>\mathbf{R}_{\mathbf{1}}>\mathbf{R}_{\mathbf{0}}$

Definition of R.T.C.: The resistance temperature coefficient at $\mathbf{t} \mathrm{C}^{0}$ is the ratio of change in resistance per degree Celsius to the resistance at $\mathbf{t} \mathrm{C}^{0}$. the unit of $\mathbf{R}$. T.C. is $1 / \mathrm{C}^{0}$.

From the Fig. 1.7, change in resistance $=\mathbf{R}_{\mathbf{2}}-\mathbf{R}_{\mathbf{1}}$, change in temperature $=\mathbf{t}_{\mathbf{2}}-\mathbf{t}_{\mathbf{1}}$

$$
\text { change in resistance per } \mathrm{C}^{\mathrm{o}}=\frac{\Delta R}{\Delta t}=\frac{R_{2}-R_{1}}{t_{2}-t_{1}}=\text { the slope of graph }
$$

Hence according to the definition of R.T.C. we can write $\boldsymbol{\alpha}_{1}$ at $\mathbf{t}_{1} \mathrm{C}^{0}$ as,

$$
\alpha_{1}=\frac{\text { change in resistance per } C^{o}}{\text { resistance at } t_{1} C^{o}}=\frac{\left(R_{2}-R_{1} / t_{2}-t_{1}\right)}{R_{1}}
$$

### 1.10.1 Use of R.T.C. In Calculating Resistance at $\mathbf{t}^{\mathbf{0}}$

Let

$$
\boldsymbol{\alpha}_{\mathbf{0}}=\text { R.T.C. at } 0 \mathrm{C}^{0}, \quad \mathbf{R}_{\mathbf{0}}=\text { Resistance at } 0 \mathrm{C}^{\mathrm{o}}, \quad \mathbf{R}_{\mathbf{t}}=\text { Resistance at } t \mathrm{C}^{0}
$$

$$
\begin{equation*}
\alpha_{0}=\frac{\left(R_{t}-R_{0} / t-0\right)}{R_{0}}=\frac{R_{t}-R_{0}}{t R_{0}} \Longrightarrow R_{t}=R_{0}\left(1+\alpha_{0} t\right) \tag{1.9}
\end{equation*}
$$

In general, above result can be expressed as

$$
\begin{equation*}
R_{\text {final }}=R_{\text {initial }}\left[1+\alpha_{\text {initial }} \Delta t\right] \tag{1.10}
\end{equation*}
$$

### 1.10.2 Effect of Temperature on R.T.C.

From the above discussion, it is clear that the value of R.T.C. also changes with the temperature. As the temperature increases, its value decreases. For any metal its value is maximize at $0 \mathrm{C}^{0}$. If starting temperature is $\mathbf{t}_{\mathbf{1}}=0 \mathrm{C}^{0}$ and a at $\mathbf{t} \mathrm{C}^{0}$ i.e. $\boldsymbol{\alpha}_{\boldsymbol{t}}$ is required then we can write,

$$
\begin{equation*}
\alpha_{t}=\frac{\alpha_{0}}{1+\alpha_{0}(t-0)}=\frac{\alpha_{0}}{1+\alpha_{0} t} \tag{1.11}
\end{equation*}
$$

### 1.10.3 Effect of Temperature on Resistivity

Similar to the resistance, the specific resistance or resistivity is a function of temperature. So similar to resistance temperature coefficient we can define temperature coefficient of resistivity as fractional change in resistivity per degree centigrade change in temperature from the given reference temperature.
if

$$
\boldsymbol{\rho}_{1}=\text { resistivity at } \mathbf{t}_{1} \mathrm{C}^{0}, \quad \boldsymbol{\rho}_{2}=\text { resistivity at } \mathbf{t}_{\mathbf{2}} \mathrm{C}^{0}
$$

Then temperature coefficient of resistivity $\boldsymbol{\alpha}$ at $\mathbf{t}_{\mathbf{1}} \mathrm{C}^{0}$ can be defined as,

$$
\begin{equation*}
\alpha_{t 1}=\frac{\left(\rho_{2}-\rho_{1}\right) /\left(t_{2}-t_{1}\right)}{\rho_{1}} \tag{1.12}
\end{equation*}
$$

Similarly we can write the expression for resistivity at time $\mathrm{t}^{0}$ as,

$$
\begin{align*}
& \rho_{\mathrm{t}}=\rho_{0}\left(1+\alpha_{0} t\right) \\
& \rho_{\mathrm{t} 2}=\rho_{t 1}\left[1+\alpha_{t 1}\left(t_{2}-t_{1}\right)\right] \tag{1.13}
\end{align*}
$$

Example 1.6: A certain winding made up of copper has a resistance of $100 \Omega$ at room temperature. If resistance temperature coefficient of copper at $0 \mathrm{C}^{0}$ is $0.00428 / \mathrm{C}^{\circ}$, calculate the winding resistance if temperature is increased to $50 \mathrm{C}^{\circ}$. Assume room temperature as $25 \mathrm{C}^{\circ}$.

Solution: $\quad t_{1}=25 C^{0}, \quad R_{1}=100 \Omega, \quad t_{2}=50 C^{0}, \quad \alpha_{0}=0.00428 / C^{0}$
Now

$$
\alpha_{t}=\frac{\alpha_{0}}{1+\alpha_{0} t}
$$

$$
\alpha_{25}=\frac{\alpha_{0}}{1+\alpha_{0} t_{1}}=\frac{0.00428}{1+0.00428 \times 25}=0.003866 / C^{o}
$$

Use

$$
\begin{aligned}
R_{50}=R_{25}\left[1+\alpha_{25}\left(\mathbf{t}_{2}-\mathbf{t}_{1}\right)\right] & =100[1+\mathbf{0 . 0 0 3 8 6 6}(50-25)] \\
& =109.6657 \Omega
\end{aligned}
$$

Example 1.7: A specimen of copper has a resistivity and a temperature coefficient of $1.6 \times 10^{-6}$ $\Omega . \mathrm{cm}$ at $0 \mathrm{C}^{0}$ and $1 / 254.5$ at $20 \mathrm{C}^{0}$ respectively. Find both of them at $60 \mathrm{C}^{\circ}$.

Solution:

$$
\rho_{0}=1.6 \times 10^{-6} \Omega . \mathrm{cm}=1.6 \times 10^{-8} \Omega . \mathrm{m}, \quad \quad \alpha_{20}=\frac{1}{254.5} / C^{o} \text { at } \mathbf{2 0} \mathrm{C}^{\mathbf{0}}
$$

Now

$$
\begin{aligned}
& \alpha_{t}=\frac{\alpha_{0}}{1+\alpha_{0} t} \quad \alpha_{20}=\frac{\alpha_{0}}{1+\alpha_{0} \times 20} \\
& \frac{1}{254.5}=\frac{\alpha_{0}}{1+20 \alpha_{0}} \Rightarrow 1+20 \alpha_{0}=254.5 \alpha_{0} \Rightarrow \alpha_{0}=\frac{1}{234.5} / C^{o} \text { at } 0 \mathrm{C}^{0} \\
& \alpha_{60}=\frac{\alpha_{0}}{1+\alpha_{0} \times 60}=\frac{1 / 234.5}{1+60 / 234.5}=\frac{1}{294.5} / C^{o} \\
& \rho_{\mathrm{t}}=\rho_{0}\left(1+\alpha_{0} \mathrm{t}\right) \\
& \rho_{60}=1.6 \times 10^{-8}\left(1+\frac{1}{234.5} \times 60\right)=2 \times 10^{-8} \Omega \cdot \mathrm{~m}
\end{aligned}
$$

### 1.10.4 R.T.C. of Composite Conductor

In many practical cases, it is necessary to manufacture conductors using two different types of materials, to achieve special requirements. Such a composite conductor is shown in the Fig.

## 1.8 .



Fig. 1.8: Composite conductors.
The material 1 has R.T.C. $\boldsymbol{\alpha}_{\mathbf{1}}$ and its contribution in composite conductor is $\mathbf{R}_{\mathbf{1}}$.
The material 2 has R.T.C. $\boldsymbol{\alpha}_{\mathbf{2}}$ and its contribution in composite conductor is $\mathbf{R}_{\mathbf{2}}$.
The combined composite conductor has resistance $\mathbf{R}_{1}+\mathbf{R}_{\mathbf{2}}$ while it's R.T.C. is neither $\boldsymbol{\alpha}_{1}$ nor $\boldsymbol{\alpha}_{\mathbf{2}}$ but it is $\boldsymbol{\alpha}_{12}$, different than $\boldsymbol{\alpha}_{1}$ and $\boldsymbol{\alpha}_{2}$.
$\mathbf{R}_{1}=$ resistance of material 1 at $\mathbf{t}_{\mathbf{1}} \mathbf{C}^{\mathbf{0}}, \quad \mathbf{R}_{\mathbf{2}}=$ resistance of material 2 at $\mathbf{t}_{\mathbf{1}} \mathbf{C}^{\mathbf{0}}$
$\mathbf{R}_{\mathbf{1 t}}=$ resistance of material 1 at $\mathbf{t}_{\mathbf{2}} \mathbf{C}^{\mathbf{0}}, \mathbf{R}_{\mathbf{2 t}}=$ resistance of material 2 at $\mathbf{t}_{\mathbf{2}} \mathbf{C}^{\mathbf{0}}$
$R_{12}=$ resistance composite of material at $t_{1} \mathbf{C o}^{0}$
$\mathbf{R}_{\mathbf{1 2 t}}=$ resistance composite of material at $\mathbf{t}_{\mathbf{2}} \mathbf{C}^{\mathbf{0}}$

$\mathbf{R}_{12}=\mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{2}}, \quad \mathbf{R}_{\mathbf{1 2 t}}=\mathbf{R}_{\mathbf{1 t}}+\mathbf{R}_{\mathbf{2 t}}$

$$
\begin{equation*}
\alpha_{12}=\frac{R_{1} \alpha_{1}+R_{2} \alpha_{2}}{R_{1}+R_{2}} \tag{1.14}
\end{equation*}
$$

Thus $\boldsymbol{\alpha}_{12}$ which is R.T.C. of composite conductor can be obtained at $\mathbf{t}_{1} \mathrm{C}^{\circ}$. once this is known, $\boldsymbol{\alpha}_{12}$ at any other temperature can be obtained as,

$$
\begin{equation*}
\alpha_{12 t}=\frac{\alpha_{12}}{1+\alpha_{12} \Delta t} \quad \text { where } \Delta t \text { is temperature rise } \tag{1.15}
\end{equation*}
$$

$$
\begin{equation*}
\frac{R_{2}}{R_{1}}=\frac{\alpha_{1}-\alpha_{12}}{\alpha_{12}+\alpha_{2}} \tag{1.16}
\end{equation*}
$$

Example 1.8: Two coils $A$ and $B$ have resistances $100 \Omega$ and $150 \Omega$ respectively at $0 C^{\circ}$ are connected in series. Coil A has resistance temperature coefficient of $0.0038 / \mathrm{C}^{0}$ while B has $0.0018 / \mathrm{C}^{0}$. Find the resistance temperature coefficient of the series combination at $0 \mathrm{C}^{0}$.

Solution: $\quad\left(\mathbf{R}_{\mathbf{A}}\right)_{0}=100 \Omega, \boldsymbol{\alpha}_{\mathbf{A} 0}=0.0038 / \mathrm{C}^{0},\left(\mathrm{R}_{\mathrm{B}}\right)_{0}=150 \Omega, \alpha_{\mathrm{B} 0}=0.0038 / \mathrm{C}^{0}$
At $0^{\prime} \mathrm{C}$, the series combination is $\left(\mathrm{R}_{\mathrm{AB}}\right)_{0}=\mathbf{R}_{\mathbf{A}}+\mathbf{R}_{\mathbf{B}}=\mathbf{1 0 0}+\mathbf{1 5 0}=\mathbf{2 5 0} \boldsymbol{\Omega}$
Now

$$
\left(R_{A B}\right)_{t}=\left(R_{A B}\right)=\left[1+\alpha_{A B 0} t\right]
$$

$\alpha_{A B}$ is resistance temperature coefficient of series combination,
Now $\quad\left(\mathbf{R}_{A}\right)_{t}=\left(\mathbf{R}_{A}\right)_{0}\left[1+\boldsymbol{\alpha}_{A 0} t\right]$ and $\quad\left(R_{B}\right)_{t}=\left(R_{B}\right)_{0}\left[1+\boldsymbol{\alpha}_{B 0} t\right]$

$$
\left(R_{A B}\right)_{t}=\left(R_{A}\right) t+\left(R_{B}\right)_{t}=(R=)_{0}\left[1+\alpha_{A 0} t\right]+\left(R_{B}\right)_{t}=\left(R_{B}\right)_{0}\left[1+\alpha_{B 0} t\right]
$$

Substituting in above, $\boldsymbol{\alpha}_{\mathrm{AB} 0}=\mathbf{0 . 0 0 2 6} / \mathbf{C}^{\mathbf{0}}$

### 1.11 Fundamental Quantities and Units

Scientists and engineers know that the terms they use, the quantities they measure must all be defined precisely. Such precise and standard measurements can be specified only if there is common system of indication of such measurements. This common system of unit is called 'SI' system i.e. International System of Units. The SI system is divided into six base units and two supplementary units. The six fundamental or base units are length, mass, time, electric current, temperature, amount of substance and luminous intensity, see Table 1.4. The two supplementary units are plane angle and solid angle. All other units are derived which are obtained from the above two classes of units. The derived units are classified into three main groups.

## 1. Mechanical units, 2. Electrical units, 3. Heat units

Table 1.4: SI base units.

| Quantity | unit | Symbol |
| :---: | :---: | :---: |
| Length | meter | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Luminous intensity | candela | cd |

### 1.11.1 Multiples and sub-multiples

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.5 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m): $600,000,000 \mathrm{~mm}=600,000 \mathrm{~m}=600 \mathrm{~km}$.

Table 1.5: SI prefixes.

| Multiplier | Prefix | Symbol |
| :---: | :---: | :---: |
| $10^{18}$ | exa | E |
| $10^{15}$ | peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{2}$ | hecto | h |
| 10 | deka | da |
| $10^{-1}$ | deci | d |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mathrm{\mu}$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

### 1.11.2 Electrical Units

The various electrical units are,

1. Electrical work: In an electric circuit, movement of electrons i.e. transfers of charge is an electric current. The electrical work is done when there is a transfer of charge. The unit of such work is Joule.

So if V is potential difference in volts and Q is charge in coulombs then we can write,

$$
\text { Electrical work }=\mathbf{W}=\mathbf{V} \times \mathbf{Q} \quad \mathbf{J} \quad \text { But } \quad \mathbf{I}=\mathbf{Q} / \mathbf{t},
$$

$$
\begin{equation*}
\mathbf{W}=\text { V.I.t } \quad \mathbf{J} \quad \text { where } \mathbf{t}=\text { time in second } \tag{1.17}
\end{equation*}
$$

2. Electrical power: The rate at which electrical work is done in an electric circuit is called an electrical power.
Electrical power $=\mathbf{P}=$ electrical work $/$ time $=\mathbf{W} / \mathbf{t}=$ V.I.t $/ \mathbf{t}$

## $P=$ V.I $\quad J / s e c$ i.e. watts

Thus power consumed in an electric circuit is 1 wall if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.
3. Electrical energy: An electrical energy is the total amount of electrical work done in an electric circuit.

Electrical energy $=\mathbf{E}=$ Power $\times$ Time $=$ V.I.t joules
The unit of energy is joule or watt-sec.
As watt-sec unit is very small, the electrical energy is measured in bigger units as watt-hour ( $\mathbf{W h}$ ) and kilo watt-hour $(\mathbf{k W h})$. When a power of 1 kW is utilized for 1 hour, the energy consumed is said to be $\mathbf{1} \mathbf{k W h}$. This unit is called a Unit.

### 1.11.3 Efficiency

The efficiency can be defined the ratio of energy output to energy input. It can be also expressed as ratio of power output to power input. Its value is always less than 1. Higher its value, more efficient is the system of equipment. Generally it is expressed in percentage, its symbol $\boldsymbol{\eta}$.

$$
\begin{align*}
\boldsymbol{\eta} \% & =\text { Energy output/Energy input } \times 100 \\
& =\text { Power output } / \text { Power input } \times 100 \tag{1.20}
\end{align*}
$$

