Ministry of Higher Education and Scientific Research

Al-Mustaqbal University College

Department of Chemical Engineering and petroleum Industrials

Mathematics II

2^{nd} Stage

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Polar Form

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or

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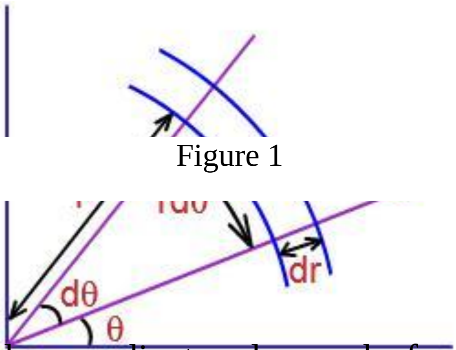


Figure 1

a- Finding Limits of Integration in polar form

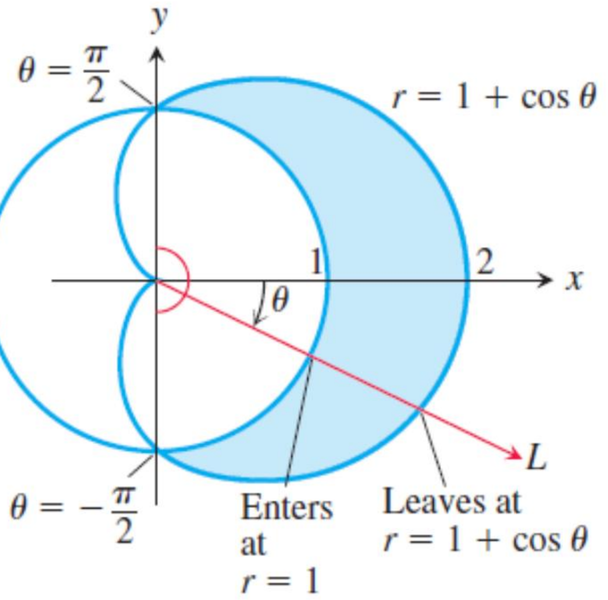
The procedure for finding limits of integration in rectangular coordinates also works for polar coordinates. To evaluate $\iint_R f(r, \theta)\, dA$ over a region R in polar coordinates, integrating first with respect to r and then with respect to , take the following steps.

1- *Sketch*. Sketch the region and label the bounding curves.

2- *Find the r-limits of integration*. Imagine a ray *L* from the origin cutting through *R* in the direction of increasing *r*. Mark the *r*-values where *L* enters and leaves *R*. These are the *r*-limits of integration. They usually depend on the angle u that *L* makes with the positive *x*-axis.

3- *Find the -limits of integration.* Find the smallest and largest -values that bound *R*. These are the -limits of integration (see figure 6). The polar iterated integral is

$$\iint_R f(r,\theta)\, dA = \int_{\theta=\pi/4}^{\theta=\pi/2} \int_{r=\sqrt{2}\csc\theta}^{r=2} f(r,\theta)\, r\, dr\, d\theta.$$



$$\int_{-\pi/2}^{\pi/2} \int_{1}^{1+\cos\theta} f(r,\theta)\, r\, dr\, d\theta.$$

Figure 2

b- **Change of variables**

Let ( ) ( ) then the formula for a change of variables in double integrals from x, y to u, v is

$$\iint (\ ) \qquad \iint (\qquad (\ )\ (\ ))\left|\frac{(\ )}{(\ )}\right|$$

that is, the integrand is expressed in terms of u and v, and dx, dy is replaced by du dv times

the absolute value of the Jacobian.

$$\left|\frac{(\ )}{(\ )}\right| \quad \left|\ \ \right|$$

For double integral transformation from the cartesian coordinates to polar coordinates ordinates as follows:

Since

using the Jacobian matrix, we find that

$$| \frac{(\quad)}{(\quad)} \quad | | \qquad | \quad ((\quad) \quad (\quad))$$

Then

$$\int \int (\quad) \qquad \int \int (\quad)$$

c- **Triple integral**

If f (x, y, z) is a function defined on a closed bounded region D in space, such as the region occupied by a solid ball or a lump of clay, then the integral of f over D may be defined in the following way.

$$\iiint \qquad \int \qquad \int_{()}^{()} \qquad \int_{(\quad)}^{(\quad)} (\qquad)$$

d- **Surface area**

Let f (x, y) be a differentiable function. As we have seen, z=f(x, y) defines a surface in x y z-space. In some applications, it necessary to know the surface area of the surface above some region R in the xy-plane. See the figure.

$$\iint \sqrt{\qquad (\quad) \quad (\quad)}$$

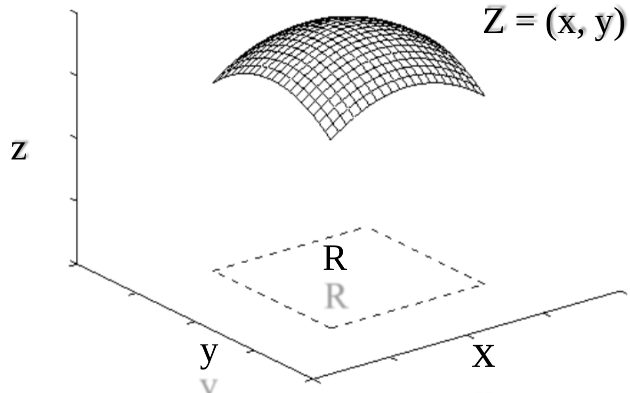


Figure 3