• Error Function.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

داله error الرئيسية

$$\operatorname{erf}_c(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du$$

Complementery
مكملة - للerror تسمى

* داله error تحتوي على الكثير من الخواص وكل خاصية لديها برهان اليكم بعض الخواص اثباتها .

ex prove $\operatorname{erf}(x^2) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t}\, t^{-\frac{1}{2}}\, dt$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

* الداله الاصليه للerror

نعوض ①

let $\boxed{u^2 = t}$ جذر $\longrightarrow$ $\boxed{u = \sqrt{t}}$

نشتق ② $\frac{dt}{du} = 2u \longrightarrow 2du\, u = dt$

$\boxed{dt = 2u\,du}$

$\boxed{du = \frac{dt}{2u}}$

$\boxed{du = \frac{dt}{2\sqrt{t}}}$

③ الحدود

$u: 0 \longrightarrow x$

$t: 0 \longrightarrow x^2$

$\sqrt{t} = t^{\frac{1}{2}}$

$$\operatorname{erf}(x) = \frac{\cancel{2}}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot \frac{dt}{\cancel{2}\sqrt{t}}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}} \cdot dt \quad \#$$

commo

② Prove : $erf(\infty) = 1$

Solution

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}}\, dt$$

اثبات السؤال السابق

$$erf(\infty) = \frac{1}{\sqrt{\pi}} \underbrace{\int_0^{\infty} e^{-t} \cdot t^{-\frac{1}{2}}\, dt}_{\Gamma(n)}$$

تكامل دالة Gramma *

$n - 1 = -\frac{1}{2}$

$n = \frac{1}{2}$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$= \frac{1}{\sqrt{\pi}} * \sqrt{\pi}$$

$$\boxed{erf(\infty) = 1} \quad \#$$

③ $erf(0) = 0$

$$erf(0) = \frac{2}{\sqrt{\pi}} \int_0^0 e^{-u^2}\, du = 0$$

④ $erf(x) + erf_c(x) = 1$

ناخذ الطرف الايسر

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \cdot du \;/\; erf_c(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} \cdot du$$

$$erf(x) + erf_c(x) = \frac{2}{\sqrt{\pi}} \left[\int_0^x e^{-u^2}\, du + \int_x^{\infty} e^{-u^2} \cdot du \right]$$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2}\, du = erf(\infty) = 1$$

$$\therefore erf(x) + erf_c(x) = 1 \quad \#$$

Ex: prove the error function is an odd function

* Function $f(x)$ is said to be odd if $f(-x) = -f(x)$

$$erf(-x) = -erf(x)$$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$erf(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-u^2} du$$

① نعوض
put $\boxed{u = -t}$, ② نشتق $\rightarrow \boxed{du = -dt}$

③ الحدود

ed

$\rightarrow x$

$(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-t^2} \, dt$

-t

$f(-x) = -$ ed(s) $\neq$