Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Syatems
Lecturer: Dr. Bayan Mahdi Sabbar & Dr. Mayasah Razzaq Abd-Ali
Lecture: 1

Lecture 1

General Review in Electrostatic & Gauss's Law

1.1 Coulomb's Law:

Records from at least 600 B.C. show evidence of the knowledge of static electricity. The Greeks were responsible for the term ``electricity," derived from their word for amber, and they spent many leisure hours rubbing a small piece of amber on their sleeves and observing how it would then attract pieces of fluff and stuff. However, their main interest lay in philosophy and logic, not in experimental science, and it was many centuries before the attracting effect was considered to be anything other than magic or a "life force".

Dr. Gilbert, physician to Her Majesty the Queen of England, was the first to do any true experimental work with this effect and in 1600 stated that glass, sulfur, amber, and other materials which he named would not only draw to themselves straws and chaff, but all metals, wood, leaves, stone, earths, even water and oil."

Shortly thereafter a colonel in the French Army Engineers, Colonel Charles Coulomb, a precise and orderly minded officer, performed an elaborate series of experiments using a delicate torsion balance, invented by himself, to determine quantitatively the force exerted between two objects, each having a static charge of electricity. His published result is now known to many high school students and bears a great similarity to Newton's gravitational law (discovered about a hundred years earlier).

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Coulomb's law : Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them:

$$F = K\frac{Q_1 Q_2}{R_{12}^2}\overrightarrow{a_{12}}$$

Where:

Q1 and Q2 are the positive or negative quantities of charge.

K: is constant

$K = \frac{1}{4\pi\epsilon_0} \qquad (\epsilon_0 = 8.85 \times 10^{-12})$

$K = 9 \times 10^9$

F: Force

$\overrightarrow{a_{12}}$: *unit vector in the direction of* R_{12}

$$\overrightarrow{a_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{r_2 - r_1}{|R_{12}|}$$

<u>Example:</u> Consider a charge of $3 \times 10^{-4}C$ at point P1(1,2,3) and charge of $-10^{-4}C$ at point P2(2,0,5), Find the F.

<u>Solution:</u>

$$R_{12} = r_2 - r_1 = (2-1)\overrightarrow{a_x} + (0-2)\overrightarrow{a_y} + (5-3)\overrightarrow{a_z} = \overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}$$

$$|R_{12}| = \sqrt{1 + (-2)^2 + 2^2} = 3$$

$$\overrightarrow{a_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{\overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}}{3}$$

$$F = K\frac{Q_1 Q_2}{R_{12}^2}\overrightarrow{a_{12}} = 9 \times 10^9 \times \frac{(3 \times 10^{-4})(-10^{-4})}{(3)^2} \times \frac{\overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}}{3}$$

$$= (-10\overrightarrow{a_x} + 20\overrightarrow{a_y} - 20\overrightarrow{a_z})N$$

1.2 Electrical Field Intensity

If we now consider one charge fixed in position, say Q1 and move the second charge slowly around ,we note that there exist everywhere a force on this second charge ,in other words this second charge is displaying the existence of a force field .call this second charge a test charge Qt. the force on its given by Coulomb's low:

$$F_t = K\frac{Q_1 Q_t}{R_{1t}^2}\overrightarrow{a_{1t}}$$

Written this force as a force per unit charge given:-

$$\frac{F_t}{Q_t} = K\frac{Q_1}{R_{1t}^2}\overrightarrow{a_{1t}}$$

The quantity on the right side of above equation is a function only of Q1 and the directed line segment from Q1 to the position of the tested charge .this describe a vector field and is called the electrical field intensity $\vec{E}$.

$$\vec{E} = \frac{F_t}{Q_t} = K\frac{Q_1}{R_{1t}^2}\overrightarrow{a_{1t}}$$

In general case:

$$\vec{E} = K\frac{Q_1}{R^2}\overrightarrow{a_r}$$

Since the Coulomb's forces are linear the electric field intensity due to two point charge Q_1 at r_1 and Q_2 at r_2 the sum of the forces on Q_t caused by Q_1 and Q_2 acting alone or

$$\vec{E} = K\frac{Q_1}{|\vec{r}-r_1|^2}\overrightarrow{a_1} + K\frac{Q_1}{|\vec{r}-r_2|^2}\overrightarrow{a_2}$$

If more than two point $\vec{E}$ will be:

$$\vec{E} = K\frac{Q_1}{|\vec{r}-r_1|^2}\overrightarrow{a_1} + K\frac{Q_1}{|\vec{r}-r_2|^2}\overrightarrow{a_2} + \cdots \ldots\ldots + K\frac{Q_1}{|\vec{r}-r_n|^2}\overrightarrow{a_n}$$

Example: Determine the electric field intensity produce by a point charge at P(1,1,1) caused by four identical 3nC point charges located at P1(1,1,0), P2(-1,1,0), P3(-1,-1,0), and P4(1,-1,0)

Solution:

$$\vec{r} - r_1 = \overrightarrow{a_z}$$

$$\vec{r} - r_2 = 2\overrightarrow{a_x} + \overrightarrow{a_z}$$

$$\vec{r} - r_3 = 2\overrightarrow{a_x} + 2\overrightarrow{a_z} + \overrightarrow{a_z}$$

$$\vec{r} - r_4 = 2\overrightarrow{a_y} + \overrightarrow{a_z}$$

$$\vec{E} = KQ_1\left[\frac{1}{|\vec{r}-r_1|^2}\overrightarrow{a_1} + \frac{1}{|\vec{r}-r_2|^2}\overrightarrow{a_2} + \frac{1}{|\vec{r}-r_3|^2}\overrightarrow{a_3} + \frac{1}{|\vec{r}-r_4|^2}\overrightarrow{a_4}\right]$$

$$\vec{E} = 27[\left[\frac{1}{(\sqrt{1})^2}(\frac{a_z}{\sqrt{1}}) + \frac{1}{(\sqrt{5})^2}(\frac{2\overrightarrow{a_x} + \overrightarrow{a_z}}{\sqrt{5}}) + \frac{1}{(\sqrt{9})^2}(\frac{2\overrightarrow{a_x} + 2\overrightarrow{a_z} + \overrightarrow{a_z}}{\sqrt{9}}) + \frac{1}{(\sqrt{5})^2}(\frac{2\overrightarrow{a_y} + \overrightarrow{a_z}}{\sqrt{5}})\right]$$

$$\vec{E} = 6.82\overrightarrow{a_x} + 6.82\overrightarrow{a_x} + 32.8\overrightarrow{a_z}$$

1.3 Electric Flux Density

About 1837 the Director of the Royal Society in London, Michael Faraday, became very interested in static electric fields and the effect of various insulating materials on these fields. This problem had been bothering him during the past ten years when he was experimenting in his now famous work on induced electromotive force. With that subject completed, he had a pair of concentric metallic spheres constructed, the outer one consisting of two hemispheres that could be firmly clamped together. He also prepared shells of insulating material (or dielectric material, or simply dielectric) which would occupy the entire volume between the concentric spheres. At that time we shall see that the materials he used will be classified as ideal dielectrics. His experiment, then, consisted essentially of the following steps:

1. With the equipment dismantled, the inner sphere was given a known positive charge.
2. The hemispheres were then clamped together around the charged sphere with about 2 cm of dielectric material between them.

3. The outer sphere was discharged by connecting it momentarily to ground.
4. The outer space was separated carefully, using tools made of insulating material in order not to disturb the induced charge on it, and the negative induced charge on each hemisphere was measured.

Faraday found that the total charge on the outer sphere was equal in magnitude to the original charge placed on the inner sphere and that this was true regardless of the dielectric material separating the two spheres. He concluded that there was some sort of "displacement" from the inner sphere to the outer which was independent of the medium, and we now refer to this flux as displacement, displacement flux, or simply electric flux.

Faraday's experiments also showed, of course, that a larger positive charge on the inner sphere induced a correspondingly larger negative charge on the outer sphere, leading to a direct proportionality between the electric flux and the charge on the inner sphere. The constant of proportionality is dependent on the system of units involved, and we are fortunate in our use of SI units, because the constant is unity. If electric flux is denoted by (psi) and the total charge on the inner sphere by Q, then for Faraday's experiment

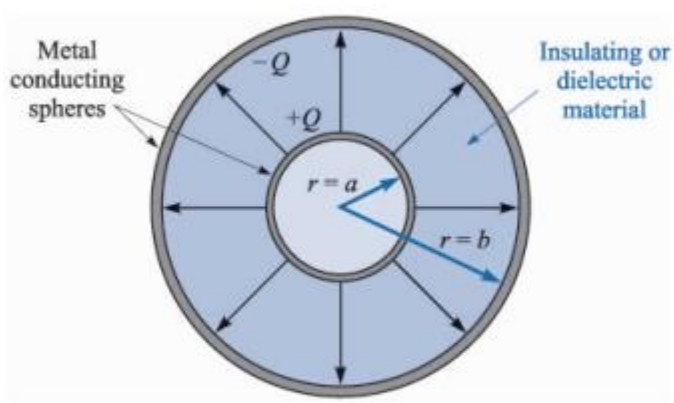
$$\psi = Q$$

and the electric flux is measured in coulombs.

We can obtain more quantitative information by considering an inner sphere of radius a and an outer sphere of radius b, with charges of $+Q$ and $-Q$, respectively (figure(1-1)). The paths of electric flux ψ extending from the inner sphere to the outer sphere are indicated by the symmetrically distributed streamlines drawn radially from one sphere to the other. At the surface of the inner sphere, ψ coulombs of electric flux are produced by the charge $Q = \psi$ coulombs distributed uniformly over a surface having an area of $4\pi a^2\ m^2$. The density of the flux at this surface is $\psi/4\pi a^2$ or $Q/4\pi a^2\ C/m^2$, and this is an important new quantity.

Electric flux density, measured in coulombs per square meter (sometimes described as "lines per square meter," for each line is due to one coulomb), is given the letter D, which was originally chosen because of the alternate names of displacement flux density or displacement density. Electric flux density is more descriptive, however, and we shall use the term consistently.

The electric flux density D is a vector field and is a member of the "flux density" class of vector fields, as opposed to the "force fields" class, which includes the electric field intensity E. The direction of D at a point is the direction of the flux lines at that point, and the magnitude is given by the number of flux lines crossing a surface normal to the lines divided by the surface area.



Figure(1-1) The electric flux in the region between a pair of charged concentric spheres. The direction and magnitude of D are not functions of the dielectric between the spheres.

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Referring again to Fig. (1-1)), the electric flux density is in the radial direction and has a value of

$$D|_{r=a} = \frac{Q_1}{4\pi a^2}\overrightarrow{a_r}$$

$$D|_{r=b} = \frac{Q_1}{4\pi b^2}\overrightarrow{a_r}$$

And at a radial distance r, where $a \leq r \leq b$,

$$D = \frac{Q_1}{4\pi r^2}\overrightarrow{a_r}$$

If we now let the inner sphere become smaller and smaller, while still retaining a charge of Q, it becomes a point charge in the limit, but the electric flux density at a point r meters from the point charge is still given by

$$D = \frac{Q_1}{4\pi r^2}\overrightarrow{a_r}$$

for Q lines of flux are symmetrically directed outward from the point and pass through an imaginary spherical surface of area $4\pi r^2$. the radial electric field intensity of a point charge in free space,

$$\vec{E} = \frac{Q_1}{4\pi \varepsilon_0 r^2}\overrightarrow{a_r}$$

In free space, therefore

$$D = \varepsilon_0 E$$ (Free Space Only)

1.4 Gauss's Law

The results of Faraday's experiments with the concentric spheres could be summed up as an experimental law by stating that the electric flux passing through any imaginary spherical surface lying between the two conducting spheres is equal to the charge enclosed within that imaginary surface. This enclosed charge is distributed on the surface of the inner sphere, or it might be concentrated as a point charge at the center of the imaginary sphere. However, since one coulomb of electric flux is produced by one coulomb of charge, the inner conductor might just as well have been a cube or a brass door key and the total induced charge on the outer sphere would still be the same. Certainly the flux density would change from its previous symmetrical distribution to some

unknown configuration, but $+Q$ coulombs on any inner conductor would produce an induced charge of $-Q$ coulombs on the surrounding sphere. Going one step further, we could now replace the two outer hemispheres by an empty (but completely closed) soup can. Q coulombs on the brass door key would produce Q lines of electric flux and would induce $-Q$ coulombs on the tin can.1

These generalizations of Faraday's experiment lead to the following statement, which is known as Gauss's law:

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

We then have the mathematical formulation of Gauss's law,

$$\psi = \oint D_s . dS = Charge\ enclosed = Q$$

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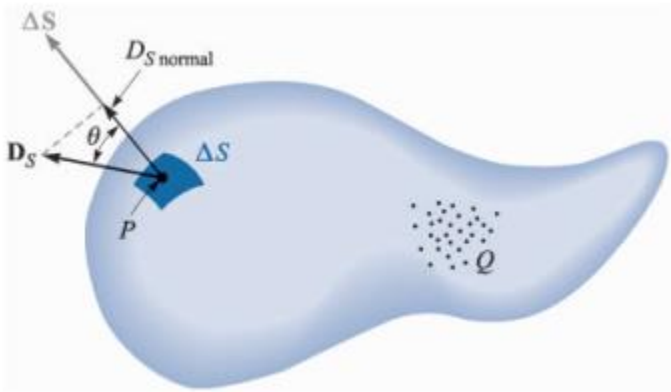

Figure (1-2): The electric flux density D_s at P due to charge Q. The total flux passing through ΔS is $D_s.\Delta S$.

Assignment:

1. A 2 mC positive charge is located in vacuum at P1(3,-2,-4) and a 5 µC negative charge is at P2(1,-4,2). Find the vector force on the negative charge.
2. Calculate $\vec{E}$ at point M(3,-4,2) in free space caused by :-

   a- A charge Q1=2µC at P1(0,0,0)

   b- A charge Q2=3µC at P2(-1,2,3)

   c- A charge Q1=2µC at P1(0,0,0) and a charge Q2=3µC at P2(-1,2,3)

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