Lecture – 10 – and Lecture – 11 -
Fan laws

The fan law equations are **used to predict the performance of a fan at some other conditions than that at which it is tested and rated.** The HVAC designer is particularly interested in the effect of power (kW), Static pressure (Ps), Volume flow rate (Q) and varying the speed (N) of the fan in a system

1-ρ=constant, N=variable

$Q \propto N$ $\Rightarrow$ $$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}$$

$Ps \propto N^2$ $\Rightarrow$ $$\frac{Ps_2}{Ps_1} = \left(\frac{N_2}{N_1}\right)^2$$

$kW \propto N^3$ $\Rightarrow$ $$\frac{kW_2}{kW_1} = \left(\frac{N_2}{N_1}\right)^3$$

2- ρ=constant, D=variable

$Q \propto D^3$ $\Rightarrow$ $$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^3$$

$Ps \propto D^2$ $\Rightarrow$ $$\frac{Ps_2}{Ps_1} = \left(\frac{D_2}{D_1}\right)^2$$

$kW \propto D^5$ $\Rightarrow$ $$\frac{kW_2}{kW_1} = \left(\frac{D_2}{D_1}\right)^5$$

Combining 1 and 2:

$$\frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3$$

$$\frac{Ps_2}{Ps_1} = \left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

$$\frac{kW_2}{kW_1} = \left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$$

3- ρ=variable, Q=constant

$$\frac{Ps_2}{Ps_1} = \frac{\rho_2}{\rho_1}$$

$$\frac{kW_2}{kW_1} = \frac{\rho_2}{\rho_1}$$

4- ρ=variable, T= variable, T(K)=T(C)+273

$\rho \propto \frac{1}{T}$ $\Rightarrow$ $$\frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}$$

$Q_2=Q_1$

$$\frac{Ps_2}{Ps_1} = \left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2\frac{\rho_2}{\rho_1}$$

$$\frac{kW_2}{kW_1} = \left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5\frac{\rho_2}{\rho_1}$$

5-ρ=variable, Ps=constant

$$\frac{Q_2}{Q_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\frac{N_2}{N_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\frac{kW_2}{kW_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

Example 1. *An air conditioning supply fan is operating at 600* **rpm** *against a static pressure of 500 Pa and requiring 4.85 kW. It is delivering 540 m^3 /min at standard conditions. In order to handle air conditioning load, heavier than originally planned, more air is desired. In order to increase volume flow rate to 610 m^s/min,* *what are the new speed, static pressure and power ?*

Sol. $\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \quad \therefore \quad N_2 = \frac{Q_2}{Q_1} \times N_1$ (Fan Law Equation 10.15A)

$\therefore \quad N_2 = \frac{610}{540} \times 600 = 677.8 \simeq \mathbf{678\ RPM}$

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^2 \qquad P_1 = 500\ Pa,\ N_1 = 600\ RPM$$
$$P_2 = ?, \qquad N_2 = 678\ RPM$$

$$\therefore \quad \frac{500}{P_2} = \left(\frac{600}{678}\right)^2 \quad \therefore \quad P_2 = \left(\frac{678}{600}\right)^2 \times 500 = \mathbf{638.5\ Pa}$$

$$\frac{KW_1}{KW_2} = \left(\frac{N_1}{N_2}\right)^3 \qquad KW_1 = 4.85\ \text{kW},\ N_1 = 600\ RPM$$
$$KW_2 = ?, \qquad N_2 = 678\ RPM$$

$$\therefore \quad \frac{4.85}{KW_2} = \left(\frac{600}{678}\right)^3 \quad \therefore \quad KW_2 = 4.85 \times \left(\frac{678}{600}\right)^3 \simeq \mathbf{7.00\ kW.}$$

Example 2. .A *fan is operating at 2700 rpm at 21 C air against 750 Pa static pressure. It is delivering 100 m^3/min and requires 2.2 BkW. A 3.75 BkW motor is powering the fan. The system is short capacity but the owner does not want to spend any money to change the motor. What is the maximum capacity from, this system with the existing 3.75 kW motor ? What is the allowable speed increase? What will flow rate and static pressure be under the new conditions ?*

$$RPM_2 = RPM_1 \times (BKW_2/BKW_1)^{1/3}$$
$$= 2700 \times (3.75/2.2)^{1/3} = \mathbf{3225.3}$$
$$Q_2 = Q_1 \times (RPM_2/RPM_1)$$
$$= 100 \times (3225.3/2700) = \mathbf{119.5\ m^3/min}$$
$$SP_2 = SP_1 \times (RPM_2/RPM_1)^2$$
$$= 750 \times (3225.3/2700)^2 = \mathbf{1070.2\ Pa.}$$

Example 3, A *fan. manufacturer wishes to project data obtained for 750 mm diameter fan to a 1500 mm diameter fan. At one operating point the 750 mm diameter fan delivers 220 m^3/min of air against a static pressure of 750 Pa. This requires 700 rpm and 1.30 BkW. What will the projected m^3/min, static pressure, BkW befor a 1500 mm fan at the same rpm ?*

$$Q_1/Q_2 = (D_1/D_2)^3 \quad \therefore \quad Q_2 = Q_1\,(D_2/D_1)^3$$
$$\therefore \quad Q_2 = 220\,(1500/750)^3 = \mathbf{1760\ m^3/min}$$

$$SP_2 = SP_1 \times (D_2/D_1)^2 = 750 \times 2^2 = \mathbf{3000\ Pa}$$

$$kW_2 = kW_1 \times (D_2/D_1)^5 = 1.3 \times (2)^5 = \mathbf{41.6\ BkW.}$$

Example 4. *A fan delivers 280 m^3/min of air against a static pressure of 500 Pa, when the speed is 500 rpm and the power input is 4.5 kW.* *What speed, static pressure and power will be obtained for a delivery of 400 m^3/min ?*

$$RPM_2 = RPM_1\,[Q_2/Q_1] = 500\,[400/280] = 714.3.$$

$$SP_2 = SP_1\,(N_2/N_1)^2 = 500 \times (714.3/500)^2 = \mathbf{1020.5\ Pa}$$

$$kW_2 = kW_1\,(N_2/N_1)^3 = 4.5\,(714.3/500)^3 = \mathbf{13.12\ kW.}$$

Example 5. *A fan delivers 230 m^3/min of air having a density of 1.2 kg/m^3 against a static pressure of 500 Pa when the speed is 600 rpm and the power input is 3.7 kW. If the inlet air temperature is changed such that the new density is 9.63 kg/m^3 at the same fan speed what is the new SP and power ?*

For constant capacity (N) and fan size (D), with variable ρ,

$$SP_2 = SP_1 \times (\rho_2/\rho_1) = 500 \times \frac{9.63}{1.2} = \mathbf{4012.5\ Pa}$$

$$kW_2 = kW_1 \times (\rho_2/\rho_1) = 3.7 \times \frac{9.63}{1.2} = \mathbf{29.7\ kW.}$$

Example 6. *For above example 5, what speed would be required to give a constant static pressure of 500 Pa at density = 9.63 kg/m^3.* *What will be. the flow rate and power ?*

Solution: For variable density, speed with constant pressure and fixed fan size, fan following relation are applicable.

$$\therefore \quad N_2 = N_1\sqrt{\rho_1/\rho_2} = 600\sqrt{1.2/9.63} = \mathbf{211.8\ RPM}$$

$$Q_2 = Q_1\sqrt{\rho_1/\rho_2} = 230\sqrt{1.2/9.63} = \mathbf{81.2\ m^3/min}$$

$$KW_2 = KW_1\sqrt{\rho_1/\rho_2} = 3.7\sqrt{1.2/9.63} = \mathbf{1.31\ kW.}$$

Example 7. *The following data is available from a test report of a centrifugal fan volume flow rate: 3 m^3/s, Shaft power = 2.6 kW. Fan static pressure = 524 Pa.* *Calculate air power and efficiency (Total) of a fan tested.* Take fan discharge area =0.3 m^2

Sol.

Discharge velocity from fan $= \frac{3}{0.3} = \frac{Q}{A} = 10$ m/s.

Velocity pressure $= \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.2 \times 100$ $\qquad \rho = 1.2$ kg/m^3 for standard air

$= 60$ N/m^2

$\therefore \quad P_T = P_S + P_V = 524 + 60 = \mathbf{584\ Pa}$

Air power $= P \times QW.$

Air Power $= 584 \times 3 = 1752$ W $=$ **1.752 kW. Ans.**

Total efficiency $\eta_T = \frac{\text{Air power}}{\text{Shaft power}} \times 100 = \frac{1.752}{2.60} \times 100 =$ **67.4%. Ans.**

Example 8. A *centrifugal fan 910 mm in diameter operates at 8.0 rpm, when handling air at a temperature of* 16 °C. *With a corresponding total pressure development of 600 N/m^2 and the shaft power is* 3 kW.

(a) If the fan is used for heating purposes and fan handles air at a temperature of 50 °C, calculate the total pressure developed and fan power under these new conditions.

(b) *If it is desired to keep the total pressure developed constant when the air handled is at a temperature 50 C, calculate the fan speed, the air volume handled and the fan power given that the volume bandied at 16 °C was 5 m^3/s. The density of air at 16 °C is 1.22 kg/m^3 at standard atmospheric pressure.*

$$\text{Density} \propto \frac{1}{T}$$

$$\therefore \quad \frac{\rho_2}{\rho_1} = \frac{T_1}{T_2} = \frac{273+16}{273+50} = \frac{289}{323}.$$

Pressure Variation (Refer Equation 10.15)

$$P \propto D^2 N^2 \rho.$$

$$P_2 = P_1\left(\frac{D_2}{P_1}\right)^2\left(\frac{N_2}{N_1}\right)^2\left(\frac{\rho_2}{\rho_1}\right) \quad \text{For constant } D \text{ and } N,$$

$$P_2 = P_1\left(\frac{\rho_2}{\rho_1}\right) = \frac{600 \times 289}{323} = \mathbf{536.84\ Pa.}$$

Power variation (Refer Equation 10.15)

$$\text{Power} \propto D^5 N^3 \rho \qquad \text{For constant } D \text{ and } N,$$

$$\text{Power} \propto \delta.$$

$$\therefore \quad kW_2 = \frac{kW_1 \times T_1}{T_2} = \frac{3 \times 289}{323} = \mathbf{2.684\ kW.}$$

If pressure is to remain constant, $P \alpha D^2 N^2 \rho$. (Refer Equation 10.15)

$\therefore \quad N^2 \propto 1/\rho.$

$$\therefore \quad N_2 = N_1\sqrt{\frac{\rho_1}{\rho_2}} = 8\sqrt{\frac{323}{289}} = 8.5 \text{ Rev/sec}$$

Also, $Q \propto D^3 N.$ (Refer Equation 10.15)

$Q \propto N.$ (with D = Constant)

$$\therefore \quad Q_2 = Q_1 \times \frac{N_2}{N_1} = \frac{5 \times 8.5}{8} = \mathbf{5.31\ m^3/sec.}$$

Power $\propto D^5 N^3 \rho$. For constant D, (Refer Equation 10.15)

$$KW_2 = KW_1\left[\frac{N_2}{N_1}\right]^3 \times \frac{T_1}{T_2} = 3\left[\frac{8.5}{8}\right]^3 \times \frac{289}{323} = \mathbf{3.22\ kW.}$$

Example 9. A *fan delivers 300 m^3/min at a static pressure of 500 Pa and 800 rpm and draws 5 kW. Calculate.(1)discharge (2)static pressure (3)power of the speed of fan is increased to 880 rpm.*

Sol. $Q_1 = 300$ m^3/min, $SP_1 = 500$ Pa, $N_1 = 800$ RPM $kW_1 = 5$

$Q_2 = ?$, $SP_2 = ?$, $N_2 = 880$ RPM $kW_2 = ?$

Speed is variable

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}, \frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^2, \left(\frac{kW_1}{kW_2}\right) = \left(\frac{N_1}{N_2}\right)^3$$

$$\therefore \quad \frac{300}{Q_2} = \frac{800}{880}, \frac{P_1}{P_2} = \left(\frac{800}{880}\right)^2 \quad \frac{15}{kW_2} = \left(\frac{800}{880}\right)^3 = 0.7513$$

$Q_2 = 330$ m^3/min,

$P_2 = 605$ Pa,

kW_2=6.66 kW

Example 10. A *centrifugal fan has a circular inlet duct 450 mm diameter and* a *rectangular duct of 450 mm* x *375 mm. The static pressure at the fan inlet is - 125 Pa and a static pressure at the fan outlet is 250 Pa when the delivers 110 m^3/min and absorbs power of 1 kW. Assume standard air,* <u>*calculate (1) Total pressure at fan inlet and outlet (2) Fan total pressure and fan static pressure (3) Fan total and fan static efficiency.*</u>

Sol. Area of inlet duct $= \pi/4\, d^2 = \pi/4 \times 0.45^2 = 0.16\ m^2$

Mean velocity in the inlet duct $= v_1 = \dfrac{Q_1}{A_1} = \dfrac{110}{0.16} = 687.5\ m/min = 11.5\ m/s.$

$\therefore$ Velocity pressure $= \frac{1}{2}\rho V^2$

$$P_V = 0.6 \times (11.5)^2 = \mathbf{79.4\ Pa}$$

$\therefore$ Total pressure at inlet $P_{T_1} = P_{V_1} + P_{S_1}$

$\therefore$ $P_{T_1} = 79.4 - 125.0 = \mathbf{-45.6\ Pa}$

Area at outlet $= 0.45 \times 0.375 = 0.16875 \approx \mathbf{0.17\ m^2}$

$\therefore$ Mean velocity at outlet $= \dfrac{Q}{A_2} = \dfrac{110}{0.17 \times 60} = \mathbf{10.8\ m/s.}$

$\therefore$ Velocity pressure $= 1/2\, \rho V^2$

$$P_{V_2} = 1/2 \times 1.2 \times (10.8)^2 = 69.984 = \mathbf{70\ Pa}$$

$\therefore$ Total pressure at outlet $P_{T_2} = P_{V_2} + P_{S_2}$

$\therefore$ $P_{T_2} = 70 + 250 = \mathbf{320\ Pa}$

$\therefore$ Fan total pressure $P_t = P_{T_2} - P_{T_1} = 320 - (-45.6) = \mathbf{365.5\ Pa}$

Now power $= P_t \times Q$

(Total air power) $= \dfrac{365.5 \times 110}{60} = 670\ W = \mathbf{0.67\ kW}$

Static air power $= P_S \times Q$ $\qquad P_S = P_t - P_{v_2}$

Shaft power

$= \dfrac{295.5 \times 110}{60} = 541.75W \qquad = 365.5 - 70 = 295.5\ Pa$

η_T = Total Power/BkW = 0.67/1.0 = 67%

η_S = Total Static Power/BkW $= \dfrac{0.54175}{1.0} = 54.2\%.$

Example 11. *Select a fan to handle 34.5 m³/min of air having a density of 0.97 kg/rn³ against a static pressure of 42.2 Pa. Find the required operating speed and power that will be needed: Use the table given below.*

Volume m^3/min	Outlet velocity m/min	42.2 Pa SP at $\rho = 1.2\ kg/m^3$. RPM	Power kW
22.6	480	919	0.10
25.4	540	962	0.13
28.2	600	1011	0.16
31.0	660	1068	0.194
34.5	720	1124	0.24

Sol.

$$\frac{Q_1}{Q_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/2} = \frac{N_1}{N_2}$$

$Q_1 = 34.5\ m^3/min,\ \rho_1 = 0.97\ kg/m^3$

$Q_2 = ?\qquad \rho_2 = 1.2\ kg/m^3$

$\therefore\quad \frac{34.5}{Q_2} = \sqrt{\frac{1.2}{0.97}} = 1.11 \qquad \therefore\ Q_2 = 31.02\ m^3/min$ at $1.2\ kg/m^3$

From above table, for Q_2=31.02 rn³/min capacity select a fan operating at 1068 rpm with outlet velocity of 660 m/min and Power of 0.194 with static pressure of 42.2 Pa. If the exact matching of (volume/static pressure) is not found in the table, linear interpolation is to be made in order to select the fan. Since the density is 0.97 kg/m³, the actual speed and power corresponding to ρ= 0.97 kg/m³ is,

$$\frac{N_1}{N_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/2} = \frac{P_1}{P_2} = \frac{1}{1.11} = \sqrt{\frac{0.97}{1.2}} = \frac{1}{1.11}.$$

$\therefore\quad N_2 = 1.11 \times 1068 = 1185.5$ RPM at $0.97\ kg/m^3$

$P_2 = 0.194 \times 1.11 = 0.215$ kW at $0.97\ kg/m^3$.

Example 12 *A fan delivers 600 m³/min against static pressure of 370 Pa with an outlet velocity of 600 m/min and a static efficiency of 75%. Calculate (1)total pressure, (2) air power, (3)Brake power (4) total or mechanical efficiency.*

Sol. For standard air,

Velocity heat $\quad P_V = \left(\frac{V}{77.5}\right)^2 Pa = \left(\frac{600}{77.5}\right)^2 = 59.937 \approx$ **60 Pa**

$\therefore$ Total pressure $\quad = P_t = P_S + P_V$

$\therefore\quad P_t = 370 + 60 = 430$ **Pa. Ans. (1)**

Air Power $\quad = \frac{Q \times P_t}{60,000}$ kW (Eq. 10.9)

$= \frac{600 \times 430}{60,000} = 4.3$ **kW. Ans. (2)**

Static efficiency $\quad \eta_s = \eta_{mech} \times \frac{P_s}{P_t}$

$\therefore\quad 0.75 = \eta_m \times \frac{370}{430}$

$\therefore\quad \eta_m = \frac{0.75 \times 430}{370} = 0.872.$ **Ans. (3)**

$= \frac{\text{Air power}}{\text{Brake power}}$

$0.872 = 4.3/BkW$

$\therefore$ Brake power $\quad = 4.3/0.872 = 4.93$ **kW. Ans. (4)**