Lecture No.3
Title: Solar Radiation Analysis

Dr. Haleemah Jaber Mohammed
alhamdaniyaha@gmail.com

3.1 The Main Parameters of the Sun

- **The Sun is the main source of light and heat in the solar system. Without the light (energy) from the Sun, there would be no life on Earth.**
- **The sun is a sphere of intensely hot gaseous matter with a diameter of 1.39 x 10^9 m (see Fig. 2.1).**
- **The sun is about 1.5 x 10^8 km away from earth, so, because thermal radiation travels with the speed of light in a vacuum (300,000 km/s), after leaving the sun solar energy reaches our planet in 8 min and 20 s.**

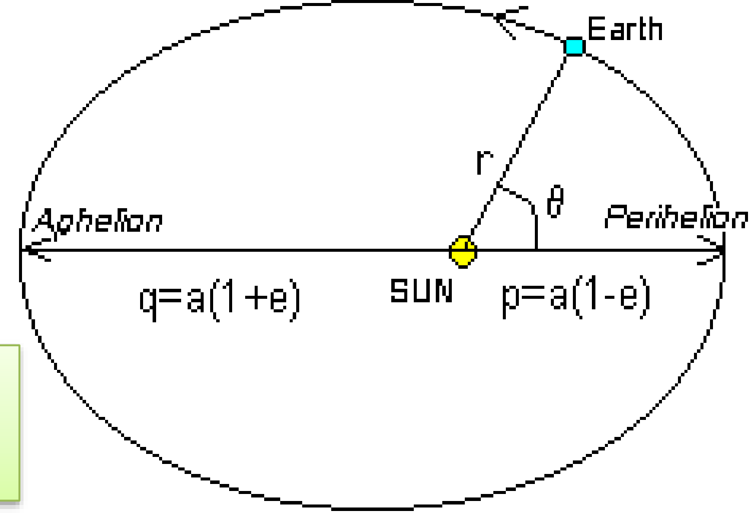


Fig. 3.1 Distance between the sun and the earth

As observed from the earth, the sun disk forms an angle of 32 min of a degree. This is important in many applications, especially in concentrator optics, where the sun cannot be considered as a point source and even this small angle is significant in the analysis of the optical behavior of the collector.
The sun has an effective black- body temperature of 5760 K. The temperature in the central region is much higher. In effect, the sun is a continuous fusion reactor in which hydrogen is turned into helium. The sun's total energy output is 3.8 x 1020 MW, which is equal to 63 MW/m2 of the sun's surface.

This energy radiates outward in all directions. The earth receives only a tiny fraction of the total radiation emitted, equal to 1.7×10^{14} kW; however, even with this small fraction, it is estimated that 84 min of solar radiation falling on earth is equal to the world energy demand for one year (about 900 EJ). As seen from the earth, the sun rotates around its axis about once every four weeks. As observed from earth, the path of the sun across the sky varies throughout the year. The shape described by the sun's position,

The most obvious variation in the sun's apparent position through the year is a north-south swing over 47° of angle (because of the 23.5° tilt of the earth axis with respect to the sun), called declination (see Section 3.2). The north-south swing in apparent angle is the main cause for the existence of seasons on earth .

The earth's orbit around the sun :

$$r = \frac{a(1-\epsilon^2)}{1+\epsilon\ COS\theta} \tag{3.1}$$

Where

a: average orbit distance = 1.5 *10^8 km ,

ϵ : Eccentricity = 0.01673

$\boldsymbol{\theta}$: is equal to the No. of the day at year and can be calculate accordant to table 3.1.

Eccentricity: deviation of a curve or orbit from circularity

- The orbit of the Earth is an ellipse not a circle, hence the distance between the Earth and Sun varies over the year, leading to apparent solar irradiation values throughout the year approximated by

$$I_o = I_{sc}\left[1 + 0.033COS\left(\frac{N}{365} \times 360°\right)\right] \tag{3.2}$$

Where the $I_{sc} = 429.5\frac{Btu}{hr.ft^2}$. ($1353\ w/m^2$)

The Earth’s closest point (about 146 million km) to the sun is called the perihelion and occurs around January 3; the Earth’s farthest point (about 156 million km) to the sun is called the aphelion and occurs around July 4.

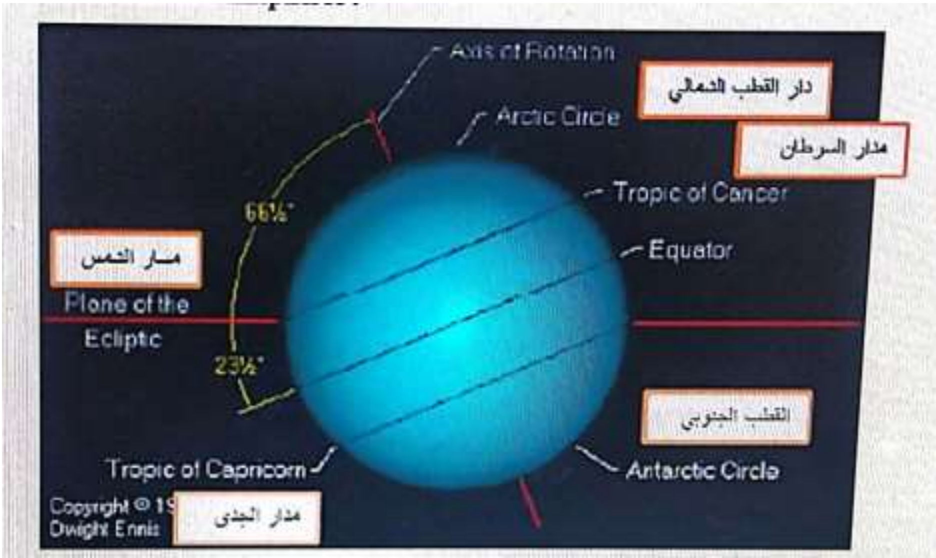
Knowledge of the sun’s path through the sky is necessary to calculate the solar radiation falling on a surface, the solar heat gain, the proper orientation of solar collectors, the placement of collectors to avoid shading, and many more factors that are not of direct interest in this book. The objective of this chapter is to describe the movements of the sun relative to the earth that give to the sun its east-west trajectory across the sky. The variation of solar incidence angle and the amount of solar energy received are analyzed for a number of fixed and tracking surfaces. The environment in which a solar system works depends mostly on the solar energy availability. Therefore, this is analyzed in some detail:

3.2 Basic Earth Sun Angles

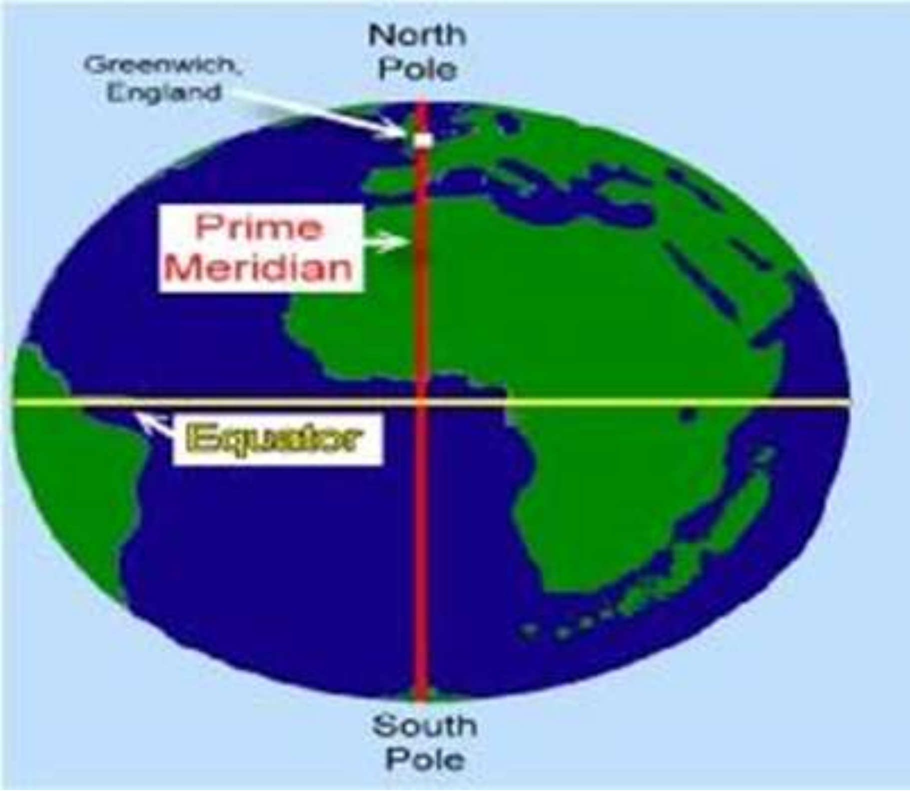
In order to understand what follows for calculations of solar radiations, the definitions of some of the basic terms are given.

Poles of the earth: The ends of the axis of rotation of the earth mark two important points on the earth's surface. They are called the poles of the earth, one as North, while the other as south .

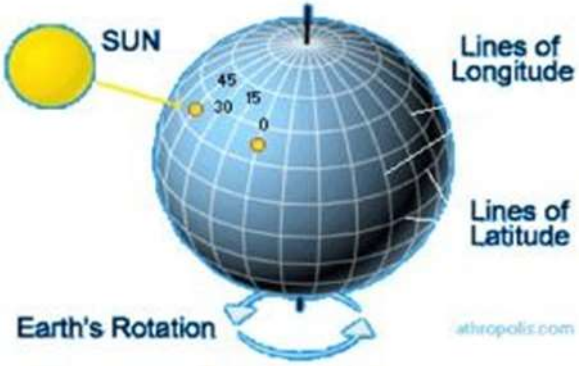
Earth's Equator: It is an imaginary great circle normal to the earth's axis, dividing the distance between the earth's poles along its surface into two equal parts. The equator divides the earth into two hemispheres called Northern and Southern hemispheres .



Meridian: It is necessary to select some reference location on the earth for helping in locating a particular position. The location of Royal Observatory Greenwich, outside of London, has been universally accepted as a reference point. An imaginary great circle passing through this point and the two poles, intersecting the equator at right angles, is called the prime (or Greenwich) meridian. Similar great circles have been drawn at intervals of 15° through the two poles .



Longitude: It is the angular distance of the location, measured east or west from the prime meridian .



Basic Earth Sun angle: The position of a point P on the earth's surface with respect on the sun's rays is known at any instant if the latitude (φ) and hour angle (ω) for the point, and the sun's declination (δ) are known. These fundamental angles are shown by Fig. 3.2. Point P represents a location on the Northern hemisphere

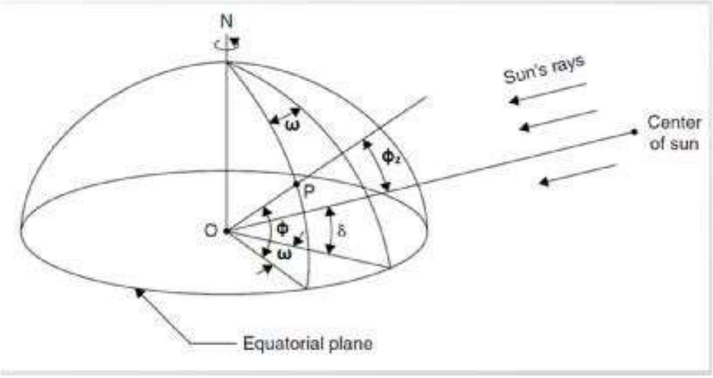


Fig 3.2. Latitude, hour angle and sun's declination

The latitude (ϕ) of a point on the surface of the earth is its angular distance north or south of the equator measured from the center of the earth. It is the angle between the line OP and the projection of OP on the
.equatorial plane. Point O represents the center of the earth

The hour angle (ω) is the angle through which the earth must turn to bring the meridian of a point directly in line with the sun's rays. It is the angle measured in the earth's equatorial plane between the projection of OP and the projection of a line from the center of the sun to the center of the earth. At solar noon the hour angle is 0 and it expresses the time of a day with respect to solar noon. It is measured positively westward from the observer and it may be expressed in,

hours, minutes and seconds
degrees, minutes, radians

ır is equivalent to $\frac{\pi 2}{24} = 0.262$ rad , or $\frac{360^\circ}{24} = 15^\circ$ and consequently 1 min = $15,4' min = 1^\circ$

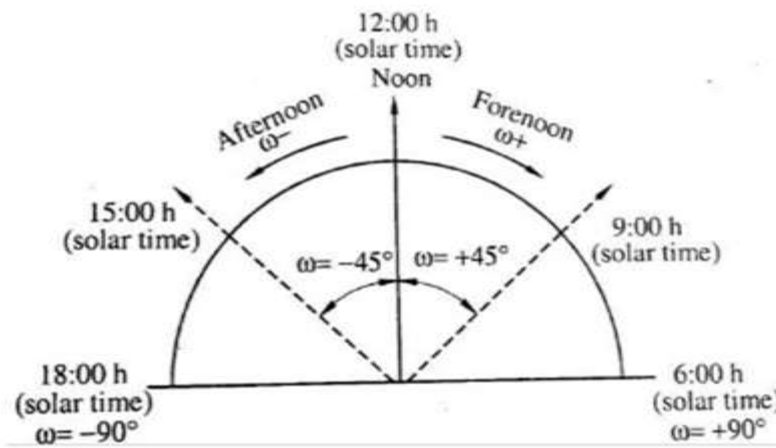
Since the Earth rotates at 360 o /24h=15 o/ h, the hour angle is given by:

$$\omega = (15^\circ h^{-1})(t_{solar} - 12\ h)$$

$$= 15^\circ h^{-1})(t_{zone} - 12h) + \omega_{eq}$$

Where t solar and zone are respectively the local solar and civil times (measured in hours), zone is the longitude where the Sun is overhead when tzone is noon (i.e. where solar time and civil time coincide). ω is positive in the evening and negative in the morning. The small correction term ωeq is called the equation of time; it never exceeds 15 min and can be neglected for most purposes.

The Sun's declination (δ): is the angular distance of the sun's rays north (or south) of the equator. It is the angle between a line extending from the center of the sun to the center of the earth, and the projection of this line upon the earth's equatorial plane. This is the direct consequence of the tilt and it would vary between 23.5o on June 22, to - 23.5° on December 22. At the time of winter solstice, the sun rays would be 23.5° south of the earth's equator (δ = -23.5o) At the time of summer solstice ,the sun's rays would be



23.5° north of the earth's equator (δ = 23 5°). At the equinoxes, the sun's declination would be zero. Fig.

2.3 shows approximately the variation of the sun's declination through the year.

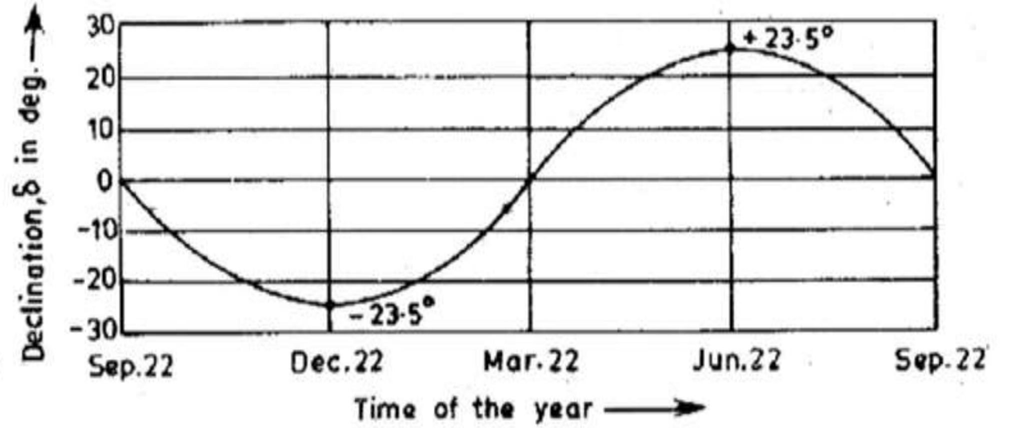


Fig 2.3 Variation of sun's declination.

.The declination, in degrees, for any given day may be calculated from the approximate equation:

$$\delta = 23.45° \sin\left[\frac{360}{365}(284 + n)\right] \qquad ((3.4$$

where n is the day of the year, [e.g. June 21, 1980 is the 173th (31 + 29 + 31 + 30 + 31 + 21) day of 1980, i.e. n = 173]. See Table 3.1.

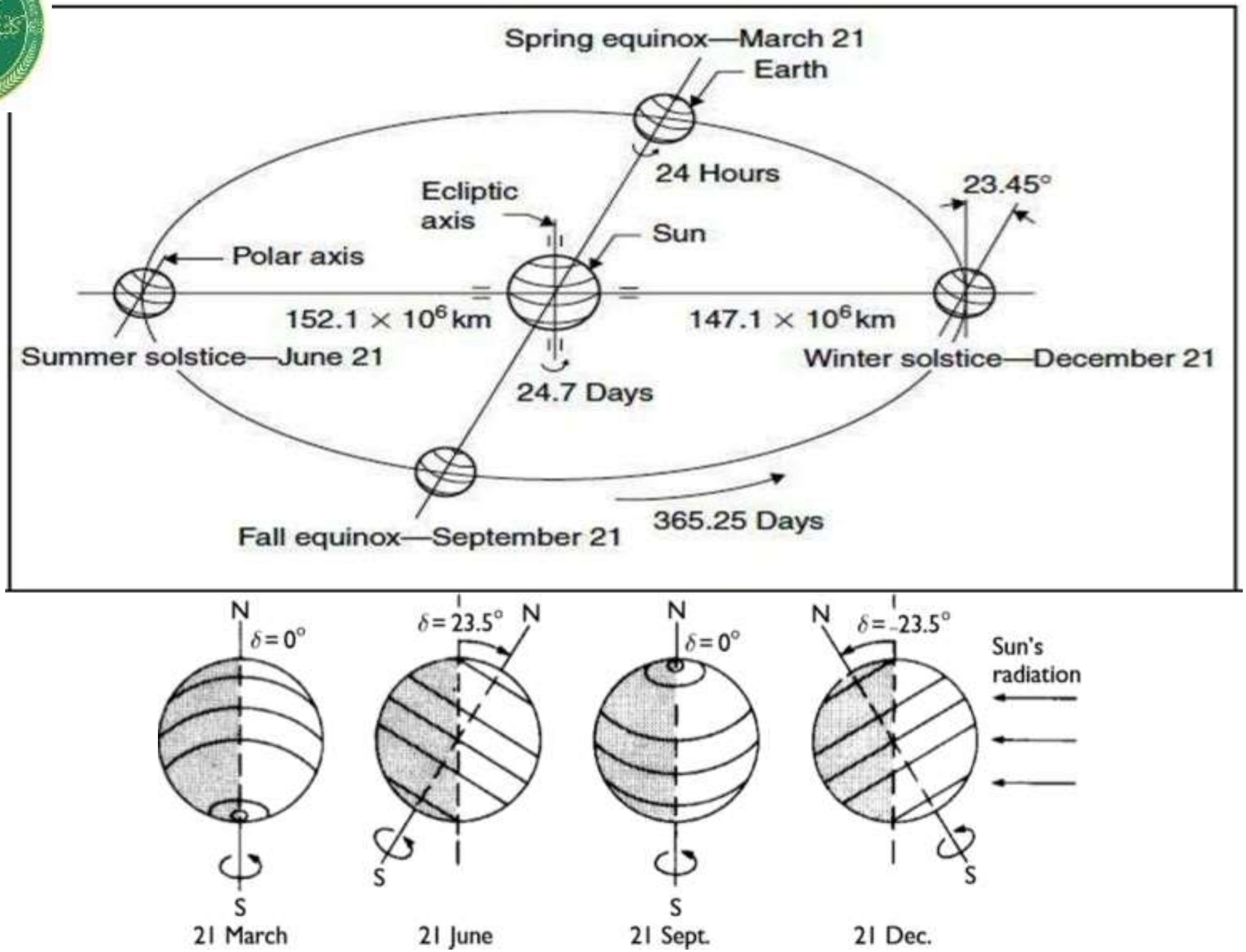


Figure 3.4 Annual motion of the earth about the sun.

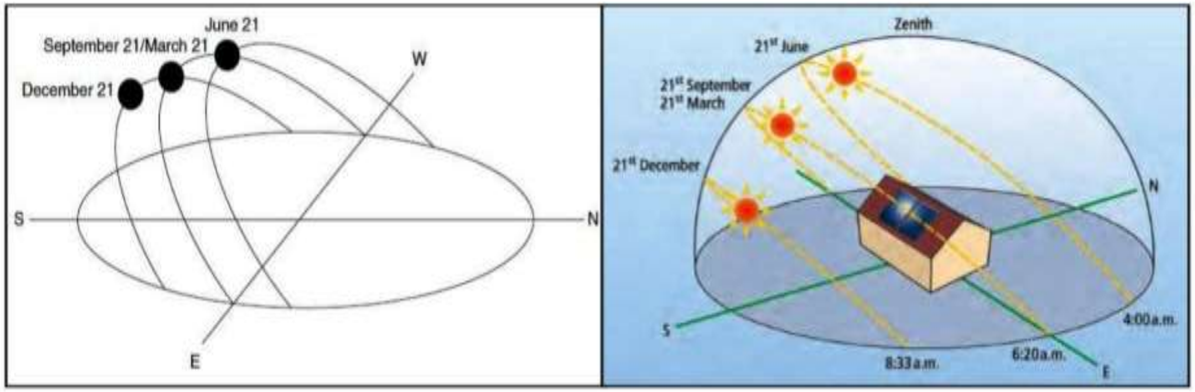


Figure 3.5 Annual changes in the sun's position in the sky (northern hemisphere.(

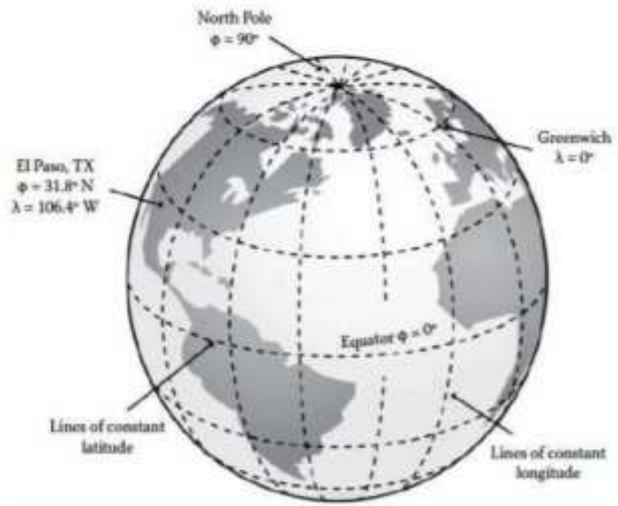
Table 3.1 Day Number and Recommended Average Day for Each Month

Month	Day number	Average day of the month		
		Date	N	δ (deg.)
January	i	17	17	−20.92
February	$31 + i$	16	47	−12.95
March	$59 + i$	16	75	−2.42
April	$90 + i$	15	105	9.41
May	$120 + i$	15	135	18.79
June	$151 + i$	11	162	23.09
July	$181 + i$	17	198	21.18
August	$212 + i$	16	228	13.45
September	$243 + i$	15	258	2.22
October	$273 + i$	15	288	−9.60
November	$304 + i$	14	318	−18.91
December	$334 + i$	10	344	−23.05

nation of Solar Time

es of time in solar energy computations are expressed in terms of apparent solar time (this is also known as true solar time). Thus we would be required to convert the clock time to the local solar time .

Greenwich meridian (zero longitude) is taken as reference for the time and time reckoned from mid night is known as universal time or Greenwich civil time (GCT or GMT). Such time is expressed on an hour scale from 0h to 24h .



Local civil time (LCT or LMT) is reckoned from the longitude of the place on any particular meridian. On a particular place LCT is more advanced than at a point westward. The difference amounts to four minutes of time for each degree difference in longitude. Time as measured by the apparent diurnal motion of the sun is called Apparent Solar Time or Solar Time. It is the time that would be shown by a sun dial whereas a civil day is precisely 24 hours, a solar day is slightly different due to irregularities of the earth's rotation, obliquity of the earth orbit and other factors, in other words due to the elliptical shape of the earth's orbit and to its increase in velocity at the perihelicon, the length of the apparent solar day,

i.e. the interval between two successive passages of the sun through the meridian, is not constant. Local civil time may deviate from true solar time by as much as 4.5o because even if the length of any apparent solar day and its corresponding mean solar day differ little, the effect is cumulative. The difference between local solar time LST and local civil time LCT is called the equation of time. Thus

$$LST = LCT + \text{Eq. of time} \qquad (3.5)$$

Table 3.2 shows weekly values of the equation of time for the year 1958, along with the values of the declination. For practical purposes, these values may be used for any year. At a given locality, watch time may differ from civil time. Clocks are usually set for the same reading through an entire zone covering about 15^o of longitude. The time kept in each zone is the local civil time of a selected meridian near the center of the zone. Such time is called standard time. Local civil time is:

LCT = Standard time ± (L_{st} - L_{local}) x 4 and solar time (3.5.1)

LST = standard time + E ± (Lst -Llocal) x 4 (+ sign for west and - ve for east((3.5.2)

Where E = the equation of time in minutes Lst = the standard meridian for the local time zone, Llocal = the longitude of the location in question in degrees west or east. Positive sign is for western and negative sign for eastern hemisphere.ش

The difference between the true solar time and the mean solar time changes continuously day-today with an annual cycle. This quantity is known as the equation of time. The equation of time, ET in minutes, is approximated by

E= 9.87 sin (2D)-7.53 cos (D)-1.5 sin (D) (3.5.3)

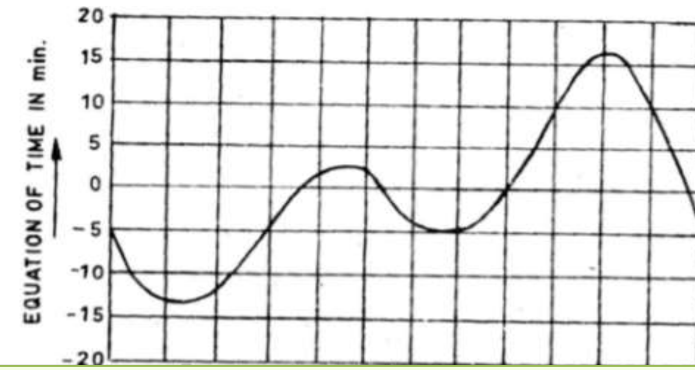
Where D = 360 $\frac{n-81}{360}$

n= is the day of the year

Lst = local longitude of standard time meridian $= 15 \times ^{\circ}$) $\frac{Long\ )}{^{\circ}15}$ (3.5.4)

$regetno\ it\ dnuor$

Fig. 3.6 Shows equation of time correction



Hence we conclude that the time specified in all the sun-angle relationship is solar time, which does not coincide with the local clock time. It is necessary to convert standard time to solar time by applying two corrections. First there is a constant correction for any difference in longitude between the location and the meridian on which the local standard time is based (e.g. 82.5oE for India). The second correction is from the equation of time which takes into account the various perturbations in the earth's orbit and the rate of rotation which affect the time, the sun appears to cross the observer's meridian. This correction is obtained from published charts.

Day→	1		8		15		22	
Month	Dec. Deg : Min	Eq. of time Min : Sec	Dec. Deg : Min	Eq. of time Min : Sec	Dec. Deg : Min	Eq. of time Min : Sec	Dec. Deg : Min	Eq. of time Min : Sec
January	– (23 : 08)	– (3 : 16)	– (22 : 20)	– (6 : 26)	– (21 : 15)	– (9 : 12)	– (19 : 50)	– (11 : 27)
February	– (17 : 18)	– (13 : 34)	– (15 : 13)	– (14 : 14)	– (12 : 55)	– (14 : 15)	– (10 : 27)	– (13 : 41)
March	– (7 : 51)	– (12 : 38)	– (5 : 10)	– (11 : 0)	– (2 : 25)	– (9 : 14)	0 : 21	–(7 : 12)
April	4 : 16	– (4 : 11)	6 : 56	– (12 : 07)	9 : 30	– (0 : 15)	11 : 57	1 : 19
May	14 : 51	2 : 50	16 : 53	3 : 31	18 : 41	3 : 44	20 : 14	3 : 30
June	21 : 57	2 : 25	22 : 47	1 : 15	23 : 17	– (0 : 09)	23 : 27	– (1 : 40)
July	23 : 10	– (3 : 33)	22 : 37	– (4 : 48)	21 : 2	– (5 : 45)	20 : 25	– (6 : 19)
August	18 : 12	– (6 : 17)	16 : 21	– (5 : 40)	14 : 17	– (4 : 35)	12 : 02	– (3 : 04)
September	8 : 33	– (0 : 15)	5 : 58	2 : 03	3 : 19	4 : 29	0 : 36	6 : 58
October	– (2 : 54)	10 : 02	– (5 : 36)	12 : 11	– (8 : 15)	13 : 59	– (10 : 48)	15 : 20
November	– (14 : 12)	16 : 20	– (16 : 22)	16 : 16	– (18 : 18)	15 : 29	– (19 : 59)	14 : 02
December	– (21 : 41)	11 : 14	– (22 : 28)	8 : 26	– (23 : 14)	5 : 13	– (23 : 27)	1 : 47

Table 3.2. Suns declination and equation of time.

Example 3.1

Determine the local solar time corresponding to 10:00 a.m. on February 8 for a location India at 87.5° east longitude. The standard meridian for the local time zone is 82o .5'. Solution

LCT = Slandered time - (Lst - Lloc) x 4 = 10.00 - (82.5 - 87.5) x 4

= 10.00 + 20' = 10:20 A.M.

From table (2.1), Eq. of time = - (14 min. 14 sec.) LST

= LCT+E = 10:20 - (14m 14s) = 10h 5m 46s

Example 3,2

Determine the LST and declination at Bhopal (latitude 23° 15' N, longitude 77° 30' E) at 12.30 on June 19. The standard meridian for the local time zone is **82° .30'**

Solution

LCT = standard time - (Lst - L1oc) x 4 = 12h 30m - (82° 30' - 77° 30') x 4

= 12h 30m - (5) x 4 = 12h 30m – 20m = 12:10

Local solar time is given by

LST = LCT + Eq. of time

From Table (3.1), equation of time E can be interpolated.

For June 19, E = (1':01")

Hence

LST = 12h 10m - (1m 1s) = 12h 8m 59s

Declination δ can be found by using the equation,

$$Ð = 23.45 \sin\left[360 \times \frac{284 + n}{365}\right] = 23.45 \sin\left[360 \times \frac{284 + 170}{365}\right]$$

$$= \mathbf{23.45 \sin[446] = 23.45 \sin 86}$$

$$= \mathbf{22.34 \times 0.9979 = 23.45^{o}}$$

Example 3.3: Find the LST for 8:00 a.m. MST on July 21 in Phoenix, AZ, which is located at 112° W longitude and a northern latitude of 33.43°.

Solution: Since Phoenix does not observe daylight savings time, it is unnecessary to make any change to the local clock time. Using Table 3.1, July 21 is the 202nd day of the year. From Equation (3.5.3), the equation of time is

$$D = 360^{\circ}\frac{n-81}{365} = 360^{\circ}\frac{202-81}{365} = 119.3^{\circ}$$

E= 9.87 sin (2D)-7.53 cos (D)-1.5 sin (D)

E= 9.87 sin (2*119.3 °)- 7.53 cos(119.3 °)-1.5 sin (119.3 °)

E= -6.05 min

LCT = standard time - (L_{st} - L_{loc}) x 4 = 8.0- (112- 33.34)

local longitude of standard time meridian= $15^{\circ} \times \left(\frac{112}{15^{\circ}}\right)_{round\ to\ integer}$ $=15^{\circ} * 7 = 105^{\circ}$

Using Equation (3.5.1), the apparent solar time (LCT)

LCT = standard time - (Lst - L1oc) x 4+ E

$=8.0-(105^{\circ} - 112^{\circ})*4(mins) + (-6.05) = 7.26$ a.m

Do You Have Any Questions?