

3 - *Rotation*

$$W = e^{i\alpha} * z \quad \text{where } \alpha = \text{rotation angle}$$

EX / what is image of rectangular Region of the Z – plane bounded by the lines $X = 0$, $y = 0$, $x = 1$, $y = 2$ under the

trans formation $W = e^{i\frac{\pi}{4}} * z$

sol /

$$W = e^{i\frac{\pi}{4}} * z$$

$$U + vi = e^{i\frac{\pi}{4}} (x + yi)$$

$$U + vi = \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) (x + yi)$$

$$U + vi = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right) (x + yi)$$

$$\sqrt{2} (u + vi) = x + yi + xi - y$$

$$\sqrt{2} * u + i\sqrt{2} v = (x - y) + i(x + y)$$

$$\sqrt{2} u = x - y \quad 1$$

$$\sqrt{2} v = x + y \quad 2$$

$$1 + 2$$

$$\sqrt{2} u + \sqrt{2} v = 2x$$

$$U + v = \sqrt{2} x$$

1 - 2

$$\sqrt{2} u - \sqrt{2} v = 2y \qquad \sqrt{2} (u - v) = 2y$$

$$U - v = \sqrt{2} y \qquad 4$$

When

$$X = 0 \quad u + v = 0 \quad u = -v$$

$$Y = 0 \quad u - v = 0 \quad u = v$$

$$X = 1 \quad u + v = \sqrt{2}$$

$$y = 2 \quad u - v = 2\sqrt{2}$$