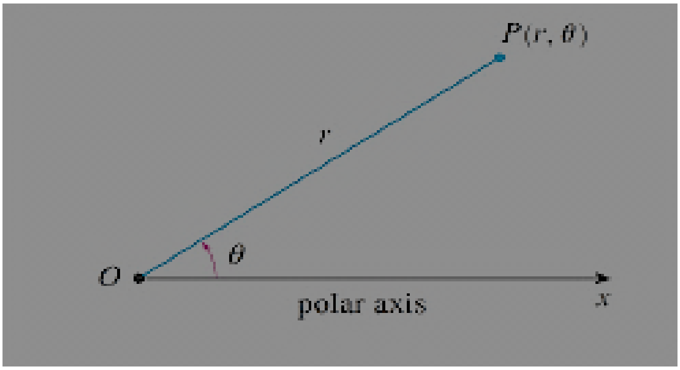
3.9 Polar Coordinates

In polar coordinates r is directed distance from origin (0) to point on to curve(p).

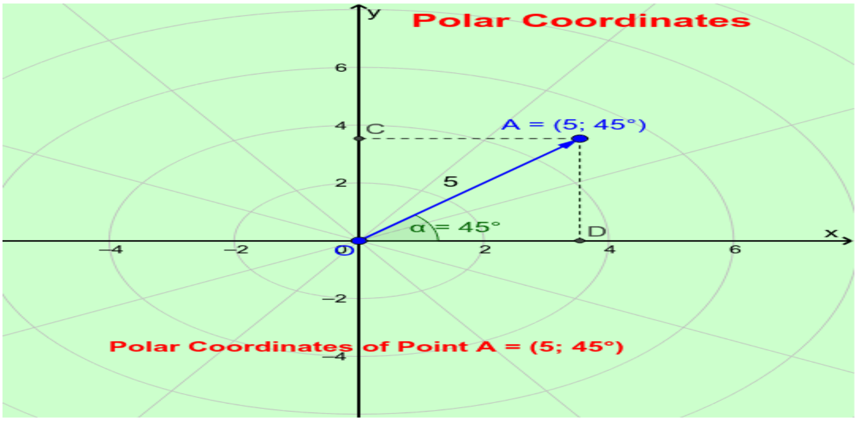
P(r,θ)$and\theta$ is $represented$ $directed$ $angle$ $from$

Initial $\theta = 0$)to $line$ $(0p)$

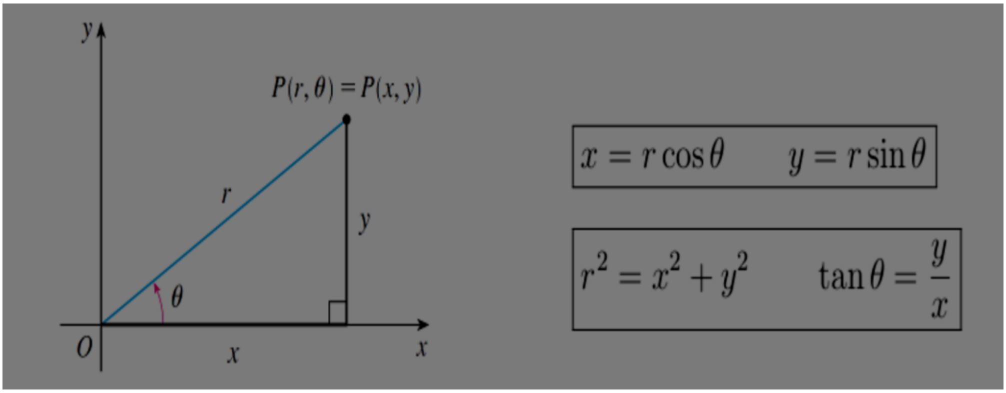


Example :Draw the following point(5,45°)

Solution//



1.Connection between polar &cartesion coordinates



Example: give the equation xy=-5 in polar coordinates.

Solution //

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

$$r\cos\theta . r\sin\theta = -5 \rightarrow r^2\cos\theta\ \sin\theta = -5$$

$$r^2\frac{\sin 2\theta}{2} = -5 \qquad [\sin 2\theta = 2\sin\theta\cos\theta]$$

Example: Find Cartesian coordinates for the curve

$$r\left[\cos(\theta - \frac{\pi}{3}\right] = 6$$

Solution//

$$r(\cos\ \theta\cos\frac{\pi}{3} + \sin\theta\sin\frac{\pi}{3}) = 6$$

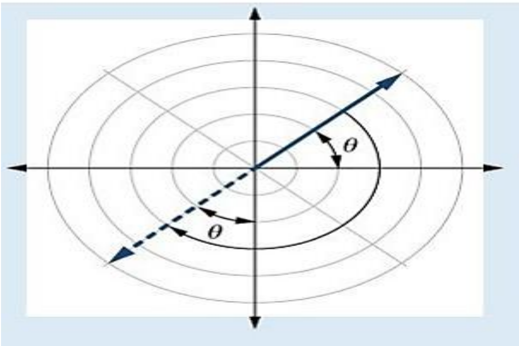
$$x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = 6$$

$$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 6 \rightarrow x + \sqrt{3}y = 12$$

2.Graphing in polar

The graph of equation f(r,$\boldsymbol{\theta}$) $= \mathbf{0}$ *consist of all points whose polar coordinates satify the equation .we look for symmetry and* max*. values of radius angle .there are three types of symmetry which are*:

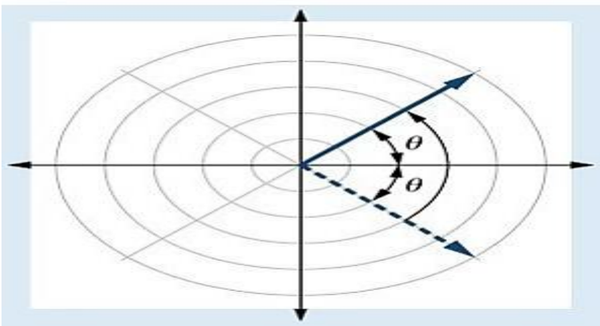
1. Symmetry about origin



a- r→ $-r$

b- $\theta \to \pi + \theta$

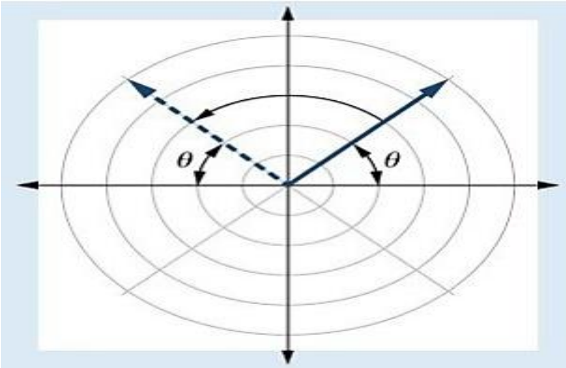
2. Symmetry about x-axis



a-$\theta \to -\theta$

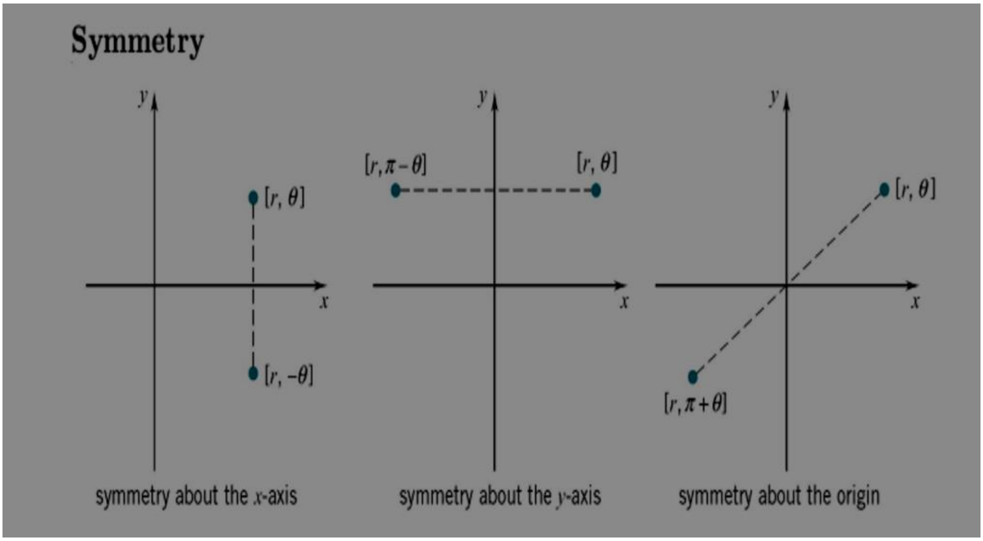
b-$\begin{cases} r \to -r \\ \theta \to \pi - \theta \end{cases}$

3. Symmetry about y-axis



a-$\theta \to \pi - \theta$

b-$\begin{cases} r \to -r \\ \theta \to -\theta \end{cases}$



Example: Graph the curve r=a (1-**cos**θ)

Solution //

1. Symmetry about origin

a- r$\rightarrow -r.\ \ -r = a(1 - cos\ \theta)\ not\ symmetry$

b-$\theta \rightarrow \pi + \theta.\ \ \ r = a(1 - \cos(\pi +\ \theta)$

$r = a(1 + \cos\theta\ )not\ symmetry$

2. Symmetry about x-axis

a-$\theta \rightarrow -\theta . r = a(1 - cos(-\ \theta) \rightarrow r = a(1 - cos\ \theta)\ symmetry$

3. Symmetry about y-axis

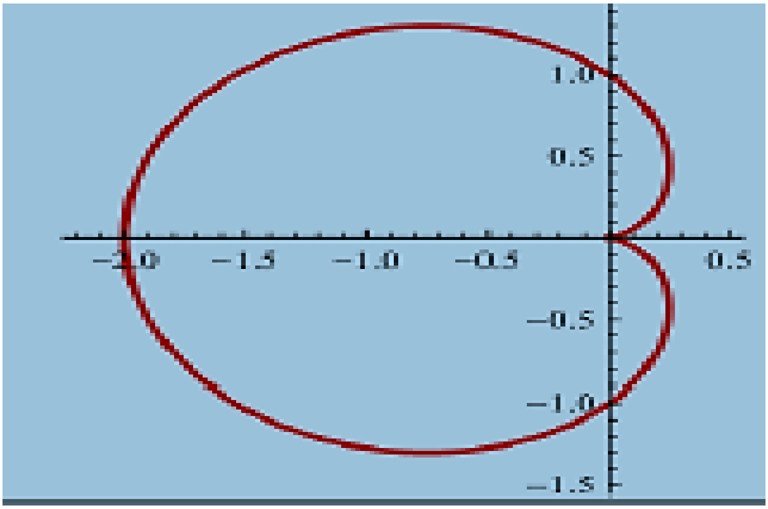
a-$\theta \rightarrow \pi - \theta . \rightarrow r = a(1 - cos(\pi -\ \ \theta)$

$\rightarrow r = a(1 + cos\ \theta)\ \ not\ symmetry$

$b - \begin{cases} r \rightarrow -r \\ \theta \rightarrow -\theta \end{cases} \rightarrow\ \ -r = a(1 - cos\ -\theta)\ \ not\ symmetry$

[r=a (1-cosθ)]

θ	**r**
0	**0**
$\boldsymbol{\frac{\pi}{4}}$	**0.3a**
$\boldsymbol{\frac{\pi}{2}}$	**a**
π	**2a**



Example: Graph the curve r^2=$4a^2$ cos θ

Solution //

1. Symmetry about origin

a- r→ $-r.\ \ r^2 = 4a^2\ cos\ \theta\ \ symmetry$

2. Symmetry about x-axis

a-$\theta \rightarrow -\theta . r^2 = 4a^2\ cos - \theta\ \ \rightarrow r^2 = 4a^2\ cos\theta\ \ symmetry$

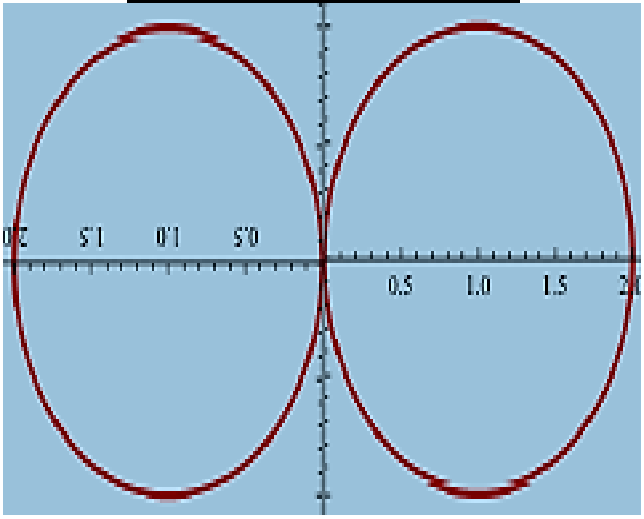
3. Symmetry about y-axis

a-$\theta \rightarrow \pi - \theta . \rightarrow r^2 = 4a^2 \cos(\pi - \theta)$

$\rightarrow r^2 = 4a^2\ cos(\theta)\ \ symmetry$

$[r^2{=}4a^2 \cos\theta]$

θ	r
$\mathbf{0}$	$\pm \mathbf{2}\boldsymbol{a}$
$\boldsymbol{\frac{\pi}{4}}$	$\pm \mathbf{1.6}\boldsymbol{a}$
$\boldsymbol{\frac{\pi}{3}}$	$\pm \sqrt{\mathbf{2}}\boldsymbol{a}$
$\boldsymbol{\frac{\pi}{2}}$	$\mathbf{0}$



Example : Graph the curve r=5

Solution //

1. Symmetry about origin

a- r→ $-r.\ -r = 5\ \ not\ \ symmetry$

b-$\theta \rightarrow \pi + \theta. \quad r = 5\ symmetry$

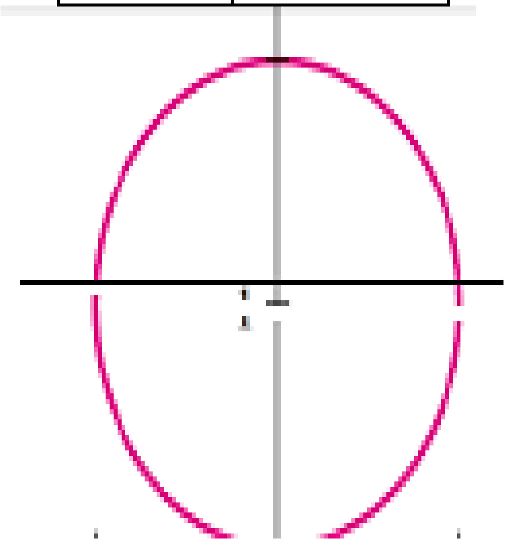
2. Symmetry about x-axis

a-$\theta \rightarrow -\theta. r = 5\ \ symmetry$

3. Symmetry about y-axis

a-$\theta \rightarrow \pi - \theta. \rightarrow r = 5\ symmetry$

θ	**r**
0	**5**
$\boldsymbol{\frac{\pi}{4}}$	**5**
$\boldsymbol{\frac{\pi}{2}}$	**5**
π	**5**



Example : Graph the curve r=2(1+sin θ)

Solution //

1. Symmetry about origin

a- r$\rightarrow -r. -r = 2(1 + sin\ \theta)\ not\ symmetry$

b-$\theta \rightarrow \pi + \theta.\quad r = 2(1 + \sin(\pi + \ \theta)$

$$r = 2(1 - \sin\theta\ ) not\ symmetry$$

2. Symmetry about x-axis

$a_{\theta} \rightarrow -\theta. r = 2(1 + \sin\ (-\theta))$

$\rightarrow r = 2(1 - sin\ \theta)\ not\ symmetry$

b-$\begin{cases} r \rightarrow -r \\ \theta \rightarrow \pi - \theta \end{cases}$

$-r = 2(1 + sin(\pi - \theta) \rightarrow -r = 2(1 + sin\theta)\ not\ symmetry$

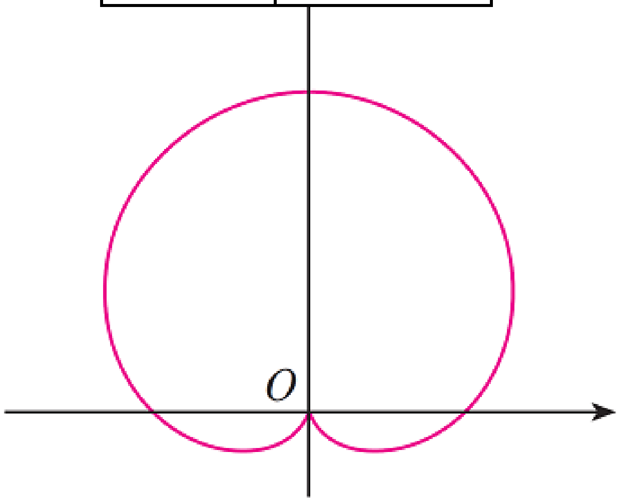
3. Symmetry about y-axis

a-$\theta \rightarrow \pi - \theta. \rightarrow r = 2(1 + \sin\ (\pi - \theta)$

$\rightarrow r = 2(1 + \sin\theta)\ symmetry$

$$[r = 2(1 + \sin\theta)]$$

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	**2**
$\frac{\pi}{4}$	**3.4**
$\frac{\pi}{2}$	**4**
$\frac{-\pi}{4}$	**0.6**
$\frac{-\pi}{2}$	**0**



Exercise: Example: Graph the curve r=2(1+cos θ)