Ex - A cylindrical steel pressure vessel 400mm in diameter with a wall thickness of 20mm, is sbjected to an internal pressure of 4.5 MPa Calculate, ⓐ The tangential and longitudinal stresses in the steel, ⓑ To what value may the internal pressure be increased if the stress in the steel is limited to 120 MPa ⓒ if the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

a * Tangential stress (hoop stress) σ_t (σ_θ)

$$\sigma_t = \frac{PD}{2t} = \frac{4.5\,MPa\,(400mm)}{2\,(20)\,mm} = 45\,MPa$$

* longitudinal stress (σ_L)

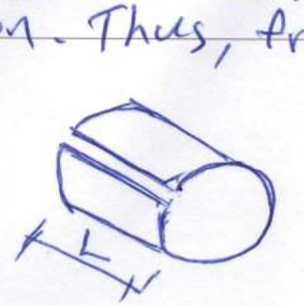
$$\sigma_L = \frac{PD}{4t} = \frac{4.5\,MPa\,(400mm)}{4\,(20\,mm)} = 22.5\,MPa$$

b $\sigma_t = 2\sigma_L$

this shows that tangential stress is critical

$$\sigma_t = \frac{PD}{2t} \Rightarrow 120\,MPa = \frac{P\,(400)}{2(20)} \Rightarrow P = 12\,MPa$$

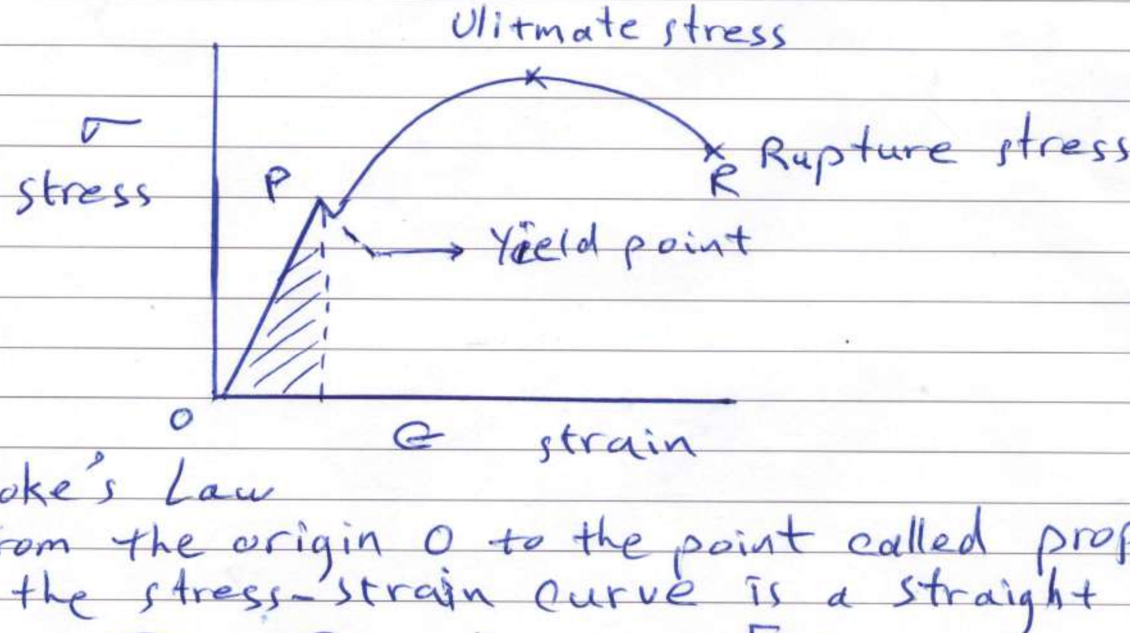
c The bursting force will cause a stress on the longitudinal section. Thus, fracture is expected as shown



Stress - Strain Diagram :-

Suppose that a metal specimen be placed in Tensile test machine. As the axial load is gradually increased in increements, the elongation over the gage length is measured at each increment of the load and this is continued until failure. Knowing the original cross-sectional area and length of specimen, the normal stress (σ) and strain (ϵ) can be obtained. The graph of these quantites with the stres σ along y-axis and strain ϵ along x-axis is called stress-strain diagram.

The stress-strain diagram differs in form for various materials.



Hooke's Law

From the origin O to the point called proportional Limit, the stress-strain curve is a straight line.

$$\sigma \propto \epsilon \Rightarrow \sigma = E\epsilon$$

E - constant = Modulus of Elasticity or Youngs Modulus is equal to the slope of stress - strain diagram from O to P

Elastic limit - it is the maximum stress that may be developed such ~~that~~ that there is No residual stress (deformation) when the load is removed.

Elastic and Plastic Ranges :-

The region in stress - strain O to P is called elastic range. The region from P to R is called plastic range

Ultimate strength :-
The maximum ordinate in the stress - strain diagram is th Ultimate strength or tensile strength.

Rupure strength :-
is the strength material at rupure, This also known as the breaking strength.

Modulus of Resilience :-
is the work done on a unit volume of material as the force is gradually increased from O to R, in $N \cdot m/m^3$. This may calculated as the ~~area~~ area under stress - strain curve from the

origin O to up the elastic limit E, The resilience of the material is its ability to absorb energy with out creating a permanent distortion.

Modulus of Toughness:-
is work done on unit volume of materail as the force is graudllally increased from O to R (Nm/m^3) this may calculated as the area under the entire stress-stain curve from (O to R) The Toughness of a material is ability to absorb energy with out causing it to break.

STiffness (k):-
is the ratio of the steady force acting on an elastic body to the resulting displacment. it has unit (N/mm)

$$F = k\delta \Rightarrow k \frac{F}{\delta}$$

Working Stress, Allowable stress And Factor of Safety :-

Working stress is defined as actual stress of material under a given loading.

Allowable stress, maximum safe stress that a material can carry. The allowable stress should be limited to values not exceeding the proportional limit. However, since proportional limit is difficult be determine accurately, the allowable stress is taken as either the yield point or ultimate strength divided by a factor of safety.

In the linear portion of the stress-strain diagram,

$$\sigma = k\,\epsilon \qquad K = \text{constant}$$

$$\Rightarrow \sigma = E\,\epsilon \qquad E = \text{elastic modulus}$$

$$\sigma = \frac{P}{A} \;,\; \epsilon = \frac{\delta}{L}$$

$$\Rightarrow \frac{P}{A} = E\,\frac{\delta}{L} \Rightarrow \boxed{\delta = \frac{PL}{AE}}$$

δ = elongation mm
P = force N
L = Length mm
A = area mm^2

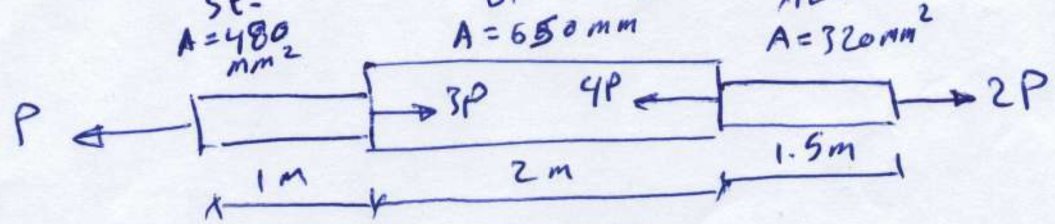
For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weights is :-

$$\delta = \frac{MgL}{2AE}$$

$g = 9.81 \ m/s^2$

M = total mass

Ex-1- Axial loads are applied at the positions indicated. Find the largest value of (P) that will not exceed on overall deformation of 3mm, or the following stresses, 140MPa in a steel, 120MPa in bronze, 80MPa in aluminum, Assume that assembly is suitably braced to prevent buckling. Use, $E_s = 200\ GPa$, $E_{al} = 70\ GPa$, $E_b = 83\ MPa$.



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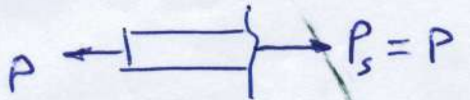
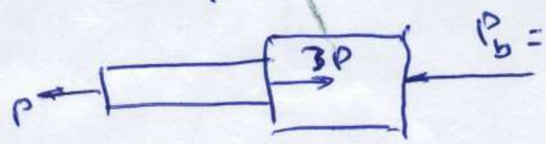


steel

$$\sigma_s = \frac{P_s}{A_s} \Rightarrow P_s = \sigma_s A_s = P$$

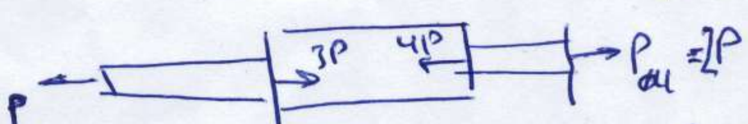
$$P = 140(480) = \underline{67.2\,kN}$$



bronze

$$P_b = \sigma_b A_b = 120(650) = 2P$$

$$\Rightarrow P = \underline{39\,kN}$$



$$P_{al} = \sigma_{al} \cdot A_{al} = 2P$$

$$2P = 80(320)$$

$$P = 12.8\,kN$$

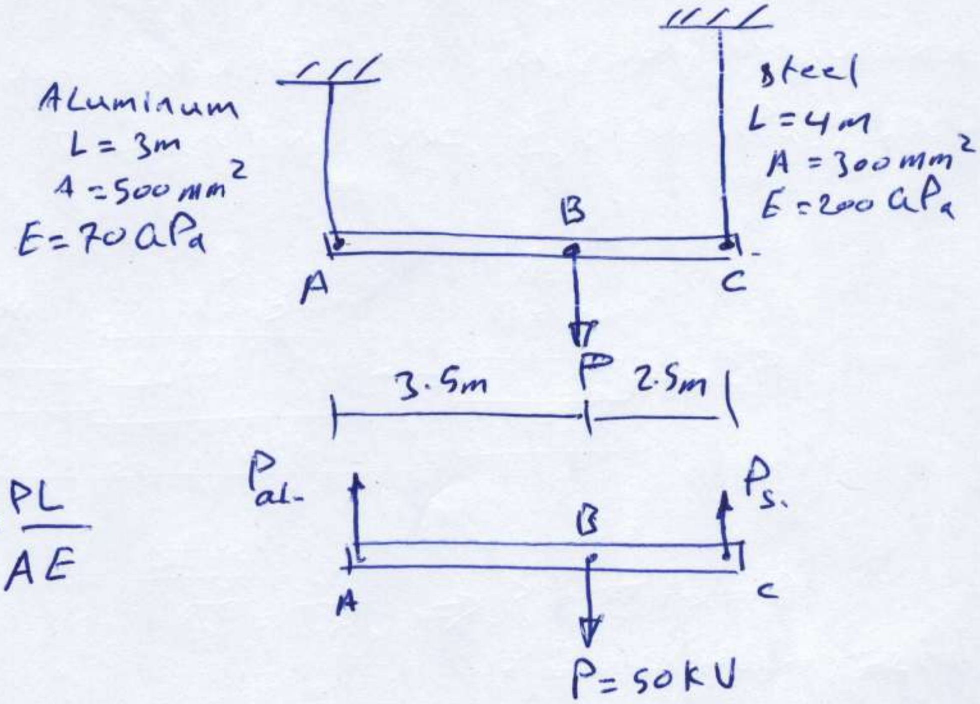
Based on allowable deformation, form $\Rightarrow$ $\delta = \frac{PL}{AE}$

$$\delta = \delta_s + \delta_b + \delta_{al}$$

$$3 = \frac{P(1000)}{480(200\times10^3)} - \frac{2P(2000)}{650(70\times10^3)} + \frac{2P(1500)}{320(83\times10^3)}$$

$$3 = \left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560}\right)P$$

Ex-2- The rigid bar AB attached to two vertical rods as shown, is horizontal before the load P is applied. Determine the vertical movement of P, if its magnitude is 50 kN.



Sol.

For aluminum

$\Sigma M_C = 0$ $\qquad \delta = \frac{PL}{AE}$

$P_{al} = \sigma_{al} \cdot A_{al}$

$6 P_{al} = 2.5 (50)$

$P_{aL} = 20.83$ kN

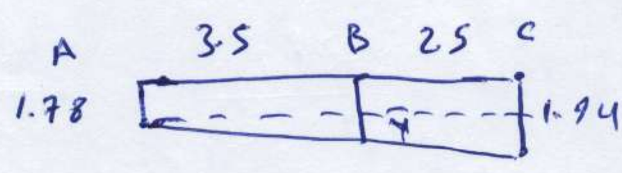
$\delta_a = \frac{20.83 \times 10^3 (3) 10^3}{500 (70 \times 10^3)} = 1.78$ mm

For steel

$\Sigma M_A = 0$

$6 P_s = 3.5 (50) \Rightarrow P_s = 29.17$ kN

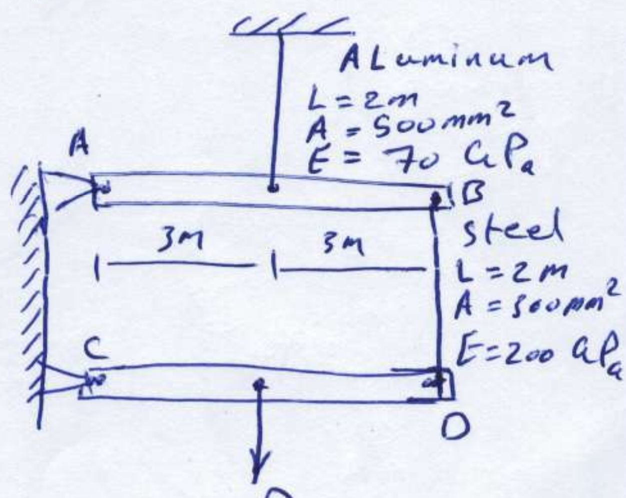
$\delta_s = \frac{29.17 \times 10^3 + 4 \times 10^3}{300 (200 \times 10^3)} = 1.94$ mm



momument diagram

$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$

Ex-3- The rigid Bars AB and CD shown are supported by pins at A and C and two rods. Determine the maximum Force P that can be applied as shown, if vertical movement is Limited to 5 mm. Neglect the weights of all members.



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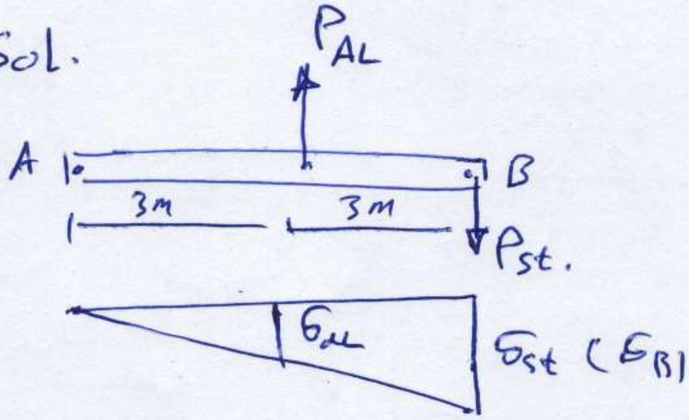


$\Sigma M_A = 0 \quad 3P_{aL} = 6P_{st.}$

$P_{al} = 2P_{st.}$

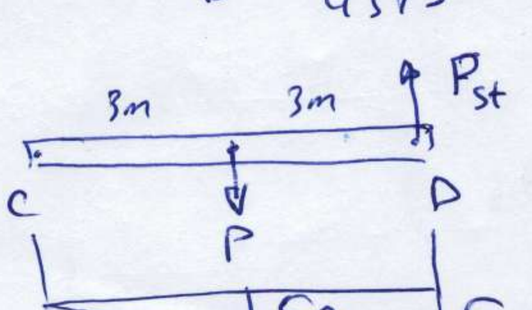
By ratio and proportion of (δ)

$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3} \qquad \delta = \frac{PL}{AE}$$

$$\delta_B = 2\delta_{aL} = 2\frac{(P_{aL}(2000))}{500(70\times10^3)} = \frac{1}{8750}P_{aL}$$

$$= \frac{1}{8750}(2P_{st.})$$

$$\Rightarrow \delta_B = \frac{1}{4375}P_{st} \quad (\text{movement of } B)$$



movement of D:-

$$\delta_D = \delta_{st.} + \delta_B$$

$$\delta_D = \frac{P_{st.}(2000)}{300(200\times10^3)} + \frac{1}{4375}P_{st.}$$

$\Sigma M = 0$

$\delta_D = \quad " \quad P$

$$\delta_P = \frac{1}{2}\delta_D = \frac{1}{2}\left(\frac{11}{42000}P_{st}\right)$$

$$\delta_P = \frac{11}{84000}P_{st}$$

$$5 = \frac{11}{84000}P_{st} \Rightarrow P_{st} = 38.181 \text{ kN}$$

$$P_{st} = \frac{1}{2}P \quad \Rightarrow P = 2(38.181) = 76.4 \text{ kN}$$