Thermal stresses (stresses due to change in Temperature) :-

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. But if the deformation is prevented, some stresses are induced in the body. Such stresses are known as thermal stresses.

let L = Original length of the body

ΔT = Change in temperature (rise or fall)

α = Coefficient of thermal expansion

$\therefore$ Increase or decrease in length $\delta_L = L \cdot \alpha \cdot \Delta T$

if the ends of the body are fixed to rigid supports, so that expansion is prevented, then compressive strain induced in the body

$$\epsilon_c = \frac{\delta_L}{L} = \frac{L \alpha \Delta T}{L} = \alpha \Delta T$$

$\therefore$ Thermal stress $\sigma_{th} = \epsilon_c \cdot E = E \alpha \Delta T$

Ex-1- A steel rod is stretched between two rigid body and carries a tensile load of 5kN at 20°C. if the allowable stress is not exceed 130MPa at -20°C, what is the minimum diameter of the rod, if $\alpha = 11.7 \ \mu m/m\,°C$ and $E = 200GPa$

$$\sigma = \alpha E \Delta T + \frac{P}{A}$$

$$130 \ MPa = (11.7 \times 10^{-6})(200 \times 10^{3} \ MPa)(20-(-20)) + \frac{5 \times 10^{3}}{A}$$

$$A = \frac{5000}{36.4} = 137.36 \ mm^2$$

$$A = \frac{\pi d^2}{4}$$

$$137.36 = \frac{\pi (d)^2}{4}$$

$$\Rightarrow d = 13.22 \ mm$$

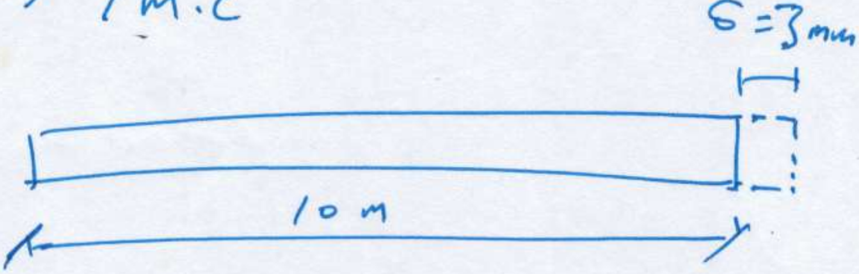
Not:-

1. When a body is composed of two or different coefficient of thermal expansions, then due to the rise in temperature, the material with higher coefficient of thermal expansion will be subjected to compressive stress whereas the with low coefficient of expansion will be Subjected to tensile stress

Ex - 2

Steel railroad reels 10m long are laid with a clearance of 3mm at a temperature of 15 °C. ① At what temperature will the rails just touch? ② What stress would be induced in the rails at temperature if there were no intial clearance? Assume $E = 200\ GPa$

$\alpha = 11.7\ \mu m / m \cdot {}^\circ C$



①

Temperature at which $\delta_T = 3mm$

$$\delta_T = \alpha\, L\, \Delta T$$

$$\delta_T = \alpha\, L\, (T_f - T_i)$$

$$3\ mm = (11.7 \times 10^{-6})(10 \times 10^{3})(T_f - 15)$$

$$T_f = 40.64\ {}^\circ C$$

②

The stress in the rail

$$\sigma_{th} = \alpha\, E\, (T_f - T_i)$$

$$= (11.7 \times 10^{-6})(200 \times 10^{3})(40.64 - 15)$$

$$= 60\ MP$$

Ex - 3 - A copper bar 50mm in diameter is placed with in a steel tube 75mm external diameter and 50mm internal diameter of exactly the same length. The Two pieces are rigidly fixed togther by two pins 18mm in diameter, one at each end passing through the bar and tube. Calculate the stress induced in the copper bar, steel tube and pins if the temperature of the combination is raised by $50^{\circ}C$, Take $E_s = 210\ GPa$, $E_c = 105\ GPa$
$\alpha_s = 11.5 \times 10^{-6} / {^{\circ}C}$ and $\alpha_c = 17 \times 10^{-6} / {^{\circ}C}$.

Sol:-

$$A_c = \frac{\pi d_c^2}{4} = \frac{\pi}{4}(50)^2 = 1964\ mm^2$$

$$A_s = \frac{\pi}{4}\left[d_{o_s}^2 - d_{i_s}^2\right] = \frac{\pi}{4}\left[75^2 - 50^2\right] = 2455\ mm^2$$

let L = length of the copper bar = length of steel tube

Free exp. for copper $\Rightarrow$ $\delta_{L_c} = \alpha_c \cdot L \cdot \Delta T = 17 \times 10^{-6} \times L \times 50 = \underline{850 \times 10^{-6} L}$

Free exp. for steel $\Rightarrow$ $\delta_{L_s} = \alpha_s \cdot L \cdot \Delta T = 11.5 \times 10^{-6} \times L \times 50 = \underline{575 \times 10^{-6} L}$

Difference in free expansion $= 850 \times 10^{-6} L - 575 \times 10^{-6} L$

$$= \underline{275 \times 10^{-6} L}$$

Since the free exp. of the copper ...

the copper bar is subjected to a compressive stress, while the steel tube is subjected to a tensile stress.

let a compressive force (P) N on the copper bar opposes the extra expansion of the copper bar and an equal tensile force (P) on the steel tube pulls the steel tube

∴ Reduction in length of coppar bar due to force (P)

$$= \frac{P.L}{A_c E_c} = \frac{P.L}{1964 \times 10^{-6} \times 105 \times 10^{9}} = \frac{P.L}{206.22 \times 10^{6}} \text{ m}$$

and increase in length of steel tube due to force (P)

$$= \frac{P.L}{A_s E_s} = \frac{P.L}{2455 \times 10^{-6} \times 210 \times 10^{9}} = \frac{P.L}{515.55 \times 10^{6}} \text{ m}$$

∴ Net effect in length $= \frac{P.L}{206.22 \times 10^{6}} + \frac{P.L}{515.55 \times 10^{6}}$

$$= 4.85 \times 10^{-9} P.L + 1.94 \times 10^{-9} PL$$

$$= 6.79 \times 10^{-9} PL \text{ m}$$

Equating this net in length to the difference in free expansion, we have

$$6.79 \times 10^{-9} P.\cancel{L} = 275 \times 10^{6} \cancel{L}$$

⇒ stress induced in the copper bar, steel tube and pins,

$$\sigma_c = \frac{P}{A_c} = \frac{40500}{1964} = 20.62 \text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{40500}{2455} = 16.5 \text{ MPa}$$

$$\tau_p = \frac{P}{2A_p} = \frac{40500}{2\frac{\pi}{4}(18)^2} = 79.57 \text{ MPa}$$