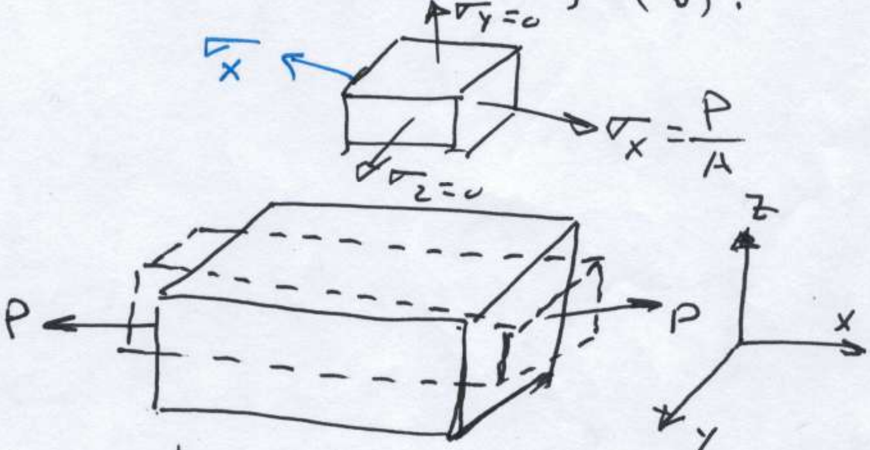
Poisson's Ratio

If a bar is subjected to a tensile loading there will be an increase in length of the bar in the direction of applied load, but there is also a decrease in a lateral dimension perpendicular to the load. It has been observed that for an elastic materials.

The ratio of lateral strain to longitudinal strain is known as Poisson's ratio and is denoted by (ν).

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$



bar under axial tensile force

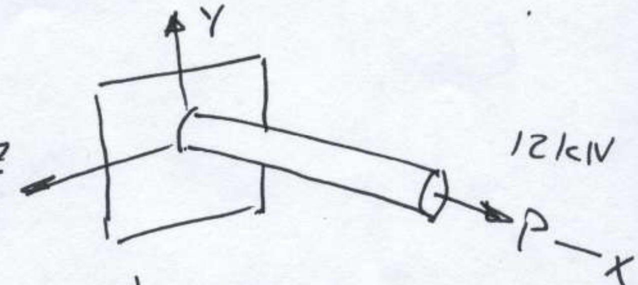
where ϵ_x is strain in the x-direction and ϵ_y and ϵ_z are strains in the perpendicular direction, The (- sign) indicates a decrease in the transverse dimension when ϵ_x is positive.

For most engineering materials the value (ν) is between (0.15 – 0.33). For most steel, it is in the range of (0.25 – 0.3)

$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -\nu\,\frac{\sigma_x}{E}$$

Ex-1- A 500mm long, 16 mm diameter rod made of homogenous, isotropic material is observed to increase in length by 300μm and to decrease in diameter by 2.4μm when subjected to an axial load (12kN). Determine the modulus of elasticity and Poisson's ratio of the material.

Sol.



$$A = \frac{\pi d^2}{4} = \frac{\pi (16)^2}{4} = 201 \ mm^2$$

$$\sigma_x = \frac{P}{A} = \frac{12 \times 10^3 \ N}{201 \ mm^2} = 59.7 \ MPa$$

$L = 500 \ mm$

$\delta_x = 300 \ \mu m$

$d = 16 \ mm$

$\delta_y = -2.4 \ \mu m$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{300 \times 10^{-9} \ mm}{500 \ mm} = 600 \times 10^{-6}$$

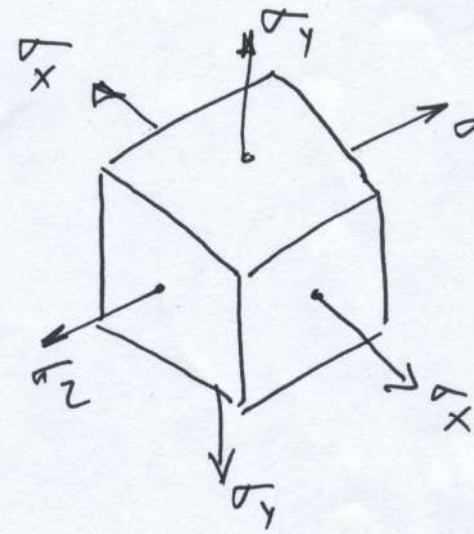
$$\epsilon_y = \frac{\delta_y}{d} = \frac{-2.4 \times 10^{-9} \ mm}{16 \ mm} = -150 \times 10^{-6}$$

$$\sigma_x = E \, \epsilon_x \Rightarrow E = \frac{\sigma_x}{\epsilon_x} = \frac{59.7 \ MPa}{600 \times 10^{-6}} = 99.5 \ GPa$$

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{-150 \times 10^{-6}}{600 \times 10^{-6}} = 0.25$$

Multiaxial Loading; Generalized Hooke's Law

Let us now consider structural elements subjected to loads acting in the directions of three coordinate axes and producing normal stresses σ_x, σ_y and σ_z which are all different from zero.



$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right)$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$

stress state for multiaxial Loading

if $\sigma_z = 0$ then, Biaxial deformation :-

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$

Relationship between, E, G and ν :-

$$G = \frac{E}{2(1+\nu)}$$

where E, modulus of elasticity

G, shear modulus

ν, Poisson's ratio

Bulk Modulus of Elasticity (K) :-

The Bulk modulus of elasticity (k) is a measure of a resistance of material to change in Volume without chang in shape or form;

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

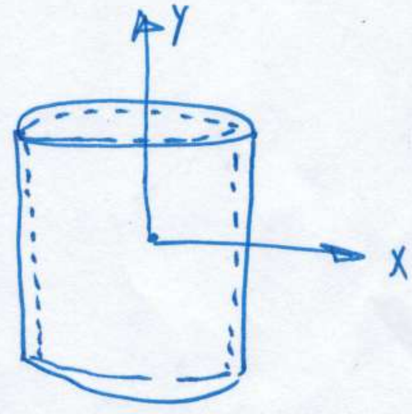
where V is the Volume and ΔV is change in volume.

$$\frac{\Delta V}{V} = \text{Volumetric strain}$$

$$= \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

Ex - A steel cylindrical drum made of a plate has internal diameter 1.2 m and 10 mm thickness, find the change in diameter that would caused by an internal pressure of 1.5 MPa, take Poisson's ratio is 0.3 and E = 200 GPa.

Sol.



σ_Y = longitudinal stress = σ_L

$$\sigma_Y = \frac{PD}{4t} = \frac{1.5\,(1200)}{4\,(10)}$$
$$= 45\,MP_a$$

σ_X = tangential stress = σ_θ

$$= \frac{PD}{2t} = \frac{(1.5)(1200)}{2(10)} = 90\,MP_a$$

$$\epsilon_X = \frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E}$$

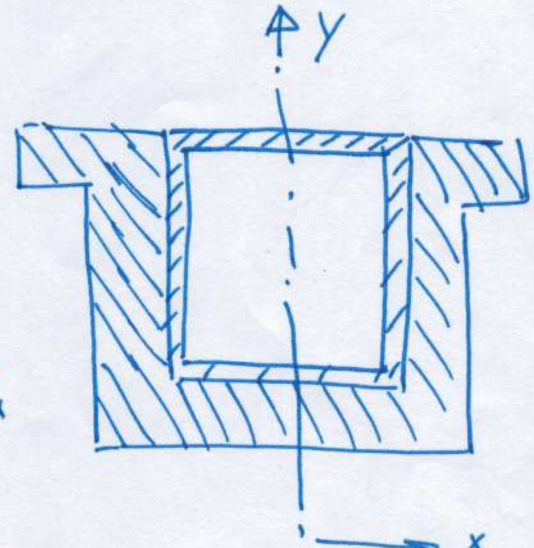
$$= \frac{90}{200\times10^3} - 0.3\,\frac{45}{200\times10^3} = 3.82\times10^{-4}$$

$$\epsilon_X = \frac{\delta_d}{d}$$

$$\Rightarrow \delta_d = d\,\epsilon_X = 1200\times 3.82\times10^{-4} = 0.458\ mm$$

Ex - A 150 mm long a closed cylindrical, is 80 mm in diameter and has a wall thickness of 3 mm. It fit without clearance in an 80 mm hole in a rigid block. It is subjected to an internal pressure of 4 MPa. take $\nu = \frac{1}{3}$ and $E = 83\,GPa$, find the tangential stress in the cylinder.

Sol.



$\sigma_y = \sigma_L$ = longitudinal stress

$$\sigma_y = \frac{PD}{4t} = \frac{4(80)}{4(3)} = \frac{80}{3}\,MPa$$

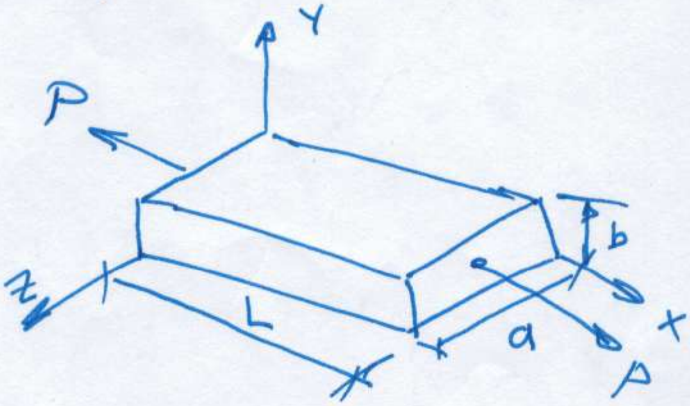
$\sigma_x = \sigma_\theta$ = tangential stress

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0 \qquad \text{(strain in x-direction)}$$

$$\Rightarrow \sigma_x = \nu\,\sigma_y$$

$$\sigma_x = \frac{1}{3}\left(\frac{80}{3}\right) = \frac{80}{9} = 8.89\,MPa$$

Ex – A steel rectangular block, $a = 2.4$ cm and $b = 1.2$ cm is subjected to an axial tensile load as shown in Fig. Measurement show block to increase in length by $\delta_x = 7.11 \times 10^{-5}$ m (intial length $L = 10$ cm) and to decrease in width by $\delta_z = 0.533 \times 10^{-5}$ m, when $P = 54$ kN. find the modulus of elasticity and Poisson's ratio for the material.



Sol.

$$A_x = 2.4 \times 1.2 = 2.88 \text{ cm}^2$$

$$\sigma_x = \frac{P_x}{A_x} = \frac{45 \times 10^3}{2.88 \times 10^{-4}}$$

$$= 196.3 \text{ MPa}$$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{7.11 \times 10^{-5}}{10^{-1}} = 7.11 \times 10^{-4}$$

$$\epsilon_z = \frac{\delta_z}{a} = -\frac{0.533 \times 10^{-5}}{2.4 \times 10^{-2}} = -2.22 \times 10^{-4}$$

$$\epsilon_y = \epsilon_z$$

$$\Rightarrow \epsilon_y = \frac{\delta_y}{b}$$

$$\Rightarrow \delta_y = (-2.22 \times 10^{-4})(1.2 \times 10^{-2}) = 2.664 \times 10^{-4} \text{ m}$$

$$\sigma_x = E\,\epsilon_x$$

$$\Rightarrow E = \frac{\sigma_x}{\epsilon_x} = \frac{156.3 \times 10^6}{\quad} = 219.8 \times 10^9 \text{ Pa} = 219.8 \text{ GPa}$$