Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: three

Signal Processing

2023-2024

Lecture (4): Discrete convolution/ Graphical method

Discrete convolution

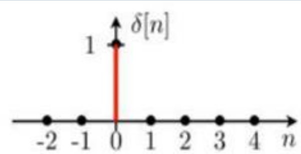
convolution is a mathematical operation that combines two functions to produce a third function that represents the amount of overlap between them. in other words, it is a way of measuring how much one signal "matches" another signal, at every point in time or space.

Convolution is a fundamental concept in many areas of science and engineering, such as signal processing, image processing, and physics. It is used to analyze and manipulate signals and images, filter out noise, extract features, and much more.

Two important definitions

1. **Delta Function** :- Symbolized by the Greek letter delta. The delta function is $\delta[n]$ a normalized impulse , that is, sample number zero has a value of one , but all other samples have a value of zero . for this reason, the delta function is frequently called the **unit impulse**

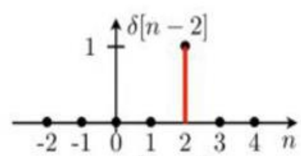
$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$



n	$\cdots$	-2	-1	0	1	2	3	4	5	6	$\cdots$
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0

Examples delta pulse:

$$\delta[n-2] = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{for } n \neq 2 \end{cases}$$



$$\delta[n+1] = \begin{cases} 1 & \text{for } n = -1 \\ 0 & \text{for } n \neq -1 \end{cases}$$

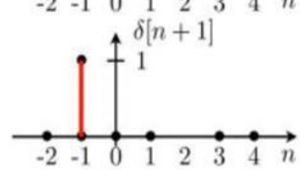


Figure (1): delta pulse

2. **Impulse Response :-** Is the signal that exits a system when a **delta function** is the input. If two systems are different in any way, they will have different impulse. The impulse response is usually given the symbol **h[n].**

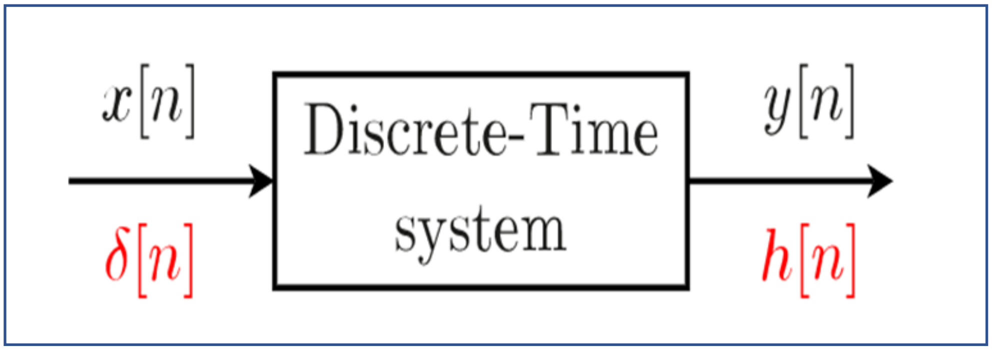
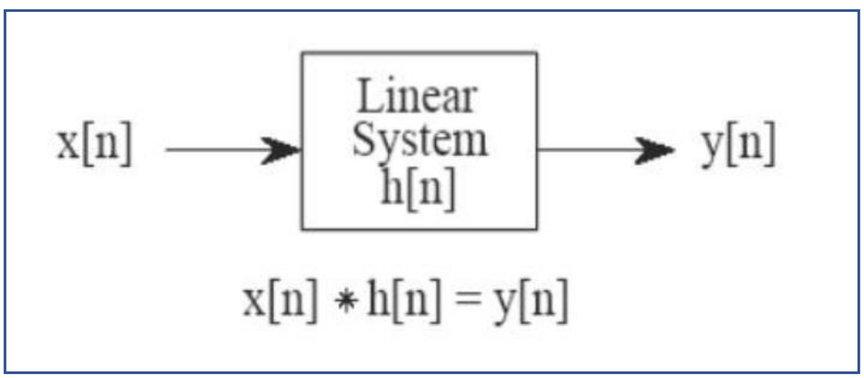


Figure (2): Impulse response h[n] of a discrete time signal

Definition of convolution

To define convolution more formally, consider an input signal, x[n], enters a linear system with an impulse response, h[n], resulting in an output signal, y[n]. In equation form:

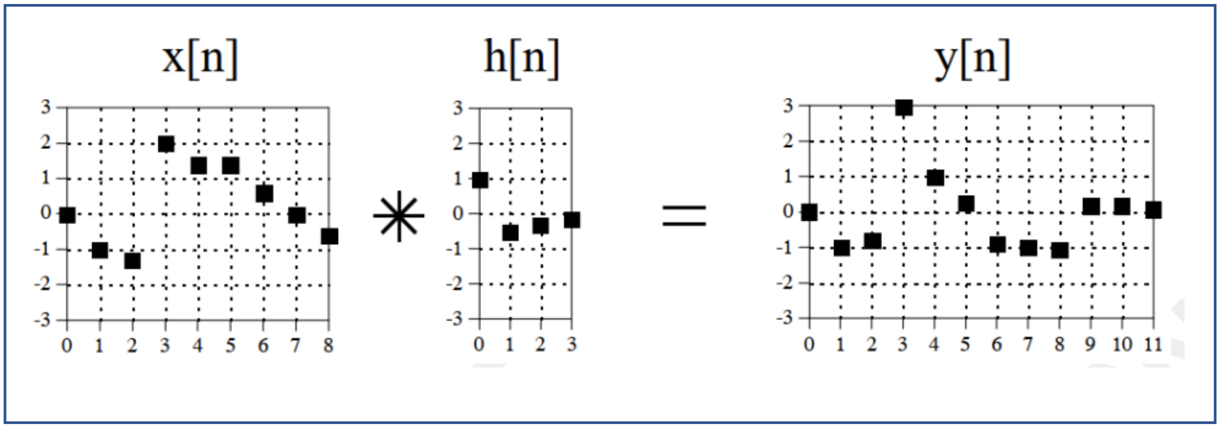
$$\mathbf{y(n)= x(n) * h(n)}=\sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$



The input signal convolved with the **impulse response** is equal to the **output signal**. Just as addition is represented by the **pulse,+,** and **multiplication** by the **cross,** ×, **convolution** is represented by the **star, *.**

Total number for samples=N_1+N_2-1

Example: Nine point input signal, x(n), is passed through a system with a four point impulse response , h(n), resulting in a 9+4-1=12 point output signal ,y(n).



The linear convolution can be performed by **direct**, **graphical, table lookup**, and **matrix by vector** methods.

Graphical Method

The convolution sum of two sequences can be found by using the following steps:

Step 1. Obtain the reversed sequence h(-k).

Step 2. Shift h(-k) by n samples to get h(n-k). If $n \geq 0$, h(-k) will be shifted to the right by n samples; but if $n < 0$, h(-k) will be shifted to the left by n samples.

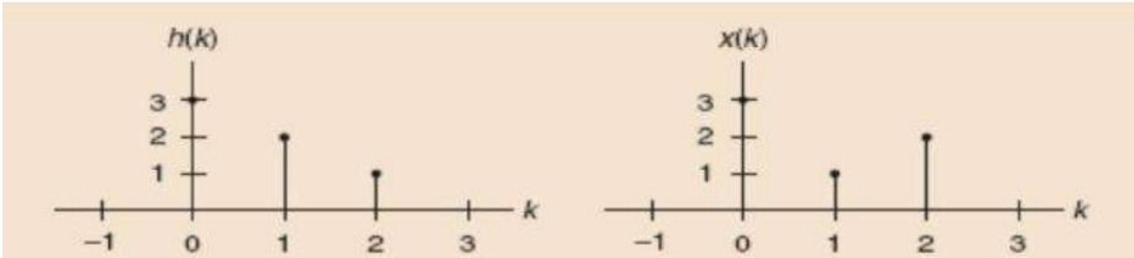
Step 3. Perform the convolution sum that is the sum of the products of two sequences x(k) and h(n-k) to get y(n).

Step 4. Repeat steps 1 to 3 for the next convolution value y(n).

Example:- Find the convolution of the two sequences x[n] and h[n] given by x[n] = {3, 1, 2} and h[n] = {3, 2, 1}.

the solution:-

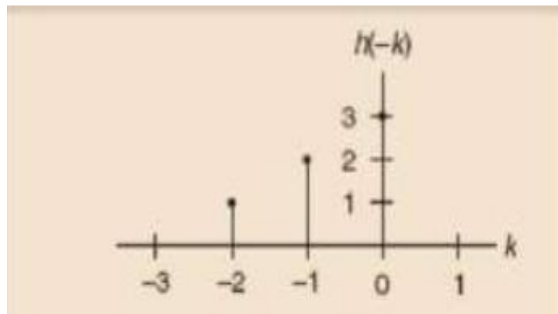
draw the signals



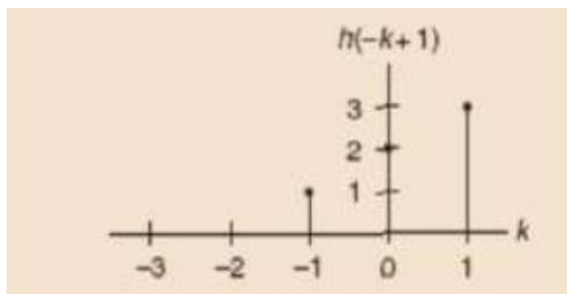
reverse h(k) and obtain

h(-k+0)

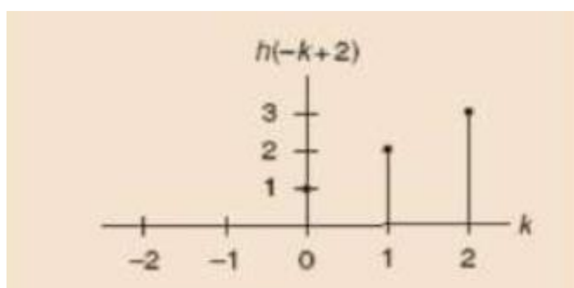
y(0)=3*3+1*0+2*0=9



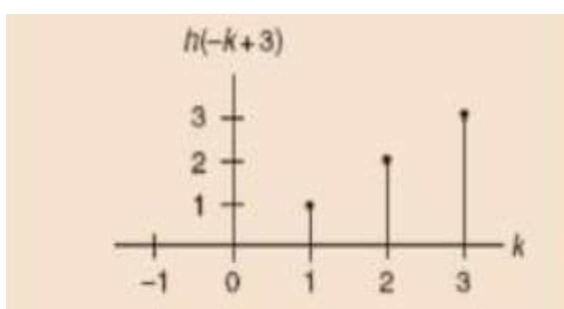
y(1)=2*3+3*1=9



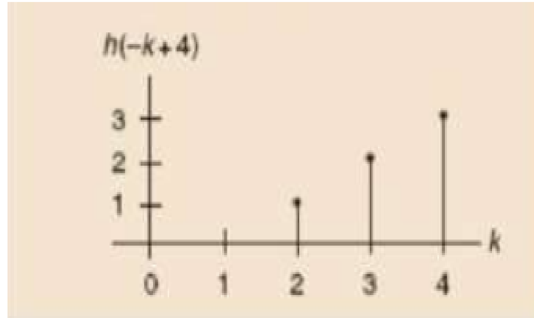
y(2)=1*3+2*1+3*2=11



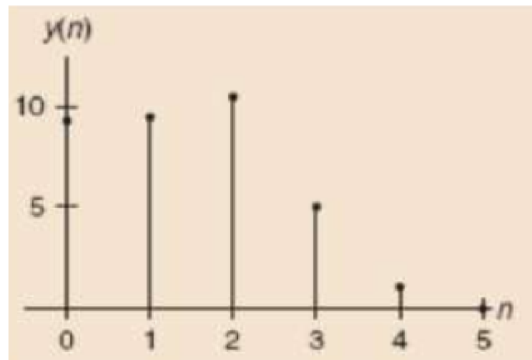
y(3)=0*3+1*1+2*2+3*0=5



y(4)=0*3+0*1+1*2+2*0+3*0=2



the output signal will be



Example / Find the convolution of the two sequences x[n] and h[n] given by;

$$x[n] = \{1, \underset{\uparrow}{2}, 2, 1, 1\}$$

$$h[n] = \{\underset{\uparrow}{3}, 2, 1\}$$