المحاضرة السادسة

* Condection - convection systems (fins) الزعانف

الهدف من الزعانف هو زيادة نقل الحرارة

fins extended surface that are designed to increase heat transfer

cas

<u>cas 1</u>: the fin is very long, and the temp at the end of the fin
is essentially that of the surrounding fluid.

$$q = \sqrt{hpkA}\,\theta_0$$

$\theta_0 = T_0 - T_\infty$

T_0: درجة حرارة (fin) او base Temp

T_∞ = fluid temp

A = fin مساحة

P = المحيط

$$\theta = T - T_\infty \quad , \quad \theta_0 = T_0 - T_\infty \qquad \frac{\theta}{\theta_0} = e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA}}$$

2-71 A long, thin copper rod 5 mm in diameter is exposed to an environment at 20 ◦ C.The base temperature of the rod is 120 ◦ C. The heat-transfer coefficient between the rod and the environment is 20 W/m2 · ◦ C. Calculate the heat given up by the rod.

P.2.71) copper rod. , $d = 5\,mm$, $h = 20\,W/m^2.°C$

$T_\infty = 20°C$ $\quad K_{copper} = 372$

$T_0 = 120°C$

Find q:-

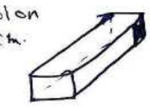
$$q = \sqrt{hPKA}\,\theta_o \qquad \left\{ \begin{array}{l} P = 2\pi r = \pi d = \pi \times 0.005 = 0.0157\ m \\ A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.005)^2 = 1.96349 \times 10^{-5}\ m^2 \end{array} \right.$$

$$q = \sqrt{20 \times 0.0157 \times 372 \times 1.96 \times 10^{-5}} \times (120 - 20)$$

$$\boxed{q = 4.7848\ W}$$

2-85 A long stainless-steel rod [k =16 W/m• ∘ C] has a square cross section 12.5 by 12.5 mm and has one end maintained at 250 ∘ C. The heat-transfer coefficient is 40 W/m2 C, and the environment temperature is 90 ∘ C. Calculate the heat lost by the rod.

stainless -steel rod. ($K = 16$) Square cross section 12.5 cm × 12.5 cm.



$T_o = 250^\circ C$
$T_\infty = 90^\circ C$
$h = 40$

find q.

بالكتاب 12.5mm
انا اعتبرتها 12.5 cm

$$q = \sqrt{hPKA}\,\theta_o \qquad \left\{ \begin{array}{l} P = 12.5 \times 4 = 50cm = 0.5\ m \\ A = (0.125)^2 = 0.015625\ m^2 \end{array} \right.$$

$$q = \sqrt{40 \times 0.5 \times 16 \times 0.015625} \times (250 - 90)$$

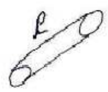
$$\boxed{q = 357.77\ W}$$

Case 2) The fin is of finite length and loses heat by convection from it's end.

$$q = \sqrt{hPKA}\,\theta_o \frac{\sinh(mL) + \left(\frac{h}{mK}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{h}{mK}\right)\sinh(mL)} \qquad m = \sqrt{\frac{hP}{KA}}$$

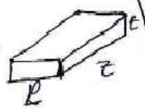
$$\frac{\theta}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh[m(L-x)] + (h/mK)\sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mK}\sinh(mL)}$$

$L \rightarrow$ fin الطول $\qquad x \rightarrow$ البعد المطلوب إيجاد T عنده



$A = \frac{\pi}{4} d^2$
$P = \pi d$ ← rod ← نحصل على case 2 ← إذا كانت له طول معين

أو نحوله إلى case3



$\boxed{\begin{aligned} P &= 2z + 2t \\ A &= zt \end{aligned}}$ ← rectangular

If $z \gg t$

$\frac{P}{A} = \frac{2\cancel{z} + 2t}{\cancel{z}t} = \boxed{\frac{2}{t}}$

2-68 An aluminum rod 2.0 cm in diameter and 12 cm long protrudes from a wall that is maintained at 250 ∘ C. The rod is exposed to an environment at 15 ∘ C. The convection heat-transfer coefficient is 12 W/m2 · ∘ C. Calculate the heat lost by the rod.

Aluminum rod $\quad d = 2\,cm \quad T_{wall} = 250\,^\circ C$
$L = 12\,cm \quad T_\infty = 15\,^\circ C$
$h = 12\ w/m^2\text{-}c$
$k = 204\ w/m\text{-}c$

find $q = ?$

$$q = \sqrt{hpkA}\,\theta_o \frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$$

rod.

$$p = \pi d = \pi * 0.02 = 0.0628\ m$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.02)^2 = 3.1415 * 10^{-4}\ m^2$$

$$m = \sqrt{\frac{hp}{KA}} = \sqrt{\frac{12 * 0.0628}{204 * 3.1415 * 10^{-4}}} = 3.429$$

$$mL = 3.429 * 0.12 = 0.41149$$

$$-\frac{h}{mk} = 0.01715$$

$$\theta_o = 250 - 15$$

$$q = \sqrt{12 * 0.628 * 204 * 3.1415 * 10^{-4}}\,(250 - 15) * \frac{\sinh(0.41149) + 0.01715\cosh(0.41149)}{\cosh(0.41149) + 0.01715\sinh(0.41149)}$$

$$q = 20.8735\ W$$

general $L_c = L + \frac{A}{P}$ للكروس سكشن

$L_c \to$ corrected length.

rectangular

$L_c = L + \frac{A}{P}$

$A = Zt$

$P = 2Z + 2t$

ثم على $\underline{case3}$

. case3 إلى case 2 تحول

rod. (circular fin)

$L_c = L + \frac{d}{4}$

$\frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d}$

$\boxed{\frac{A}{P} = \frac{d}{4}}$

mL_c

ثم على $\underline{case3}$

قانون

رسم

* $\underline{case\ 3}$ The end of the fin is insulated so that $\frac{dT}{dx} = 0$ at $x = L$

$$q = \sqrt{hPKA}\ \theta_o \tanh(mL_c)$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\theta}{\theta_o} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}$$

$\boxed{\text{at the end of fin } x = L_c}$

هنا يطلب درجة الحرارة عند

طول معين هو x إذا طلب عند

آخر الـ fin تكون $\underline{L_c = x}$

$m = \sqrt{\frac{hP}{KA}}$

L_c ... rod / rectangular

التحول

P.2-68 Aluminum rod, $d = 2\,cm$, $L = 12\,cm$, $T_w = 250°C$, $T_\infty = 15°C$, $h = 12$, $K = 204$

sol:-

$L_c = L + \frac{d}{4} = 12 + \frac{2}{4} = 12.5\,cm$

$A = \frac{\pi}{4}d^2 = 3.1415 \times 10^{-4}\,m^2$

$P = 0.0628\,m$

$m = \sqrt{\frac{hP}{KA}} = 3.429$

$mL_c = 3.429 \times 0.125 = \boxed{0.428625}$

$q = \sqrt{hPKA}\ \theta_o \tanh(mL_c) = \sqrt{12 * 0.0628 * 204 * 3.1415 \times 10^{-4}}\,(250 - 15) * \tanh(0.428625)$

$$\boxed{q = 20.87313\,W}$$

Case 3

Rectangular.

L → length
t → thickness.

$L_c = L + \frac{t}{2}$ — ① نجد L_c

$\boxed{A_m = t L_c}$

② نجد $L_c^{3/2}\left(\frac{h}{k A_m}\right)^{1/2}$

③ من طريقة الرسمة ① Fig 2-11 نجد η_f

④ نعوض في المعادلة لنجد q

$\boxed{q = \eta_f h A \theta_0}$

$\boxed{A = 2 L_c W}$

2-74 A straight fin of rectangular profile has a thermal conductivity of 14 W/m · ∘C, thickness of 2.0 mm, and length of 23 mm. The base of the fin is maintained at a temperature of 220 ∘C while the fin is exposed to a convection environment at 23 ∘C with h = 25 W/m2 · ∘C. Calculate the heat lost per meter of fin depth.

$k = 14$ W/m.c , $t = 2$ mm $L = 23$ mm

$T_0 = 220°C$ $T_\infty = 23°C$ $h = 25$ find q ?

Solve

$$L_c = L + \frac{t}{2} = 23 + \frac{2}{2} = 24 \text{ mm} = \boxed{0.024 \text{ m}}$$

$$L_c^{\frac{3}{2}}\left(\frac{h}{kA_m}\right)^{\frac{1}{2}} = (0.024)^{\frac{3}{2}}\left(\frac{25}{14 * 0.002 * 0.024}\right)^{\frac{1}{2}} = 0.717$$

from fig 2-11 $\eta_f = 0.75$

$$q = \eta_f h A \theta_0 = \eta_f h \, 2 L_c W \theta_0$$

$$\frac{q}{W} = 0.75 * 25 * 2 * 0.024 * (220 - 23)$$

$$= 177.3 \text{ W/m}$$

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Case 3 triangular.

$L_c = L$ ①

$\boxed{A_m = \frac{t}{2} L_c}$ $L_c^{3/2}\left(\frac{h}{KA_m}\right)^{\frac{1}{2}}$ ②

③ عن طريق الرسمة 11 Fig 2-11 نجد η_f

$q = \eta_f h A \theta_o$ ④

$A = 2w\sqrt{L^2 + \frac{t^2}{4}}$

2-77 A triangular fin of stainless steel (18% Cr, 8% Ni) is attached to a plane wall maintained at 460 ◦ C. The fin thickness is 6.4 mm, and the length is 2.5 cm. The environment is at 93 ◦ C, and the convection heat-transfer coefficient is 28 W/m2 · ◦ C. Calculate the heat lost from the fin.

P.2-77) Stainless steel (18% Cr, 8% Ni) ⇒ $K = 16.3$, $T_o = 460°C$

الحل $t = 6.4$ mm, $L = 2.5$ cm, $T_\infty = 93°C$, $h = 28$, find q:

Sol: Assume $\boxed{w = 1m}$

$L_c = L = 0.025$ m

$A_m = \frac{t}{2} L_c = \frac{0.0064}{2} * 0.025 = 8*10^{-5}\ m^2$

$L_c^{3/2}\left(\frac{h}{KA_m}\right)^{\frac{1}{2}} = \boxed{0.579}$

from fig 2-11 → $\eta_f = 0.85$

$q = \eta_f h A \theta_o$

$q = \eta_f h * \left(2w\sqrt{L^2 + \frac{t^2}{4}}\right)(T_o - T_\infty) \Rightarrow \boxed{q = 440.29\ W}$

* لاحظ أن المساحة مضروبة في 2 في القانون النهائي لأن انتقال الحرارة يكون على سطحي ال fin

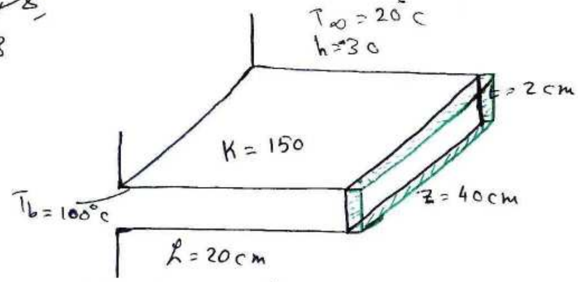
Example :- Determain the heat transfer rate from the rectangular fin The tip of the fin is not insulated, and the fin has thermal conductivity of 150 w/m.c, The Base temp is 100°c and the the fluid is @ 20°c, the heat transfer coefficient between the fin and air is 30 w/m².c°

يمكن حله مباشرة على case 2 أو تحويله إلى case3 .

هنا سنحوله ونحله على case3 .

Sol\y:

①



corrected length

$$L_c = L + \frac{A}{P} = L + \frac{zt}{2z+2t}$$

$$L_c = 0.2 + \frac{0.4 * 0.02}{2 * 0.4 + 2 * 0.02} = 0.2095$$

$$P = 2z + 2t = 0.84$$

$$A = zt = 0.008$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{30 * 0.84}{150 * 0.008}} = \boxed{4.58}$$

$$mL_c = 4.58 * 0.2095 = \boxed{0.96}$$

$$q = \sqrt{hPkA}\ \theta_0 \tanh(mL_c)$$

$$\Rightarrow q = \sqrt{30 * 0.84 * 150 * 0.008} * (100 - 20) \tan(0.96)$$

$$= 327.43w$$

② Determain the temp of the end of the fin. $\boxed{x = L_c}$

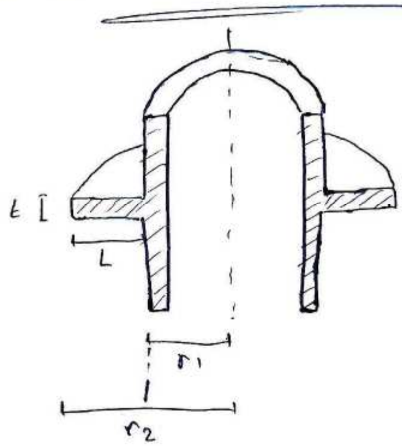
$$\frac{\theta}{\theta_o} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(0)}{\cosh(0.96)} \Rightarrow \frac{T - 20}{100 - 20} = \frac{1}{1.49729}$$

$$\boxed{T_{\boxed{x = L_c}} = 73.43^\circ C}$$

* ملاحظة: بإمكانك حل السؤال على case2

case 3 circumferential fin of rectangular profile.



$r_1 = r_2 - L$ ①

$L_c = L + \frac{t}{2}$ ②

$r_{2c} = r_1 + L_c$ ③

$A_m = t L_c$ ④

r_{2c}/r_1 ⑤

$L_c^{3/2}\left(\frac{h}{K A_m}\right)^{1/2}$ ⑥

$q = \eta_f h A \theta_o$ — نجد η_f ونعوض في ② من Fig 2-12 ⑦

$\boxed{A = 2\pi(r_{2c}^2 - r_1^2)}$

2-75 A circumferential fin of rectangular profile is constructed of a material having $k = 55 W/m \cdot {}^\circ C$ and is installed on a tube having a diameter of 3 cm. The length of fin is 3 cm and the thickness is 2 mm. If the fin is exposed to a convection environment at 20 ∘ C with a convection coefficient of 68 W/m2 ⋅ ∘C and the tube wall temperature is 100 ∘ C, calculate the heat lost by the fin.

$K = 55$, $d = 3 cm$, $r_1 = 1.5 cm$, $L = 3 cm$, $t = 2 mm$
$T_\infty = 20^\circ C$, $h = 68$, $T_w = 100^\circ C$, find q.

$$- L_c = L + \frac{t}{2} = 0.03 + \frac{0.002}{2} = 0.031\ m$$

$$- r_{2c} = r_1 + L_c = 0.015 + 0.031 = \boxed{0.046\ m}$$

$$- A_m = t L_c = 0.002 * 0.031 = \boxed{6.2 * 10^{-5}\ m^2}$$

$$- r_{2c}/r_1 = \boxed{3.0667}$$

$$- L_c^{3/2}\left(\frac{h}{K A_m}\right)^{\frac{1}{2}} = (0.031)^{3/2}\left(\frac{68}{55 * 6.2 * 10^{-5}}\right)^{\frac{1}{2}} = \boxed{0.77}$$

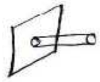
$\eta_f \rightarrow$ from fig 2-12 $\Rightarrow \boxed{\eta_f = 0.6}$
2 25

$$q = \eta_f\ h A\ \theta_o = \eta_f h\ (2\pi(r_{2c}^2 - r_1^2))\ (T_o - T_\infty)$$

$$\boxed{q = 38.78\ W}$$

2-89 An aluminum block is cast with an array of pin fins protruding like that shown in Figure 2-10*d* and subjected to room air at 20 ∘ C. The convection coefficient between the pins and the surrounding air may be assumed to be *h* = 13.2 W/m2 ・ ∘ C. The pin diameters are 2 mm and their length is 25 mm. The base of the aluminum block may be assumed constant at 70 ∘ C. Calculate the total heat lost by an array of 15 by 15, that is, 225 fins

Pin fins
rod



, $T_\infty = 20°C$, $h = 13.2$, $d = 2mm$, $L = 25mm$, $T_o = 70°C$
find q, q for 225 fins

$\boxed{K = 204 \text{ Aluminum}}$

$$q = \sqrt{hPKA}\ \theta_o\ \tanh(m L_c)$$

Sol:

$$L_c = L + \frac{d}{4} = 0.025 + 0.002/4 = 0.0255$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h \pi d}{K \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{Kd}} = \sqrt{\frac{4 * 13.2}{204 * 0.002}} = \boxed{11.38}$$

$$q_{(one\ pin)} = \sqrt{13.2 * \pi * 0.002 * 204 * \frac{\pi}{4}(0.002)^2}\ \overset{(70-20)}{}\ \tanh(11.38 * 0.0255)$$

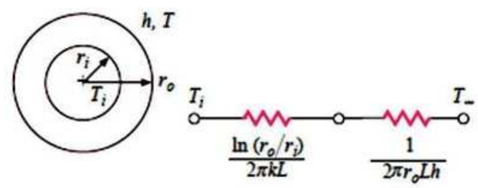
$\boxed{q = 0.102911\ W}$ for one pin.

$$q_{(225\ pin)} = 225 * 0.102911 = \boxed{23.15\ W}$$

CRITICAL THICKNESS OF INSULATION

* لدينا اسطوانة (circular pipe) بنصف قطر (r_i) ودرجة حرارة (T_i) ، تم إضافة مادة عازلة على الإسطوانة بنصف قطر (r_o) تم أيضاً تعريض السطح الخارجي إلى مائع (fluid) عند درجة حرارة T_∞.

Figure 2-7 | Critical insulation thickness.



$$q = \frac{\Delta T}{\sum R_{th}} = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{hA_o}}$$

$$\boxed{A_o = 2\pi r_o L}$$

$$q = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{2\pi r_o Lh}} \Rightarrow q = \frac{2\pi L\,(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}}$$

* r_o هو نصف القطر الذي يحصل عنده أقصى إنتقال للحرارة .

لإيجاد r_o نشتق المعادلة بالنسبة لـ r_o وتكون المشتقة تساوي صفر عند أقصى إنتقال حرارة .

$$\frac{dq}{dr_o} = 0 \quad @ \text{ maximum Heat Transfer.}$$

$$\frac{dq}{dr_o} = \frac{\cancelto{0.0}{\left(\frac{\ln(r_o/r_i)}{k} + \frac{1}{h\,r_o}\right)}(0) - 2\pi L\,(T_i - T_\infty)\left[\frac{\frac{1}{r_i}/\frac{r_o}{r_i}}{k} + \left[\frac{-h}{r_o^2 h^2}\right]\right]}{\left[\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}\right]} = 0$$

$$-2\pi L\,(T_i - T_\infty)\left[\frac{1}{r_o k} - \frac{1}{r_o^2 h}\right] = 0$$

$$\therefore \frac{1}{r_o k} - \frac{1}{r_o^2 h} = 0 \Rightarrow r_o k = r_o^2 h$$

$$\boxed{r_o = \frac{k}{h}}$$

The Equation expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be *increased* by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of h the convection heat loss may actually increase with the addition of insulation because of increased surface area.

EXAMPLE 2-6

Calculate the critical radius of insulation for asbestos [$k = 0.17$ W/m · °C] surrounding a pipe and exposed to room air at 20°C with $h = 3.0$ W/m^2 · °C. Calculate the heat loss from a 200°C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution:
$K_{(asbestos)} = 0.17$, $T_\infty = 20°C$, $h = 3\ W/m^2.C°$, $T_i = 200°C$
$D = 5\ cm$, $r_i = 2.5\ cm.$

Find ① $\frac{q}{L}$ with insulation , ② $\frac{q}{L}$ without insulation

① $r_o = \frac{K}{h} \Rightarrow r_o = \frac{0.17}{3} \Rightarrow \boxed{r_o = 5.67\ cm = 0.0567\ m}$

$$q = \frac{2\pi L (T_i - T_\infty)}{\frac{Ln(r_o/r_i)}{K} + \frac{1}{r_o h}} \Rightarrow \frac{q}{L} = \frac{2\pi (T_i - T_\infty)}{\frac{Ln(r_o/r_i)}{K} + \frac{1}{r_o h}}$$

$$\frac{q}{L} = \frac{2\pi(200 - 20)}{\frac{Ln(5.67/2.5)}{0.17} + \frac{1}{0.0567 * 3}} \Rightarrow \boxed{\frac{q}{L} = 105.7\ W/m}$$

② هنا يكون حل مع المحيط الخارجي والـ pipe

$$q = h A (T_w - T_\infty) \Rightarrow q = h (2\pi r L)(T_w - T_\infty)$$

المساحة الجانبية للإسطوانة.

$$\therefore \frac{q}{L} = h (2\pi r)(T_w - T_\infty) \Rightarrow \frac{q}{L} = 3 * 2 * \pi * 0.025 * (200 - 20) \Rightarrow \boxed{\frac{q}{L} = 84.8\ W/m}$$

نتيجة
$$\begin{cases} r < r_{(critical)} \Rightarrow r\uparrow\ q\uparrow \\ r > r_{(critical)} \Rightarrow r\uparrow\ q\downarrow \end{cases}$$

CH. 4 Unsteady state conduction.

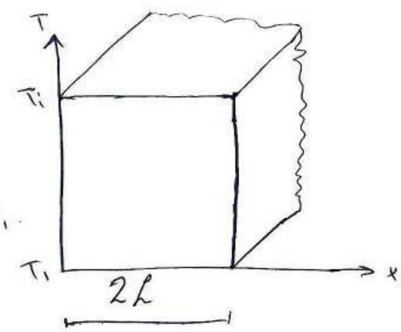
- Unsteady heat transfer.
- Transient heat "

* For plane wall.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q^\circ}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

In This case temp. is not only a function of x, it is also a function of time. $T(x, \tau)$

* Assumption:



1) Infinite plate with thickness 2L
2) T_i is the initial temp @ $\tau = 0.0$
3) sudden change while cooling to T_1.
4) constant thermal conductivity.
5) Unsteady. (درجة الحرارة تتغير مع الزمن)

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{0.0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0.0} + \cancel{\frac{q^\circ}{k}}^{0.0} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}} \quad ---(1) \qquad \boxed{\theta = T - T_1}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial \tau}$$

Boundary and initial condition.

$\theta = \theta_i = T_i - T_1 \quad @ \quad \tau = 0 \quad$ and $0 \le x < 2L$

$\theta = 0 \quad @ \quad \tau > 0 \quad x = 0 \quad ----(*)$

$\theta = 0 \quad @ \quad \tau > 0 \quad x = 2L \quad ----(**)$

$\theta(x, \tau)$

$\theta = X , H(\tau)$

$$\frac{\partial \theta}{\partial \tau} = x_{(x)} \frac{\partial H}{\partial \tau} \Rightarrow \frac{\partial \theta}{\partial x} = H(\tau) \frac{\partial x_{(n)}}{\partial x} \Rightarrow \frac{\partial^2 \theta}{\partial x^2} =$$

$$H(\tau) \frac{\partial^2 X_m}{\partial x^2}$$

$$H(\tau) \frac{\partial^2 X(m)}{\partial x^2} = \frac{1}{\alpha} X_{(x)} \frac{\partial H}{\partial \tau}$$

$$\frac{1}{x_{(x)}} \frac{\partial^2 X(x)}{\partial x^2} = \frac{1}{\alpha} \frac{1}{H(\tau)} \frac{\partial H}{\partial \tau} = -\lambda^2$$

λ^2: separation parameter

$$\frac{\partial^2 x_{(x)}}{\partial x^2} + \lambda^2 x_{(x)} = 0$$

$$\frac{\partial H}{\partial \tau} + \alpha \lambda^2 H(\tau) = 0$$

$$\theta = [c_1 \cos \lambda x + c_2 \sin \lambda x] e^{-\alpha \lambda^2 \tau}$$

using B-C $\Rightarrow 0 = (c_1 \cos 0 + c_2 \sin(0)) e^{-\alpha \lambda^2 \tau}$

$c_1 = 0$

$\theta = (c_2 \sin \lambda x) e^{-\alpha \lambda^2 \tau}$ ——(2)

using B-C $\Rightarrow 0 = (c_2 \sin 2L\lambda) e^{-\alpha \lambda^2 \tau}$

$\sin 2\lambda L = 0$ $\qquad 0, \lambda, 2\pi, 3\pi$

$2\lambda L = n\lambda$

$$\lambda = \frac{n\pi}{2L}$$

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$$\boxed{\lambda = \frac{n\pi}{2L}}$$

$$\theta = \sum_{n=1}^{\infty}\left(C_n \sin\frac{n\pi x}{2L}\right) e^{-\alpha\left(\frac{n\pi}{2L}\right)^2 \tau}$$

substibute in equ. ②.

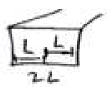
Infinite plate having thickness ... = 2L

$$\frac{T-T_1}{T_i-T_1} = \frac{\theta}{\theta_i} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} e^{-\alpha\left(\frac{n\pi}{2L}\right)^2\tau} \sin\left(\frac{n\pi x}{2L}\right) \qquad n = 1, 3, 5, 7, 9, \ldots$$

هنا الزوايا بالراديان

$$\alpha = \frac{k}{\rho c}$$

٤-٢ An infinite plate having a thickness of ٢,٥ cm is initially at a temperature of ١٥٠◦C, and the surface temperature is suddenly lowered to ٣٠◦C. The thermal diffusivity of the material is ١.٨×١٠−٦ m٢/s. Calculate the center-plate temperature after ١ min by summing the first four nonzero terms of Equation (٤-٣). Check the answer using the Heisler charts.



thickness = 2L = 2.5 cm , $T_i = 150°C$, $T_1 = 30°C$

thermal diffusivity $(\alpha) = 1.8 * 10^{-6}$ m²/s , find $T|_{\tau = 60s,\; x = L}$

— calculate the center plate ($x = L$) temp. after 1 min. using the first 4 nonzero terms of equation.

solv :- $n = 1, 3, 5, 7$

$$\frac{T-30}{150-30} = \frac{4}{\pi}\left[\frac{1}{1} e^{-1.8\times10^{-6}\times 60\left[\frac{\pi}{0.025}\right]^2} \sin\frac{1\pi L}{2L} + \frac{1}{3} e^{-1.8\times10^{-6}\times60\left[\frac{3\pi}{0.025}\right]^2} \sin\frac{3\pi L}{2L} + \frac{1}{5} e^{-1.8\times10^{-6}\times60\left[\frac{5\pi}{0.025}\right]^2} \sin\frac{5\pi}{2} + \frac{1}{7} e^{-1.8\times10^{-6}\times60\left[\frac{7\pi}{0.025}\right]^2} \sin\frac{7\pi}{2}\right]$$

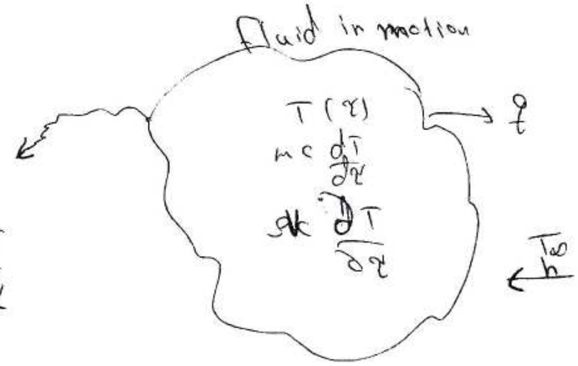
$$\frac{T-30}{120} = \frac{4}{\pi}\left[0.1818 - 7.1912 * 10^{-8} + 6.0834 * 10^{-20} - 7.27 * 10^{-38}\right]$$

$$\frac{T-30}{120} = 0.231 \Rightarrow \boxed{T = 57.777°C} \quad \#$$

* lumped - Heat-capacity system.

- The system is assumed to be uniform in Temp. This means that temp defference has been decayed. (فتذبذب متقطع)
- The internal thermal conductive resistance is smaller than the external convective resistance.

جسم غمر في مائع.



* Energy Balance.

$$q = hA(T - T_\infty) = -\rho c V \frac{dT}{d\tau}$$

$V \Rightarrow$ Volume
$A \Rightarrow$ surface area
$c \Rightarrow$ spesific heat transfer (KJ/Kg) - c
$T_\infty \Rightarrow$ Temp fluid

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\Theta}{\Theta_0} = e^{\frac{-hA\tau}{\rho c V}}$$

$A = m^2$, T_0 = Temp, intial
τ = scond T = center temp , T_∞ = fluid Temp (air, water)
$c = J/kg\text{-}c$ $V = m^3$ T_0 = initial temp

٤-٦ A piece of aluminum weighing ٦ kg and initially at a temperature of ٣٠٠◦C is suddenly immersed in a fluid at ٢٠◦C. The convection heat-transfer coefficient is ٥٨W/m٢◦C.Taking the aluminum as a sphere having the same weight as that given, estimate the time required to cool the aluminum to ٩٠◦C, using the lumped-capacity method of analysis.

$m = 6\,kg$, $T_o = 300^\circ C$, $T_\infty = 20^\circ C$, $h = 58\ W/m^2.^\circ C$.
Aluminum as a sphere. , $T = 90^\circ C$, find τ (time) using lumped capacity method.

تذكر.
sphere $\Rightarrow$ $A = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Sol:- $$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}}$$

from Table A-2 :- Aluminum. $\rho = 2707\ kg/m^3$
$c = 0.896\ kJ/kg.^\circ C$
$\boxed{c = 896\ J/kg.^\circ C}$

To find volume and Area. we must to find (r)

$\boxed{\rho = \frac{m}{v}}$

$m = \rho v$

$6 = 2707 \times \frac{4}{3}\pi r^3 \Rightarrow \boxed{r = 0.0807\ m}$

$A = 4\pi r^2 = 4 \times \pi \times (0.0807)^2 = 0.0822\ m^2$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}} = e^{-\frac{hA\tau}{cm}}$$

$$\ln \frac{T - T_\infty}{T_o - T_\infty} = \ln e^{-\frac{hA\tau}{cm}}.$$

$$\ln \frac{90 - 20}{300 - 20} = -\frac{58 \times 0.0822 \times \tau}{896 \times 6}$$

$$\boxed{\tau = 1563\ sec}$$

٤-١٠ A stainless-steel rod (١٨٪ Cr, ٨٪ Ni) ٦,٤ mm in diameter is initially at a uniform temperature of ٢٥◦C and is suddenly immersed in a liquid at ١٥٠◦C with *h*= ١٢٠ W/m٢ ◦C. Using the lumped-capacity method of analysis, calculate the time necessary for the rod temperature to reach ١٢٠◦C.

stainless - steel rod (cylinder) (18% Cr, 8% Ni) $d = 6.4$ mm.

$T_o = 25$, $T_\infty = 150°c$, $h = 120$, using lumped - capacity method.

find $\tau|_{T=120°c}$.

sol :-

from table A-2. (page 651) $\rho = 7817$ kg/m³
$c = 460$ J/kg.c

cylinder.

$A = \pi d L$

$V = \frac{\pi}{4} d^2 L$

Note :- L is unknown. so ... ?

$$\frac{A}{V} = \frac{\pi d L}{\frac{\pi}{4} d^2 L} = \boxed{\frac{4}{d}}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}}$$

$$\frac{120 - 150}{25 - 150} = e^{\frac{-120 \times 4 \times \tau}{7817 \times 460 \times 6.4 \times 10^{-3}}}$$

$$\boxed{\tau = 13.16 \text{ sec}}$$

Ln
للطرفين

$\ast$ Time constant $\tau = \frac{\rho c V}{h A}$.

$\boxed{\text{special case.}}$ IF time is equal to time constant. in lumped capacity method.

$$\frac{\theta}{\theta_o} = e^{\frac{-hA\tau}{\rho c V}} = e^{-\frac{hA}{\rho c V}\left(\frac{\rho c V}{hA}\right)} = e^{-1}$$

$$\frac{\theta}{\theta_o} = 0.368. \qquad \therefore \boxed{T - T_\infty = 36.8\% \, [T_o - T_\infty]}$$

في جميع الاسئلة السابقه كان السؤال محدد طريقه الحل و هي (lumped) . لكن في حاله عدم تحديد طريقه الحل نستخدم biot number لتحديد طريقه الحل.

$Bi < 0.1 \Rightarrow$ lumped can be used
$Bi > 0.1 \Rightarrow$ from figures (Heisler charts)

$$Bi = \frac{h\,(V/A)}{K} = \frac{hs}{K}$$

$s \rightarrow$ characteristic length. $\frac{V}{A}$

$$s\,(\text{plate}) = \frac{V}{A} = \frac{L^3}{L^2} = L$$

$$s\,(\text{cylinder}) = \frac{V}{A} = \frac{\frac{\pi}{4}d^2L}{\pi d L} = \frac{d}{4} = \frac{R}{2}$$

$$s\,(\text{sphere}) = \frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{d}{6}$$

٤-١٦ A ١٢-mm-diameter aluminum sphere is heated to a uniform temperature of ٤٠٠◦C and then suddenly subjected to room air at ٢٠◦C with a convection heat-transfer coefficient of ١٠ W/m٢ ⊠◦C. Calculate the time for the center temperature of the sphere to reach ٢٠٠◦C.

$d = 12$ mm. (Aluminum sphere). , $T_o = 400°C$, $T_\infty = 20°C$

$h = 10\ W/m^2.°C$, $\tau|_{T=200°C}$: ?

Sol: Table: $K = 204$, $\rho = 2707$, $c = 896$.

check. sphere $\left(\frac{V}{A}\right) = \frac{d}{6}$ $\quad \frac{A}{V} = \frac{6}{d}$

$$Bi = \frac{h}{K}\frac{V}{A} = \frac{10}{204} * \frac{12 * 10^{-3}}{6} = 9.8 * 10^{-5} < 0.1 \therefore \text{lumped can use.}$$

$$\therefore \frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA\tau}{\rho c V}} \Rightarrow \frac{200-20}{400-20} = e^{\frac{-10 * 6 * \tau}{2707 * 896 * 12 * 10^{-3}}}$$

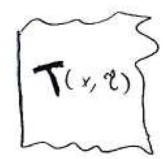
$$\boxed{\tau = 362\ sec}$$

$F_o \rightarrow$ Fourier's Number. $\qquad F_o = \dfrac{\alpha \tau}{s^2} = \dfrac{K \tau}{\rho c s^2}$

$$\frac{\theta}{\theta_o} = e^{-Bi F_o}$$

* semi-infinite plate. (slab).

Is a solid with a single plane surface with it's other three surfaces being far enough to be ignored.



① semi-infinite plate, (very larg slab) (very thicknes) (larg thick layer)

sudden change in temp.

$$\ast \quad \frac{T(x,\tau) - T_o}{T_i - T_o} = erf\left(\frac{x}{2\sqrt{\alpha \tau}}\right)$$

$erf \rightarrow$ Table A-1 .

خطوات الحل ← ① نجد $\dfrac{x}{2\sqrt{\alpha\tau}}$

② نجد ما يقابله من الجدول.

$x \rightarrow$ depth
$\tau \rightarrow$ Time $\Big] \rightarrow$ @ T
$T_i \rightarrow$ initial temp
$T_o \rightarrow$ final temp

$$\ast \quad q = \frac{-K\, A\, (T_i - T_o)}{\sqrt{\alpha \pi \tau}}\, e^{\left(\frac{-x^2}{4\alpha\tau}\right)} \quad \Big\{ \text{ at surface } x = 0 \quad q = -\frac{KA\,(T_i - T_o)}{\sqrt{\pi \alpha \tau}}$$

٤-٢١ A very large slab of copper is initially at a temperature of ٣٠٠°C. The surface temperature is suddenly lowered to ٣٥°C. What is the temperature at a depth of ٧,٥ cm ٤ min after the surface temperature is changed?

very larg slab of copper. // suddenly lowered //.

$T_i = 300°c$, $T_o = 35$. find T | $x = 7.5\ cm = 0.075\ m$
$\tau = 4\ min = 240\ s$

$$\frac{T - T_o}{T_i - T_o} = erf\left(\frac{X}{2\sqrt{\alpha \tau}}\right)$$

$$\frac{X}{2\sqrt{\alpha \tau}} = \frac{0.075}{2\sqrt{11.234 \times 10^{-5} \times 240}} = 0.2284$$

$\frac{X}{2\sqrt{\alpha \tau}}$	$erf \frac{X}{2\sqrt{\alpha \tau}}$
0.22	0.24430
0.2284	??
0.24	0.2657

erf $\frac{X}{}$

$$\frac{T - 35}{300 - 35} = 0.2533 \Rightarrow T = 102.1249\ c^{\circ}$$

٤-٢٦ A large slab of copper is initially at a uniform temperature of ٩٠◦C. Its surface temperature is suddenly lowered to ٣٠◦C. Calculate the heat-transfer rate through a plane ٧,٥ cm from the surface ١٠ s after the surface temperature is lowered.

large slab. copper. $T_i = 90^{\circ}C$ (suddenly), $T_o = 30^{\circ}C$, find $\frac{q}{A}$ (Heat T. rate)

$X = 7.5$ cm. from the surface.

$\tau = 10$ s.

copper $K = 386$. $\alpha = 11.23 * 10^{-5}\ m^2/s$

sol:-

$$q = \frac{-KA\ (T_i - T_o)}{\sqrt{\alpha \pi \tau}}\ e^{\left(-\frac{x^2}{4\alpha\tau}\right)} \Rightarrow \frac{q}{A} = \frac{-K(T_i - T_o)}{\sqrt{\alpha \pi \tau}}\ e^{\left(-\frac{x^2}{4\alpha\tau}\right)}$$

$$\frac{q}{A} = \frac{-386\ (90 - 30)}{\sqrt{11.23 * 10^{-5} * \pi \times 10}}\ e^{\left(\frac{-(0.075)^2}{4 \times 11.23 \times 10^{-5} * 10}\right)} = -111.46\ kW/m^2.$$

② constant H. Flux on semi-Infinite solid.

$$T - T_i = \frac{2 q_o}{A K}\sqrt{\frac{\alpha \tau}{\pi}}\ \exp\left(\frac{-x^2}{4\alpha\tau}\right) - \frac{q_o\, X}{K A}\left(1 - \mathrm{erf}\ \frac{X}{2\sqrt{\alpha \tau}}\right)$$

انتبه . $\underline{erf}$ لجميع القيم التي فوق 3.6 لـ $\frac{x}{2\sqrt{\alpha\tau}}$ يساوي ١ هذا مهم لأنه
الحد الثاني من المعادلة قد اصبح صفر في هذه الحالة، فانتبه.

٤-٢٧ A large slab of aluminum at a uniform temperature of ٣٠◦C is suddenly exposed to a constant surface heat flux of ١٥ kW/m٢. What is the temperature at a depth of ٢,٥ cm after ٢ min?

large slab Aluminum. , $T_i = 30^\circ C$, $\frac{q_o}{A} = 15\ KW/m^2$

find $T|_{x = 2.5\,cm,\ \tau = 120\,s}$

Sol:- from table $K = 204.$, $\alpha = 8.42 * 10^{-5}$

$$- X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.025}{2\sqrt{8.42*10^{-5}*120}} = 0.124355$$

$- \mathrm{erf}\ (0.124355) \Rightarrow$

x	erf
0.12	0.13476
0.124355	ZZ
0.14	0.15695

$\Rightarrow \mathrm{erf}(X) = 0.13959$

$$T - 30 = \frac{2*15000}{204}\sqrt{\frac{8.42*10^{-5}*120}{\pi}}\ \exp\left(\frac{-(0.025)^2}{4*8.42*10^{-5}*120}\right) - \frac{15000*0.025}{204}\left(1 - 0.13959\right)$$

$$\boxed{T = 36.59^\circ c}$$

③ Energy pulse @ surface.

$$T - T_i = \frac{Q_o}{A \rho C \sqrt{\pi \alpha \tau}}\ \exp\left(\frac{-x^2}{4\alpha\tau}\right)$$

energy.
Q_o :- in (J)

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٤-٤٥ A semi-infinite solid of stainless steel (١٨٪ Cr, ٨٪ Ni) is initially at a uniform temperature of ٠◦C. The surface is pulsed with a laser with ١٠ MJ/m٢ instantaneous energy. Calculate the temperature at the surface and depth of ١ cm after a time of ٣ s.

semi-infinite solid of stainless steel (18% Cr, 8% Ni), $T_i = 0°C$

$\frac{Q_o}{A} = 10 * 10^6 \ J/m^2$ Find $T \Big|_{\substack{x = 0.01m \\ \tau = 3s}}$?

sol:-
from Table. $\rho = 7817$, $C = 460$, $\alpha = 0.444 * 10^{-5} \ m^2/s$

$$T - 0 = \frac{10 * 10^6}{7817 * 460 * \sqrt{\pi * 0.444 * 10^{-5} * 3}} \exp\left(-\frac{(0.01)^2}{4 * 0.444 * 10^{-5} * 3}\right)$$

$T = 64.99°C$

$\boxed{\text{see P.4.46}}$ ~~P.4.48~~, 49, 50.

(4) semi-infinite slab with convection from the surface

$$\frac{T - T_i}{T_\infty - T_i} = 1 - \text{erf}\, X - \left[\exp\left(\frac{hx}{K} + \frac{h^2 \alpha \tau}{K^2}\right)\right] * \left[1 - \text{erf}\left(X + \frac{h\sqrt{\alpha \tau}}{K}\right)\right]$$

$X = \frac{x}{2\sqrt{\alpha \tau}}$

T_i :- initial temp of solid
T_∞ :- environment temp.

٤-٢٤ A semi-infinite slab of material having k =٠,١ W/m ٠季C and α=١,١×١٠−٧m٢⁄s is maintained at an initially uniform temperature of ٢٠◦C. Calculate the temperature at a depth of ٥ cm after ١٠٠ s if (a) the surface temperature is suddenly raised to ١٥٠◦C, (b) the surface is suddenly exposed to a convection source with h=٤٠W/m٢ ◦C and ١٥٠◦C, and (c) the surface is suddenly exposed to a constant heat flux of ٣٥٠ W/m٢.

P.4-24 semi-infinite slab. , $K = 0.1$, $\alpha = 1.1 \times 10^{-7}$, $T_i = 20^\circ C$

find $T|_{x = 0.05\,m,\ \tau = 100\,s}$ IF

① suddenly raised to $150^\circ C \rightarrow T_o$

② suddenly exposed to convection source.
$h = 40$, $T_\infty = 150^\circ C$

③ suddenly exposed to C.H.F. $\frac{q}{A} = 350$

Sol:-

① $$X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.05}{2\sqrt{1.1 \times 10^{-7} \times 100}} = 7.53$$

$erf(7.53) = 1 \rightarrow$ Table A-1 (The error function)

$$\frac{T - T_o}{T_i - T_o} = erf(X) = 1$$

$$T - \cancel{T_o} = T_i - \cancel{T_o} \Rightarrow \boxed{T = T_i = 20^\circ C} \quad \#$$

② $$\frac{T - T_i}{T_\infty - T_i} = 1 - erf\,X - \left[\exp\left(\frac{hx}{K} + \frac{h^2 \alpha \tau}{K^2}\right)\right]\left[1 - erf\left(X + \frac{h\sqrt{\alpha\tau}}{K}\right)\right]$$

هنا واضح ان erf هنا لان $X = 7.53$ وسنجد لها قيمة وستعطي $erf = 1$ لذلك الحد الأخير صفر.

$$\frac{T - T_i}{T_\infty - T_i} = 0 \Rightarrow T - T_i = 0 \quad \boxed{T = T_i = 20^\circ C}$$

③ $$T - 20 = \frac{2 \times 350}{0.1} \times \sqrt{\frac{1.1 \times 10^{-7} \times 100}{\pi}} \times \exp\left(\frac{-(0.05)^2}{4 \times 1.1 \times 10^{-7} \times 100}\right) - \frac{q_o x}{KA}\left(1 - erf\,X\right)$$

$\boxed{erf(X) = 1}$

$$T - 20 = 2.76 \times 10^{-24} \Rightarrow \boxed{T = 20^\circ C} \quad \#$$