Complex function

Definition of functions

A complex variable

*Let **S** be a set of compiex number.*

*A function **S** defined on **S** is arule which assocates with $z \in$ **S**,*

*A uniqne $w \in$ **S** written as $w=f(z)$*

*w=number called the value of f at z and is denoted by $f(z)$; that is $w=f(z)$. the set **S** iscalled the domain of definition of f.*

$$z=x+yi$$

$$w=u+iv$$

Eeach of the real numbers (u) and (v)depends on the real variables x and y where $u=u\ (x,y)$and $v=v\ (x,y)$

Where u and v function of x and y

$w=f(z) \quad \rightarrow w=u\ (x,y)+v\ (x,y)$

EX/ for what values of z is the complex function w=1/z defined, also find u, v and w when z=3-2:

Sol.

z=x+yi

$w=\frac{\mathbf{1}}{\mathbf{z}} \quad \rightarrow \quad w=\frac{1}{x+yi}$

$w=\frac{1}{x+yi} \cdot \frac{x-yi}{x-yi}$

$=\frac{x-yi}{x^2+y^2} \quad \rightarrow \quad w=\frac{x}{x^2+y^2}-\frac{yi}{x^2+y^2}$

$u=\frac{x}{x^2+y^2} \qquad v=\frac{-y}{x^2+y^2}$

z=3-2i

z=x+yi *x=3 , y=-2*

$u=\frac{x}{x^2+y^2}=\frac{3}{9+4}=\frac{3}{13}$

$v=\frac{-y}{x^2+y^2}=\frac{-2}{13}$

w=u+iv

$w=\frac{3}{13}+\frac{-2}{13}i$

EX / find u (x,y) and v(x,y) for the complex function $f(z)=z^3$

sol.

$z=x+yi$

$f(z)=(x+yi)^3$

$=x^3+i^3y^3+3xyi.(x+iy)$

$=x^3+i^3y^3+3xyi(x+yi)$ $\qquad i^3=-i$

$=x^3-iy^3+3ixy^2-3xy^2$

$=(x^3-3xy^2)+i(3xy^2-y^3)$

$w=f(z)$

$w=u+iv$

$u(x,y)=x^3-3xy^2$

$v(x,y)=3xy^2-y^3$

EX/Find the solution of $f(z)=z^2-z$

Sol.

$z=x+yi$

$w=f(z)$

$w=(x+yi)^2-(x+yi)$

$=x^2+2xyi+y^2i^2-(x+yi)$

$=x^2+2xyi-y^2-x+yi$

$=x^2-y^2-x+2xyi+yi$

$u=x^2-y^2-x$

$v=2xy+y$

EX /Consider $f(z)=z^2 + z\,i$ and express it in to terms of real and imaginary parts.

Sol.

$z=x+yi$

$f(z)= (x+yi)^2 +i(x+yi)$

$= x^2 + 2xyi + y^2i^2 +xi+yi^2$

$= x^2 + 2xyi - y^2 + xi - y$

$= x^2 - y^2 - y + i(x + 2xy)$

$w=f(z)$

$u+vi = x^2 - y^2 - y +i (x + 2xy)$

$u= x^2 - y^2 - y$

$v= x + 2xy$