Some Important functions

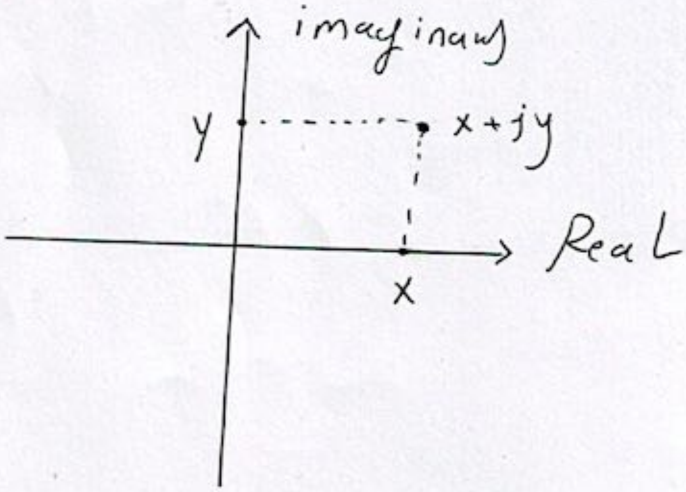
* العدد المركب في الاحداثيات الديكارتيه

$$\boxed{Z = x + jy} \rightarrow \text{complex number in cartesian form}$$

1- A complex number is the combination of a real part (x) and an imaginary part (jy).

2- when plotting the location on the cartesian form the coordinate is (x, y) ← عند رسم الموقع على المستوى الديكاري يكون الاحداثي

3- The real part is plotted on the real-axis and the imaginary part is plotted on the imaginary-axis.



$$\begin{bmatrix} X = r\cos\theta \\ y = r\sin\theta \end{bmatrix} \text{Polar coordin}$$

$$\Downarrow$$

$$Z = x + jy$$

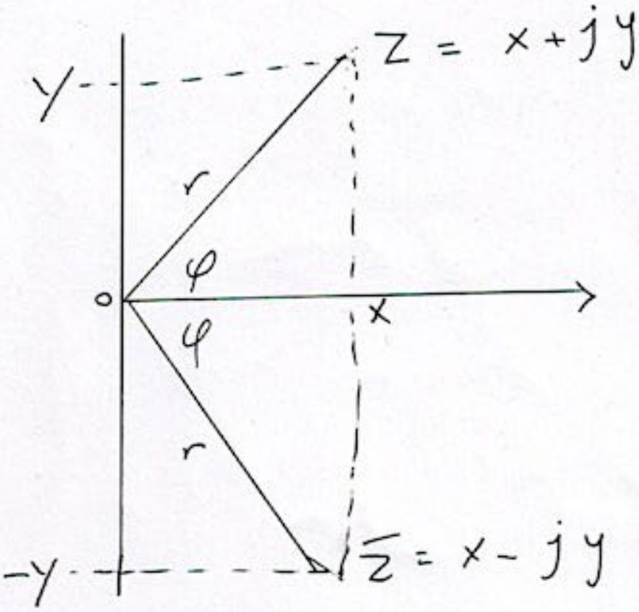
العدد المركب في الاحداثيات القطبية

* $Z = r[\cos\theta + j\sin\theta] \rightarrow$ complex number in Polar cordinates

* $\bar{Z} = x - jy \longrightarrow$ complex conjugate مرافق العدد

A complex conjugate is a complex number that has the Same read part as the orginal number, but it has an imaginay part that is equal to the imaginary part of the orginal number in absolute-value, and different from it in term of sign

$$\bar{Z} = x - jy$$



*

$$Z = r[\cos\Theta + j\sin\Theta]$$
$$\bar{Z} = r[\cos\Theta - j\sin\Theta]$$
complex number in Polar coordinate

$$\left.\begin{array}{l} Z = re^{j\Theta} \\ \bar{Z} = re^{-j\Theta} \end{array}\right]$$ complex number in Exponential form
الصيغة الاسية

$$e^{j\theta} = \cos\Theta + j\sin\Theta$$
$$e^{-j\Theta} = \cos\Theta - j\sin\Theta$$

$$\begin{aligned} e^{j\theta} &= \cos\theta + j\,\cancel{\sin\theta} \\ e^{-j\theta} &= \cos\theta - j\,\cancel{\sin\theta} \end{aligned} \quad + \text{ جمع}$$

$$2\cos\theta = e^{j\theta} + e^{-j\theta} \Rightarrow \cos = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\begin{aligned} e^{j\theta} &= \cancel{\cos\theta} + j\sin\theta \\ e^{-j\theta} &= \cancel{\cos\theta} - j\sin\theta \end{aligned} \quad - \text{ طرح}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\left[\begin{aligned} \cos\theta &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta &= \frac{e^{j\theta} - e^{-j\theta}}{2j} \end{aligned}\right]$$

By the same way

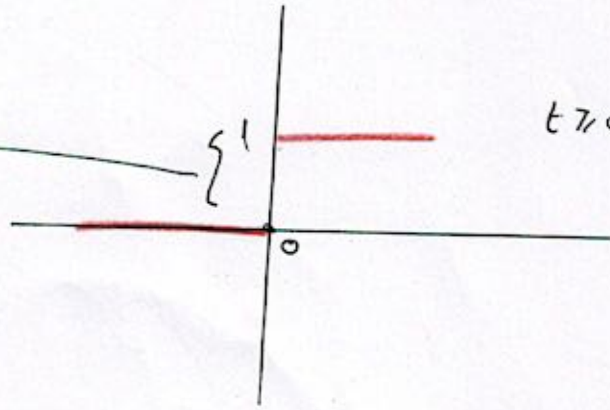
$$\left[\begin{aligned} \cosh\theta &= \frac{e^{\theta} + e^{-\theta}}{2} \\ \sinh\theta &= \frac{e^{\theta} - e^{-\theta}}{2} \end{aligned}\right]$$

unit step function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

هنا يكون مقدار القفزة ,1,

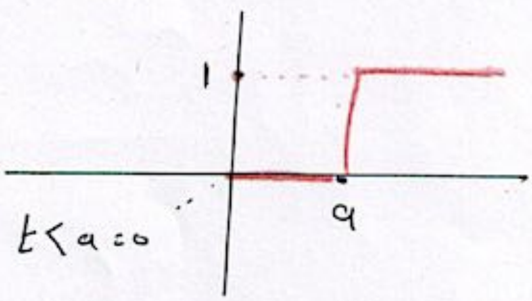
$t < 0 = 0$



$t \geq 0 = 1$

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$

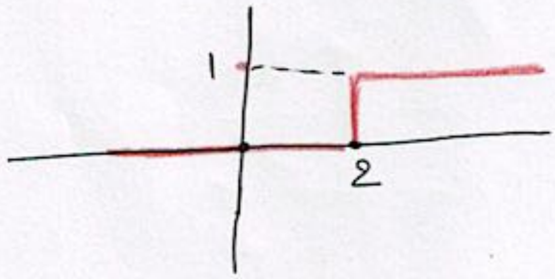
$a > 0$ positive



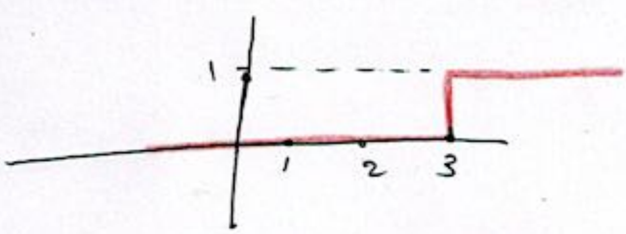
$t \geq a = 1$

$t < a = 0$

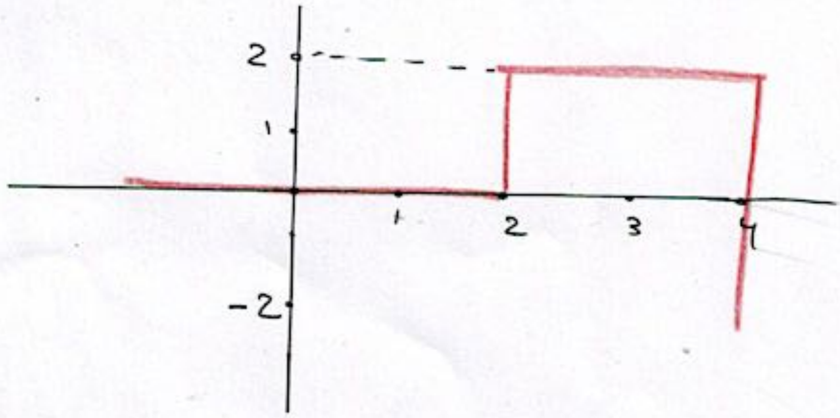
Ex:- $f(t) = u(t-2) \Rightarrow 1\,u(t-2)$



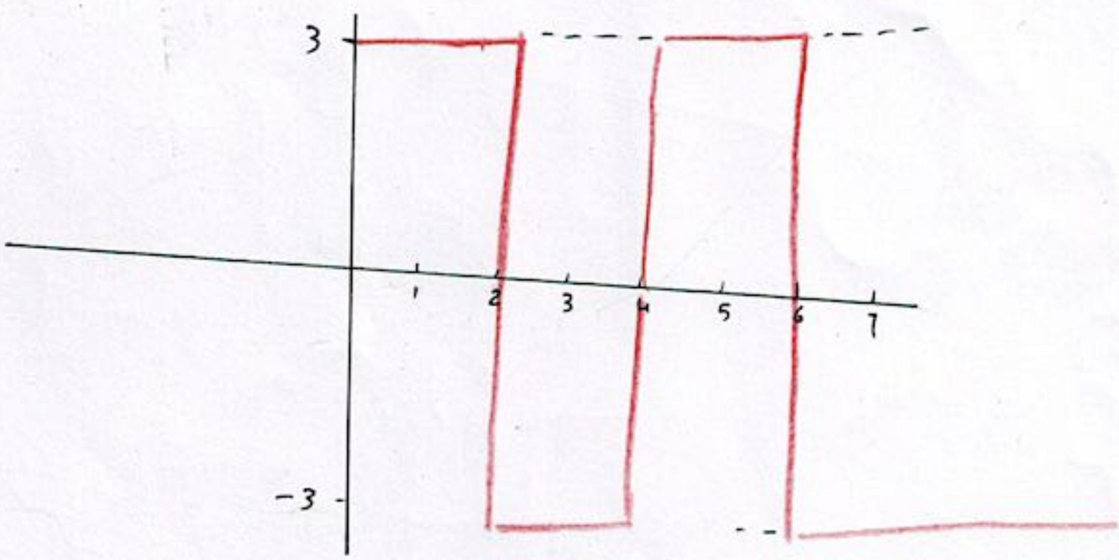
Ex:- $f(t) = u(t-3)$



$f(t) = 2[u(t-2) - u(t-4)$



$f(t) = 3[u(t) - 2u(t-2) + 2u(t-4) - 2u(t-6)]$



5,

faictorial function

$$n! = \int_0^\infty t^n e^{-t}\, dt \quad n \geqslant 0$$

$$n! = n(n-1)! = n(n-1)(n-2)!$$

$$3! = 3(3-1)! = 3(3-1)(3-2)!$$

$$= 3 \times 2 \times 1 \Rightarrow 3! = 6$$

$$\left[\begin{array}{ll} 0! & = 1 \\ 1! & = 1 \end{array}\right]$$

Famma function

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t}\, dt \quad n \geqslant 0$$

$$\left[\begin{array}{l} 1\text{- } \Gamma(n+1) = n\,\Gamma(n) = n! \\ 2\text{- } \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \\ 3\text{- } \Gamma(0) = 0 \\ 4\text{- } \Gamma(1) = 1 \end{array}\right]$$

Laplace transform

تحويل لابلاس

هو طريقة رياضية للتحويل بين مجالين مختلفين غالباً
بين الزمن والتردد

$f(t) \longrightarrow$ in time domain

by using $\mathcal{L}$ (Laplace transform)

$F(s) \longrightarrow$ in frequency domain

الصيغة العامة للتحويل

$$\boxed{F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \, dt}$$

ويستخدم للدوال المستمرة

Then $S \Rightarrow$ is parametric real or complex

فائدة تحويل لابلاس :- في حل المعادلات التفاضلية والمعادلات
التفاضلية الجزئية بواسطة Laplace (نتخلص من التوابع)

Laplace transform

طريقة رياضية للتحويل بين مجالين مختلفين غالباً بين الزمن , التردد.

$$\underset{\text{in Time domain}}{\underline{f(t)}} \xrightarrow{\mathcal{L}} \underset{\text{in frequency domain}}{\underline{F(s)}}$$

الصيغة العامة للتحويل

$$F(s) = \int_0^{\infty} f(t) \cdot e^{-st} \, dt$$

مجموعة من الدوال المشهورة.

1- $f(t) = a$

$$\mathcal{L}\, a = \frac{a}{s}$$

2- $f(t) = t^n$

$$\mathcal{L}\, t^n = \frac{n!}{s^{n+1}}$$

3- $f(t) = e^{\mp at}$

$$\mathcal{L}\, e^{\mp at} = \frac{1}{s \pm a}$$

عكس الاشارة

4- $f(t) = \sin wt$

$$\mathcal{L}\, \sin wt = \frac{w}{s^2 + w^2}$$

5- $f(t) = \cos wt$

$$\mathcal{L}\, \cos wt = \frac{s}{s^2 + w^2}$$

6- $f(t) = \sinh wt$

$$\mathcal{L}\, \sinh wt = \frac{w}{s^2 - w^2}$$

7- $f(t) = \cosh wt$

$$\mathcal{L}\, \cosh wt = \frac{s}{s^2 - w^2}$$

1. $f(t) = a$

Find $F(s) \Rightarrow F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$

$$F(s) = \int_0^\infty a\, e^{-st}\, dt \Rightarrow a \int_0^\infty e^{-st}\, dt$$

$$= a \left[\frac{e^{-st}}{-s} \right]_0^\infty \Rightarrow a \left[0 - \frac{1}{-s} \right]$$

$$F(s) = a \frac{1}{s} \Rightarrow \boxed{\frac{a}{s}}$$

So $F(s) \Rightarrow \frac{a}{s}$ when $f(t) = a$

2. $f(t) = e^{\pm at}$

$$F(s) = \int_0^\infty e^{+at} \cdot e^{-st}\, dt$$

$$= \int_0^\infty e^{-(a-s)t}\, dt \Rightarrow \left[\frac{e^{-(a-s)t}}{-(a-s)} \right]_0^\infty$$

$$= \left[0 - \frac{1}{-(a-s)} \right] \Rightarrow \frac{1}{a-s}$$

So if we have $f(t) = e^{\pm at} \Rightarrow F(s) = \dfrac{1}{s \mp a}$

متطابقات للحفظ

$$\begin{bmatrix} \sin 2x = 2\sin x \cos x \\ \sin^2 x = \frac{1}{2}(1 - \cos 2x) \\ \cos^2 x = \frac{1}{2}(1 + \cos 2x) \end{bmatrix}$$

Ex: -

1- $\mathcal{L}\, 5 \Rightarrow \boxed{F(s) = \frac{5}{s}}$

2- $\mathcal{L}\, 3t^4 \Rightarrow 3\,\mathcal{L}\, t^4 \Rightarrow F(s) = 3\left[\frac{4!}{s^{4+1}}\right] = 3\left[\frac{24}{s^5}\right]$

$\boxed{F(s) = \frac{27}{s^5}}$

3- $\mathcal{L}\, 2e^{-6t} \Rightarrow 2\,\mathcal{L}\, e^{-6t} \Rightarrow F(s) = 2\left[\frac{1}{s+6}\right]$

$\boxed{F(s) = \frac{2}{s+6}}$

4- $\mathcal{L}\, 4 \sin 6t \Rightarrow 4\,\mathcal{L} \sin 6t \Rightarrow F(s) = 4\left[\frac{6}{s^2+36}\right]$

$\boxed{F(s) = \frac{24}{s^2+36}}$

5- $\mathcal{L}\, 5 \cos 4t \Rightarrow 5\,\mathcal{L} \cos 4t \Rightarrow F(s) = 5\left[\frac{s}{s^2+16}\right]$

$\boxed{F(s) = \frac{5s}{s^2+16}}$

6- $\mathcal{L} \sinh 2t \Rightarrow \boxed{\frac{2}{s^2-4}}$

7- $\mathcal{L} \cosh 9t \Rightarrow \boxed{\frac{s}{s^2-18}}$

Ex: -

$$f(t) = (t+3)^2$$

$$\mathcal{L}(t^2 + 6t + 9) \Rightarrow \mathcal{L}t^2 + \mathcal{L}6t + \mathcal{L}9$$

$$F(s) = \frac{2!}{s^{2+1}} + 6\frac{1!}{s^{1+1}} + \frac{9}{s}$$

$$= \frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}$$

Ex:- $f(t) = (t-3)^3$

$$= \mathcal{L}(t-3)^3 \Rightarrow \mathcal{L}(t-3)^2(t-3)$$

$$= \mathcal{L}(t^2 - 6t + 9)(t-3)$$

$$= \mathcal{L}(t^3 \underbrace{-3t^2 - 6t^2} + \underbrace{18t + 9t} - 27)$$

$$= \mathcal{L}\, t^3 - 9t^2 + 27t - 27$$

$$= \mathcal{L}t^3 - 9\mathcal{L}t^2 + 27\mathcal{L}t - \mathcal{L}27$$

$$= \frac{3!}{s^{3+1}} - 9\frac{2!}{s^{2+1}} + 27\frac{1!}{s^{1+1}} - \frac{27}{s}$$

$$= \frac{6}{s^4} - \frac{18}{s^3} + \frac{27}{s^2} - \frac{27}{2}$$

Ex:- $\mathcal{L}\ \sin 2t \ \cos 2t$

$$\boxed{\sin 2t = 2 \sin t \cos t}$$ متطابقة

$$\sin 4t = 2 \sin 2t \cos 2t$$

$$F(s) = \frac{1}{2}\mathcal{L}\ \cancel{2} \sin 2t \cos 2t$$

$$F(s) = \frac{1}{2}\mathcal{L}\ \sin 4t$$

$$= \frac{1}{2} \cdot \frac{4}{s^2+16}$$

$$= \mathcal{L}\ \sin^2 3t \Rightarrow \mathcal{L}\ (\sin 3t)^2$$

$$\boxed{\sin^2 t = \frac{1}{2}(1 - \cos 2t)} \quad \times 3$$

$$\sin^2 3t = \frac{1}{2}(1 - \cos 6t)$$

$$= \mathcal{L}\ \frac{1}{2}(1 - \cos 6t)$$

$$F(s) = \frac{1}{2} \mathcal{L}\ 1 - \mathcal{L}\ \cos 6t$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2 + 36}\right]$$

$$= \frac{1}{2s} - \frac{s}{2(s^2+36)}$$

$\mathcal{L}\cos^2 5t$

$F(s) = \mathcal{L}\cos^2 5t$

$\boxed{\cos^2 t = \frac{1}{2}(1 - \cos 2t)}$ $\times 5$

$\boxed{\cos^2 5t = \frac{1}{2}(1 - \cos 10t)}$

$F(s) = \mathcal{L}\,\frac{1}{2}(1 - \cos 10t)$

$= \frac{1}{2}\mathcal{L}1 - \mathcal{L}\cos 10t$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{10}{s^2+100}\right]$