Fundamental of Control Engineering

Systems Engineering Department
Second Class II semester

Introduction:

Control Theory: It is that part of science which concern control problems.

Control Problem: If we want something to act or vary according to a certain performance specification, then we say that we have a control problem.

Ex. We want to keep the temperature in a room at certain level and as we order, then we say that we have temperature control problem.

***Plant*:** A piece of equipments the purpose of which is to perform a particular operation (we will call any object to be controlled a plant).

Ex. Heating furnace, chemical reactor or space craft.

Process: A progressively continuing operation (natural or artificial) that consist of a series of actions or changes in a certain way leading towards a particular result or end. We will call any operation to be controlled a process. Processes could be chemical, economic, or biological.

System: A combination of components that act together and perform a certain objective (could be physical, biological, or economic).

Disturbance: A signal which tends to conversely affect the value of the output of a system (of course it is undesired signal).

Command input i/p: The motivating input signal to the system which is independent of the output of the system.

Reference i/p elements: An element which modifies the command i/p into suitable signal (called reference i/p) for the controlled system.

Reference input: It is almost the desired output**.**

Actuating signal**:** The difference between the reference input and feed back (f/b) signals. It actuates the control unit (controller) to maintain the output at the desired value.

Control unit: The unit which receives the actuating signal and delivers the control signal.

Controlled variable (actual o/p)*:* The variable which we need actually to control it.
Ex. temperature, pressure, liquid level, flow rate, etc.

Feedback signal: A signal representing a measure of the actual o/p which is fed back into control system for purpose of comparison with the reference signal.

Feedback element: Usually it represents a transducer, the purpose of which is to convert the o/p of the system in to a signal of suitable physical nature for the next stage in the system (error detector).

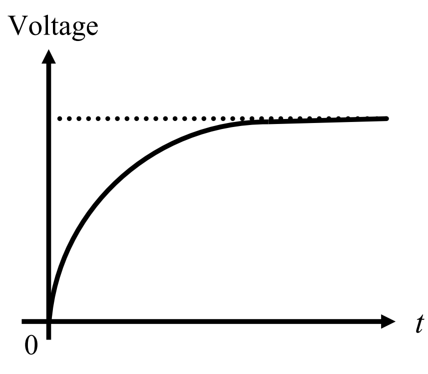
Feedback control: An operation which tends to reduce the difference between the o/p of the system and the reference i/p.

***Servomechanism control system*:** A feedback control system in which the o/p is mechanical variable (position, speed, acceleration).

Process control system: A feedback control system in which the o/p is a variable such as temperature, pressure liquid level.

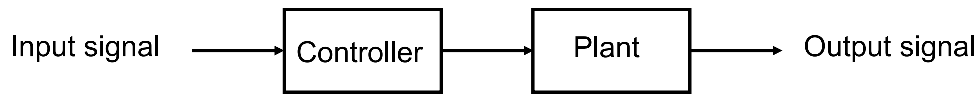
***Automatic regulating system**:* A feedback control system in which the reference i/p (desired output) is either constant or slowly varying with time and the primary task is to maintain the o/p at the desired value in the presences of disturbance.

***Close loop control system**:* A control system in which the o/p signal has a direct effect upon the control action.



Open loop control system: A control system in which the o/p signal has no effect upon the control action.

Ex. heater, light, washing machine



Open loop control system

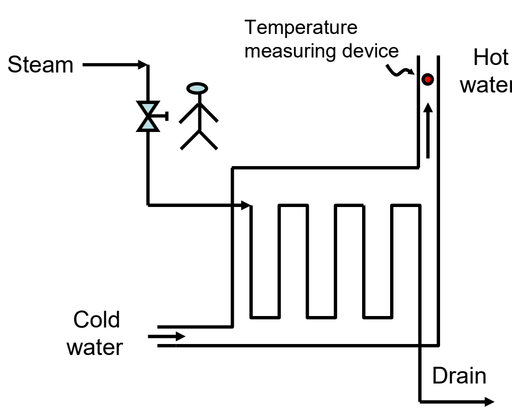
C/L control system versus O/L control system:

1) F/b control system is relatively insensitive to external disturbance and internal variation in system parameters. So we can use relatively inexpensive components with close loop control.

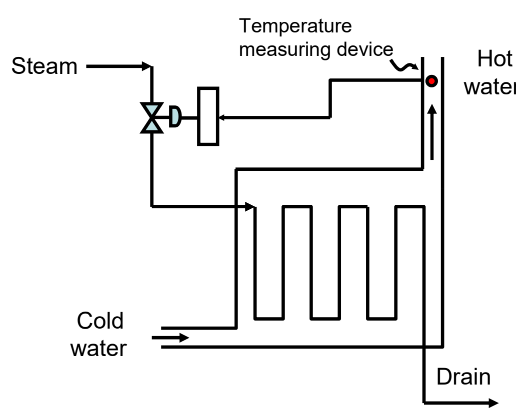
2) The required power of the system is less in O/L than in C/L control system.

Note: finally, which one to be used depends on the situation, sometimes we might use both of them in a certain way to get optimum case.

Manual and automatic feedback control:



Manual F/b control system of a steam thermal system



Automatic F/b control system of a steam thermal system

Classification of control system:

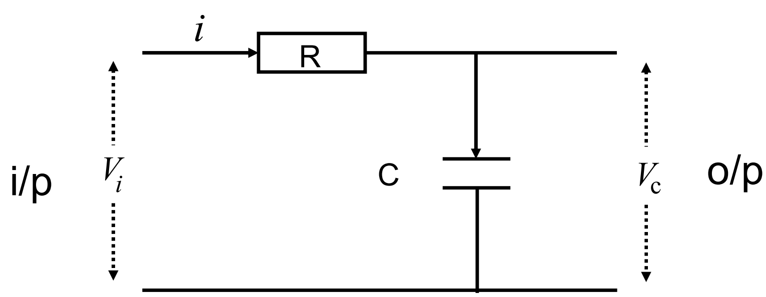
1. linear or nonlinear
2. C/L or O/L
3. Electrical. mechanical,…, etc
4. Continuous or discrete
5. Time variant or time invariant.

Mathematical Representation of Control System:

1. ***Electrical system:***

Ex(1).

$$V_c = \frac{1}{c}\int i\ dt = \frac{1}{cD} i$$

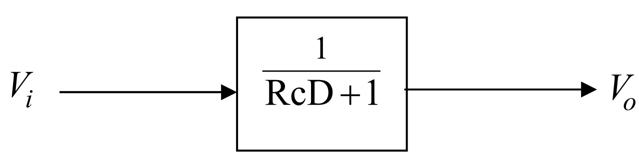


$$i = \frac{V_i}{R + \frac{1}{cD}}$$

$$V_c = \frac{\frac{1}{cD}}{R + \frac{1}{cD}} V_i$$

$$\boxed{V_c = \frac{1}{RcD + 1} V_i}$$

Differential equation



Transfer function (*T.f*): the T.f of a linear time invariant system is defined to be the relation of the laplace transform of the o/p (response function) to the laplace transform of the i/p (deriving force) under the assumption that all initial conditions are zero.

by L.T, $V_c = \frac{1}{RcD+1} V_i$ $\Rightarrow$ $\boxed{\frac{V_c(s)}{V_i(s)} = \frac{1}{Rcs+1}}$

Ex(2).

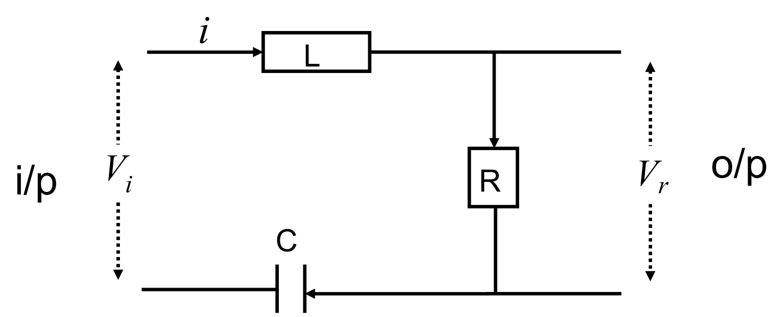
$$V_i = i\,(LD + R + \frac{1}{cD})$$

$$i = \frac{1}{LD + R + \frac{1}{cD}} V_i$$

$$V_r = i\,R$$

$$V_r = \frac{R}{LD + R + \frac{1}{cD}} V_i$$

$$V_r = \frac{RcD}{LcD^2 + RcD + 1} V_i$$



$$V_i \longrightarrow \boxed{\frac{RcD}{LcD^2 + RcD + 1}} \longrightarrow V_r$$

2. *Mechanical system:*

a) Translational mechanical system:

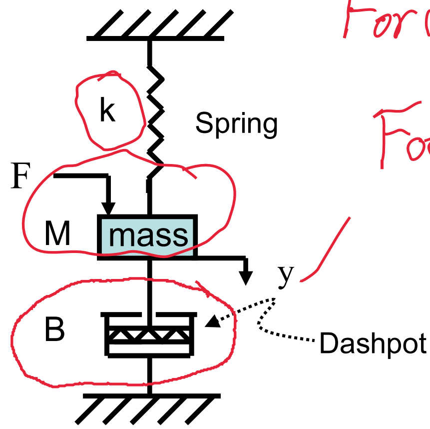
Ex(1).

$$\sum F = ma$$

$$F = ky + mD^2y + BDy$$

$$F = mD^2y + BDy + ky$$

$$\frac{y(s)}{F(s)} = \frac{1}{ms^2 + Bs + k}$$

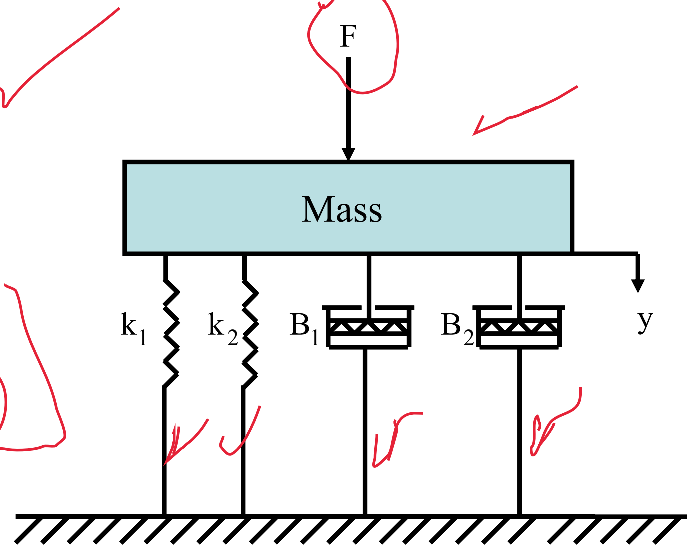


k: spring constant (stiffness coefficient)
B: viscous friction coefficient

Ex(2).

$$F = mD^2y + (k_2 + k_1)y + (B_1 + B_2)Dy$$

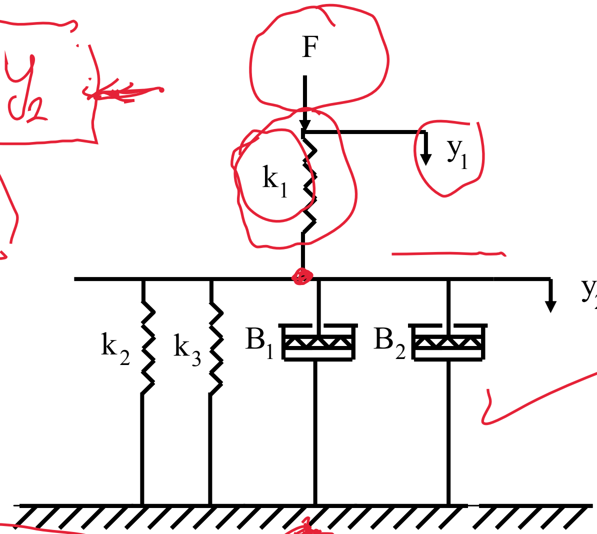
$$F = mD^2y + (B_1 + B_2)Dy + (k_2 + k_1)y$$



Ex(3).

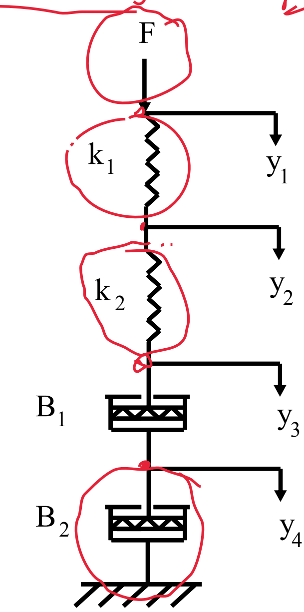
$$F = k_1(y_1 - y_2)$$

$$k_1(y_1 - y_2) = (k_2 + k_3)y_2 + (B_1 + B_2)Dy_2$$



Ex(4).

$$F = k_1(y_1 - y_2) = k_2(y_2 - y_3) = B_1D(y_3 - y_4) = B_2Dy_4$$



b) Rotational mechanical systems:

$J\alpha = \sum T$

J: Moment of inertia

α : Rotational acceleration

T: Torque

Ex(1).

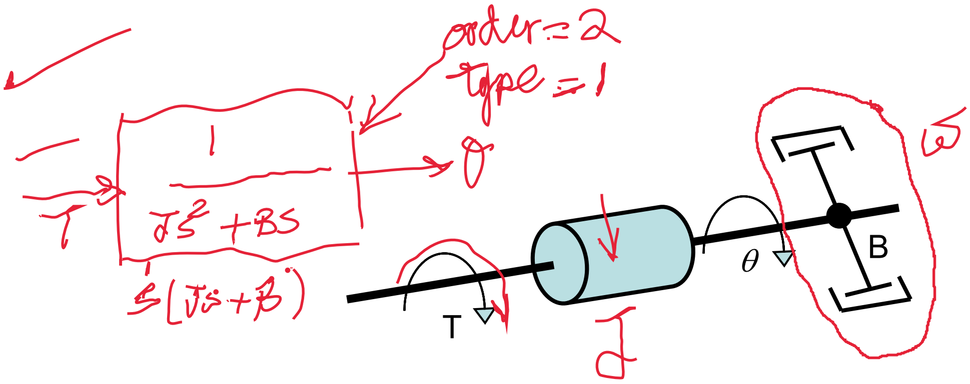
$$T = JD^2\theta + BD\theta$$

$$T = JD\omega + B\omega$$

where,

$$\omega = \dot{\theta} = D\theta$$

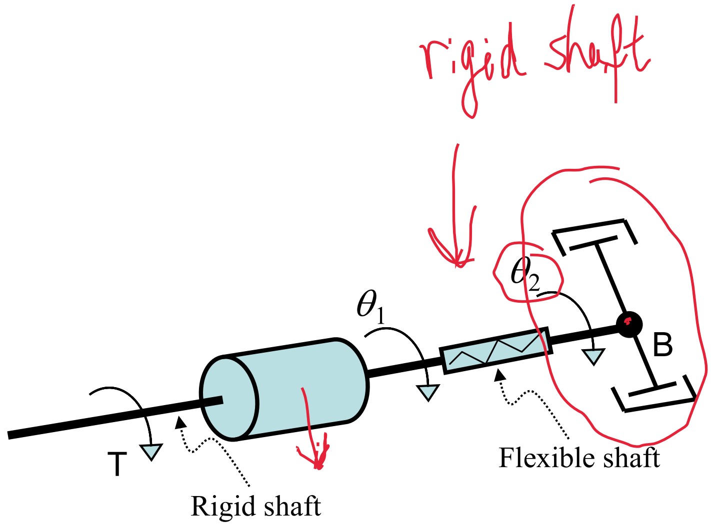
$$\alpha = \dot{\omega} = \ddot{\theta} = D^2\theta$$



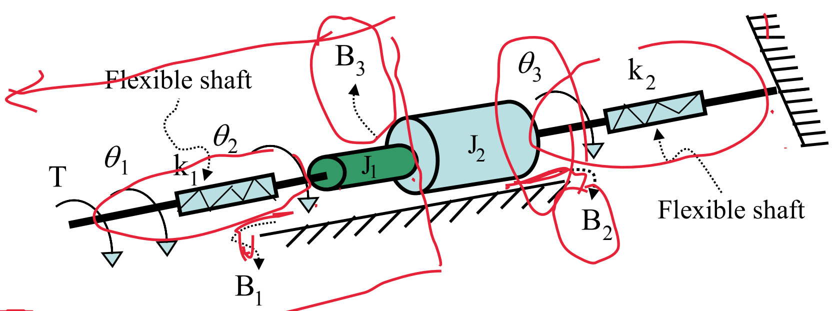
Ex(2).

$$T = JD^2\theta_1 + k(\theta_1 - \theta_2)$$

$$k(\theta_1 - \theta_2) = BD\theta_2$$



Ex(3).



$$T = k_1(\theta_1 - \theta_2)$$

$$k_1(\theta_1 - \theta_2) = J_1D^2\theta_2 + B_3D(\theta_2 - \theta_3) + B_1D\theta_2$$

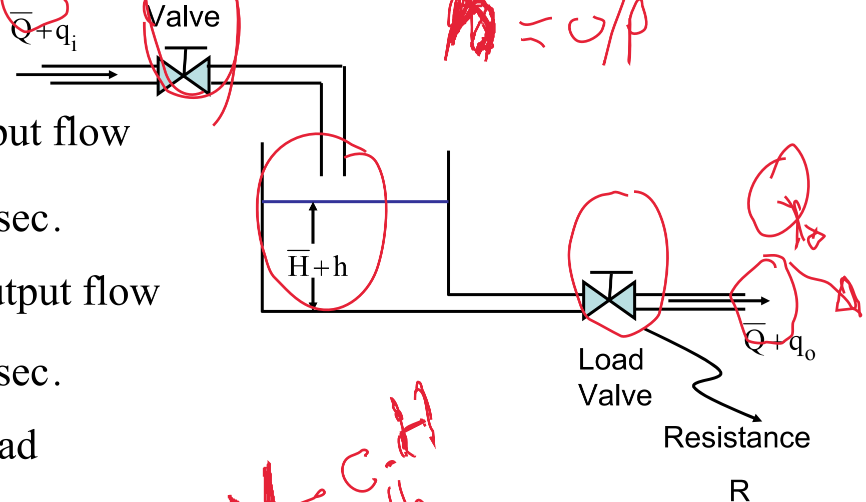
$$B_3D(\theta_2 - \theta_3) = J_2D^2\theta_3 + B_2D\theta_3 + k_2\theta_3$$

3. Liquid level systems:

Ex(1).

$\bar{Q}$**:** S.S liquid flow rate ft^3/sec

$\bar{H}$**:** S.S head.ft



q_i : Small deviation of the input flow rate from its S.S value ft^3/sec.

q_o : Small deviation of the output flow rate from its S.S value ft^3/sec.

h : Small deviation of the head from its S.S value ft.

c : *area*

$$q_i - q_o = c\frac{dh}{dt}$$

$$q_o = \frac{h}{R} \quad \text{(for laminar flow)}$$

$$q_o = k\sqrt{h} \quad \text{(for turbulent flow)}$$

$$q_i - \frac{h}{R} = c\frac{dh}{dt}$$

$$\boxed{Rc\frac{dh}{dt} + h = Rq_i}$$ Differential equation

By , laplace transform.

$$(Rcs+1)H(s) = RQ_i(s)$$

$$\boxed{\frac{H_o(s)}{Q_i(s)} = \frac{R}{Rcs+1}}$$

We can find that:-

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{Rcs+1}$$

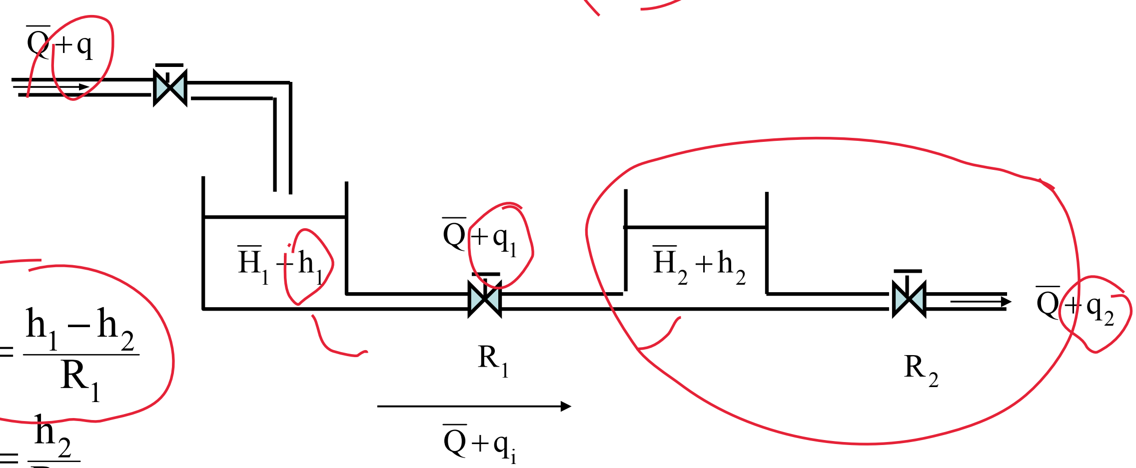
Transfer function in case when Q_o =o/p and Q_i = i/p

where, $q_o = \frac{h}{R}$

by L.T $Q_o(s) = \frac{H(s)}{R}$

$H(s) = RQ_o(s)$

Ex(2). Liquid level systems with interaction



$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$q_2 = \frac{h_2}{R_2}$$

$$c_1 \frac{dh_1}{dt} = q - q_1$$

$$c_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1c_1R_2c_2s^2 + s(R_1c_1 + R_2c_2 + R_2c_1) + 1}$$

H.W find the T.Fs ; $\frac{Q_1(s)}{Q(s)}, \frac{H_1(s)}{Q(s)}, \frac{H_2(s)}{Q(s)}$

Ex(3). Non interaction liquid level system:

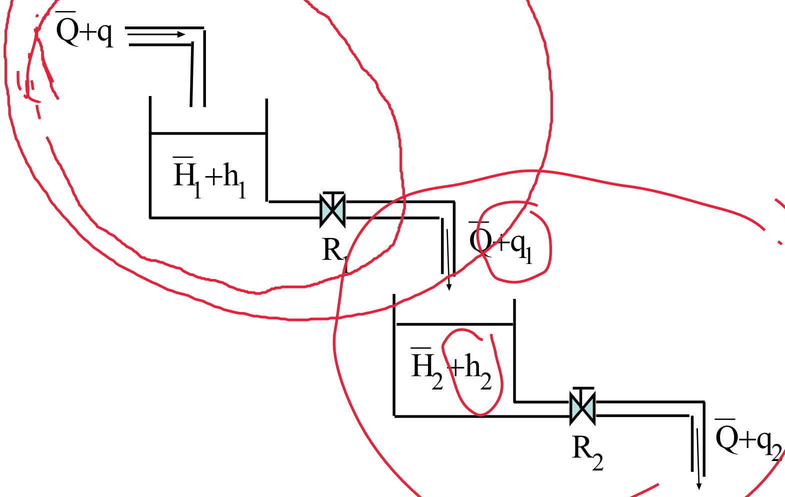
$$q_1 = \frac{h_1}{R_1}$$

$$q_2 = \frac{h_2}{R_2}$$

$$q - q_1 = c_1 \frac{dh_1}{dt}$$

$$q_1 - q_2 = c_2 \frac{dh_2}{dt}$$

H.w. Find $\frac{Q_2(s)}{Q(s)}$,



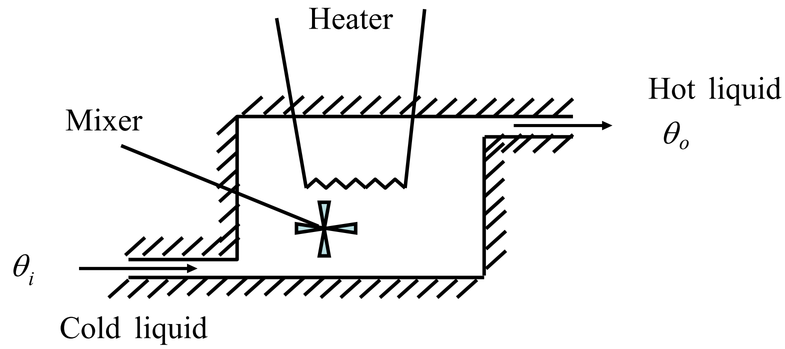
$\mathcal{L}^{-1}\, Y(s) = y(t)$

partial Fraction Expansion

Control Theory

Second Class **Control and System Engineering Department**
By M. J. Mohamed

4. Thermal system



θ_i : S.S Temperature of inflowing liquid, F^o
θ_o : S.S Temperature of outflowing liquid, F^o
G : S.S liquid flow rate lb/sec.
M : mass of liquid in tank , lb
c : specific heat of liquid , B tu/lb. F^o
R : thermal resistance , F^o sec/B tu.
Q : thermal capacitance, B tu/F^o.
$\overline{H}$: S.S heat i/p rate , B tu/sec.

Consider that heat input rate changes from $\overline{H}$ to $\overline{H}+h_i$ then heat outflow will change from $\overline{H}$ to $\overline{H}+h_o$ also the temperature of the outfollwing liquid will change from $\overline{\theta}_o$ to $\overline{\theta}_o+\theta$.
Considering change only:

$$h_i - h_o = Q\frac{d\theta}{dt} \quad , \quad \theta = h.R$$

or, $$RQ\frac{d\theta}{dt} + \theta = Rh_i$$

Note: $h_o = G.c.\theta$, $G.c = \frac{1}{R}$, Q=M.c

By L.T : $\frac{\theta(s)}{H_i(s)} = \frac{R}{RQs+1}$ ⟹ This is the T.f between changes in h and θ

If we consider that the driving function (i.e) i/p was a change in θ_i then:-

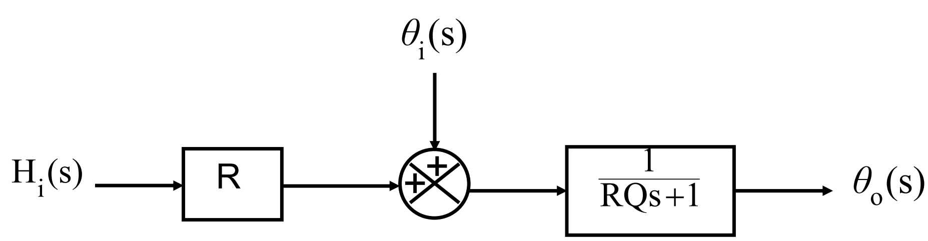
$$Q\frac{d\theta}{dt} = G.c.\theta_i - h_o$$

$$Q\frac{d\theta}{dt} = \frac{1}{R}.\theta_i - \frac{\theta}{R}$$

$$RQ\frac{d\theta}{dt} + \theta = \theta_i \implies \frac{\theta(s)}{\theta_i(s)} = \frac{1}{RQs+1}$$

In case of changes in both h_i and θ_i then we have:-

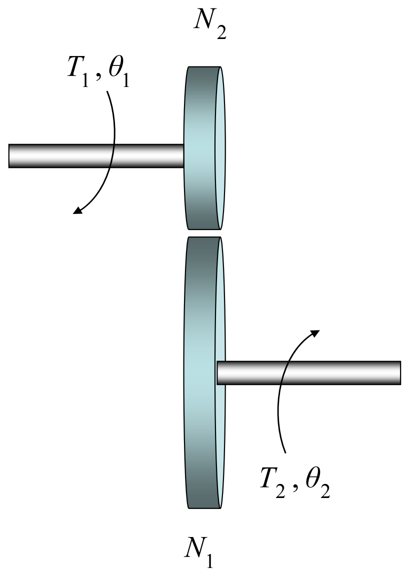
$$R.c\frac{d\theta}{dt} + \theta = \theta_i + Rh_i$$



5) ***Gear Trains:***

A gear train is a mechanical device that transmits energy from one part of a system to another in such a way that force, torque, speed, and displacement are altered. Two gears are shown coupled together in following figure. The inertia and friction of the gears are neglected in the ideal case considered.

The relationships between the torque T_1 and T_2 , angular displacements θ_1 and θ_2, and the teeth numbers N_1 and N_2 , of the gear train are derived from the following facts.



1- The number of teeth on the surface of the gear is proportional to the radius r_1 and r_2 of the gears, that is.

$$r_1 N_2 = r_2 N_1$$

2- The distance traveled along the surface of each gear is the same. Therefore,

$$\theta_1 . r_1 = \theta_2 . r_2$$

3- The work done by one gear is equal to that of the other since there is assumed to be no loss, thus

$$T_1 \theta_1 = T_2 \theta_2$$

If the angular velocities of the two gears are ω_1 and ω_2.

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$$