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المرحلة :الثانية
السنة الدراسية :2023
عنوان المحاضرة

Mo Tu We Th Fr Sa Su

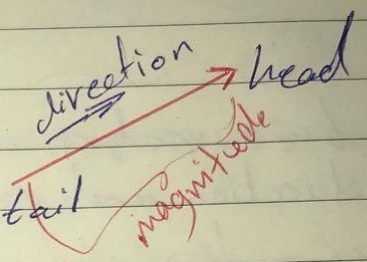
Memo No. Lec 1
Date 2021 / 10 / 23

"Vector analysis and vector fields"

A vector:- is an objct that has both magnitude and direction

magnitude = length

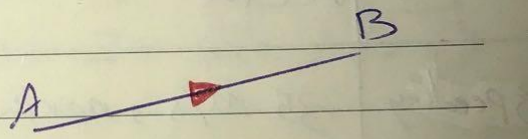
direction= tail to head



A vector is a combination of three things :-

* a positive number ((magnitude))
* a direction in space
* a sence

A vector in the dia →



$\overrightarrow{AB}$:-

* direction A to B → indicating the sence
* the magnitude of $\overrightarrow{AB}$ = length of AB

The magnitud of $\overrightarrow{AB} = |\overrightarrow{AB}|$

vectors
used to discrib (velocity, acceleration, force)

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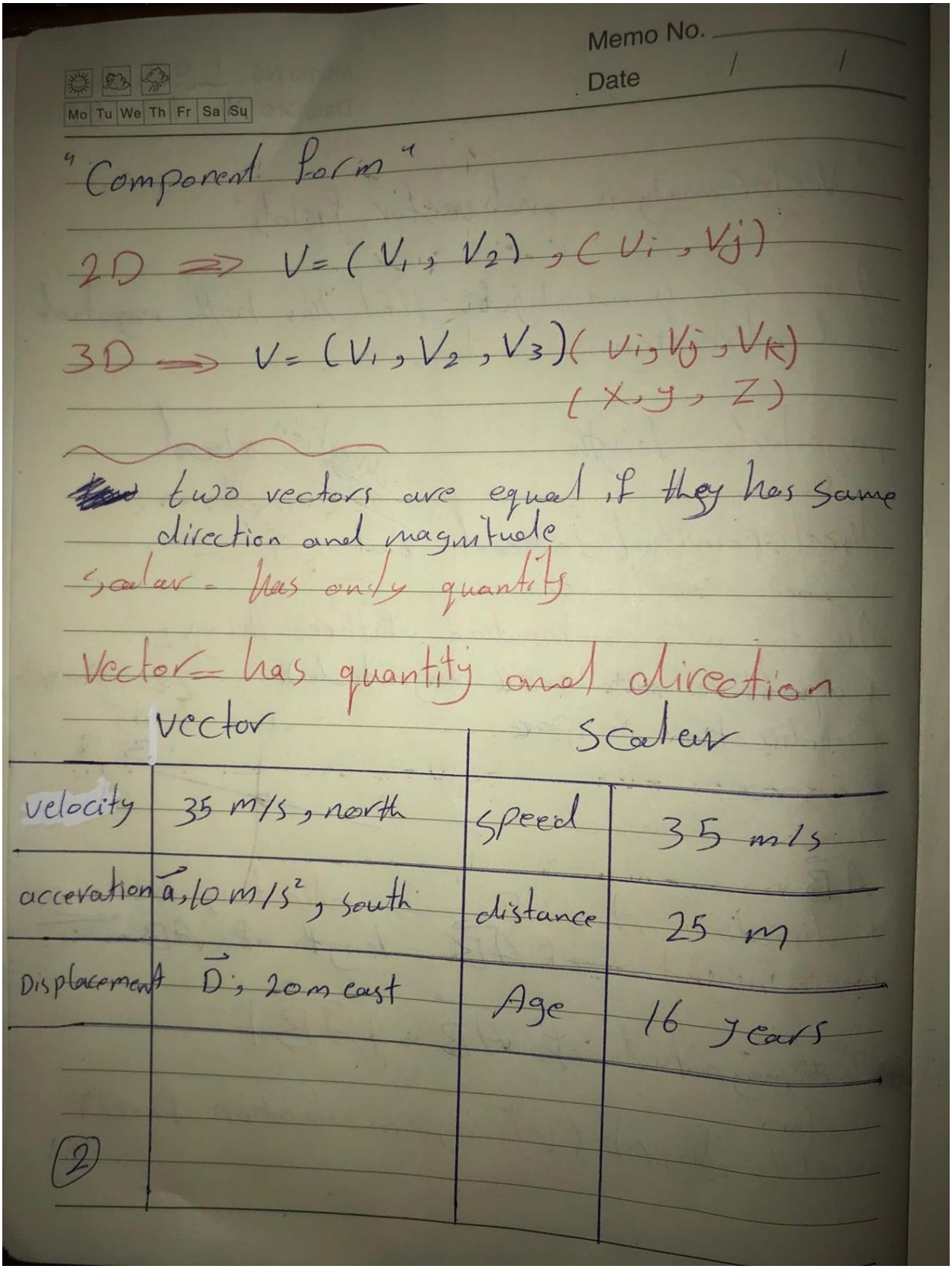

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"Component form"

2D ⇒ $V = (V_1, V_2)$, (V_i, V_j)

3D ⇒ $V = (V_1, V_2, V_3)$ (V_i, V_j, V_k)
(x, y, z)

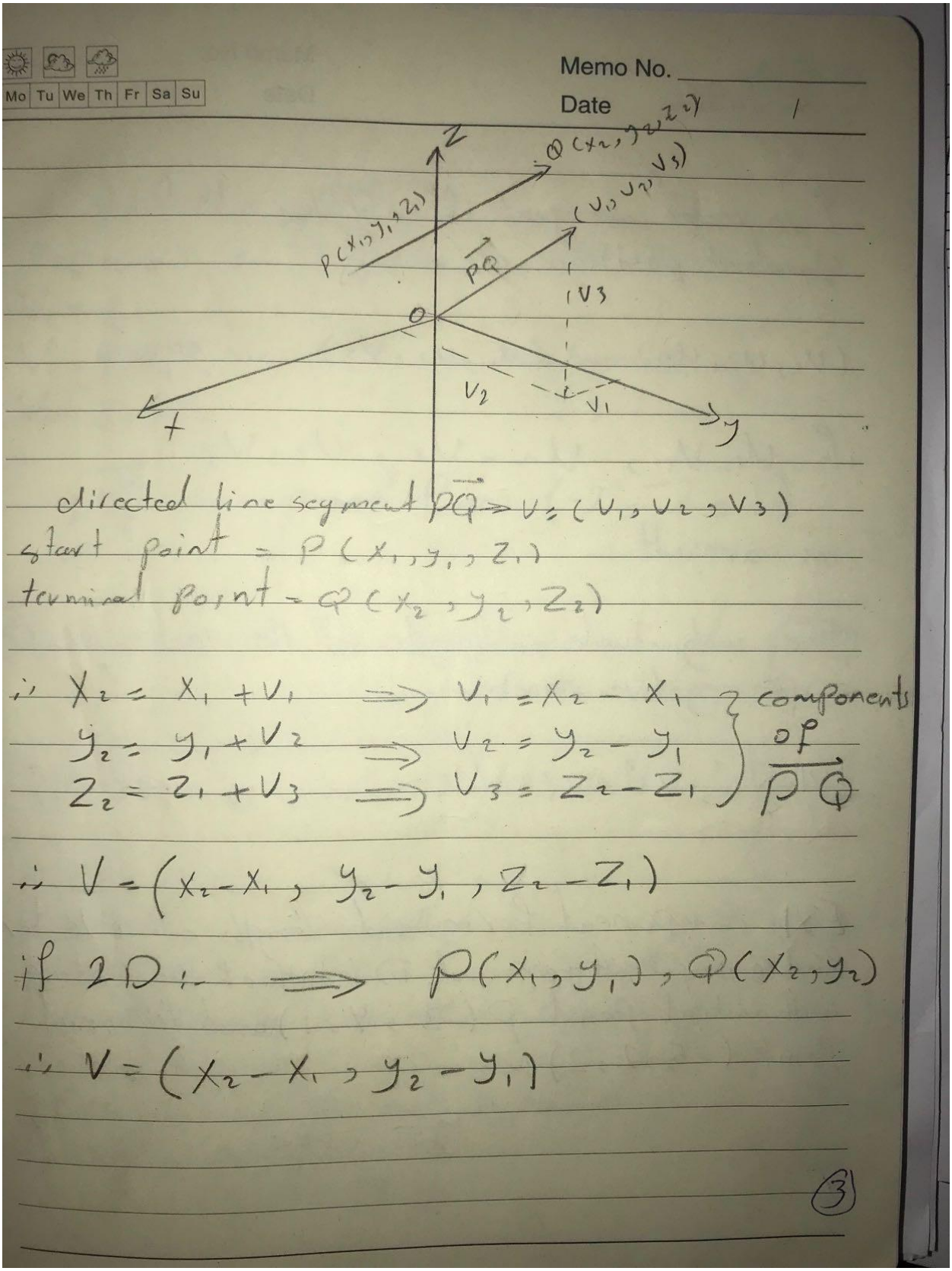
two vectors are equal if they has same direction and magnitude

Scalar - has only quantity

Vector - has quantity and direction

vector		scaler	
velocity	35 m/s, north	speed	35 m/s
accervation	$\vec{a}$, 10 m/s^2, south	distance	25 m
Displacement	$\vec{D}$, 20m east	Age	16 years

(2)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Z
Q(x2, y2, z2)
V3
(V1, V2,
P(x1, y1, z1)
PQ
V3
O
V2
V1
x
y
directed line segment $\overrightarrow{PQ} \rightarrow V = (V_1, V_2, V_3)$
start point = $P(x_1, y_1, z_1)$
terminal point = $Q(x_2, y_2, z_2)$
$\therefore x_2 = x_1 + V_1 \Rightarrow V_1 = x_2 - x_1$
$y_2 = y_1 + V_2 \Rightarrow V_2 = y_2 - y_1$
$z_2 = z_1 + V_3 \Rightarrow V_3 = z_2 - z_1$
components of $\overrightarrow{PQ}$
$\therefore V = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
if 2D :- $\Rightarrow P(x_1, y_1), Q(x_2, y_2)$
$\therefore V = (x_2 - x_1, y_2 - y_1)$
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Two vector are equal if only has identical standard position.

(u_1, u_2, u_3) and (v_1, v_2, v_3) are equal

if $u_1 = v_1$, $u_2 = v_2$, $u_3 = v_3$

as a result :-

The magnitude or length of the vector $V = \overrightarrow{PQ}$ is non negative number :-

$$|V| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Ex1:- Component form and length of a Vector

Find (a) component form and (b) length of the vector with initial point $P(-3, 4, 1)$ and terminal point $(-5, 2, 2)$:-

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(a) the standard position vector V representing $\overrightarrow{PQ}$ has components :-

$P=(-3, 4, 1)$, $Q=(-5, 2, 2)$

$V_1 = x_2 - x_1 = (-5-(-3)) = -2$

$V_2 = y_2 - y_1 = (4 - 2) = 2$

$V_3 = z_2 - z_1 = (2 - 1) = 1$

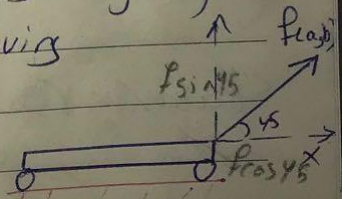
$\therefore \overrightarrow{PQ}$ in component form $(-2, 2, 1)$.

(b) The length or magnitude of $V = \overrightarrow{PQ}$:-

$$|V| = \sqrt{(-2)^2 + (2)^2 + (1)^2} = \sqrt{9} = 3$$

ex2 :- Force moving a cart :-

A small cart being pulled along a smooth horizontal floor with a 20 lb force F making a 45° angle to the floor, what is the effective force moving the cart forward?



$\because F=(a, b)$ and the cart is moving forward i.e $a = |F| \cos 45$

$= 14.14$ lb

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EX3: A man travels 120 km east and then 160 km north, calculat his resultant displacement? and direction?

(200km, 53.1° N to E)

Sol:-

$c^2 = a^2 + b^2$

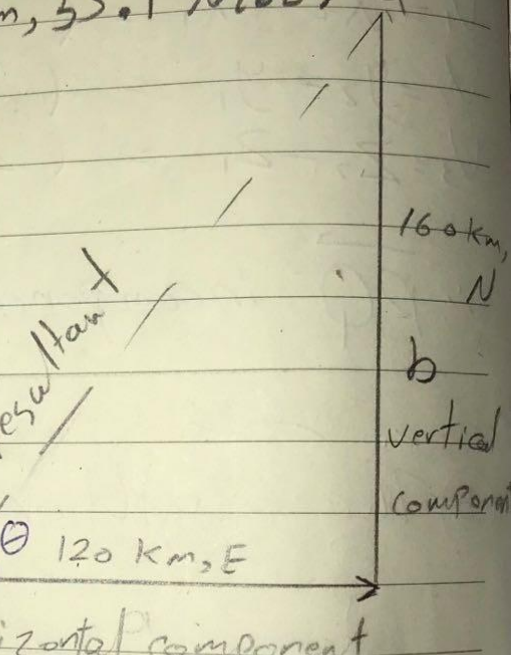
c- resultant $= \sqrt{(120)^2 + (160)^2}$

$= 200 km \ N \to E$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{160}{120} \Longrightarrow$

$\theta = \tan^{-1}\left(\frac{160}{120}\right) = 53.1$



"Vectors algebra operations"

- vector addition
- substraction
- scalar multiplication

scalar = real number, +, −, 0

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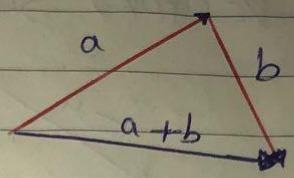
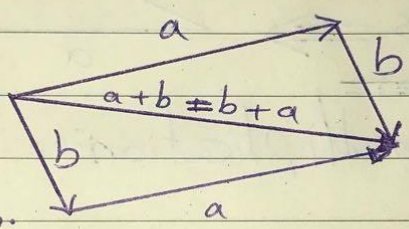
A- Addition of vectors satisfies two properties :-



① The commutative law, (The order of addition doesn't matter)

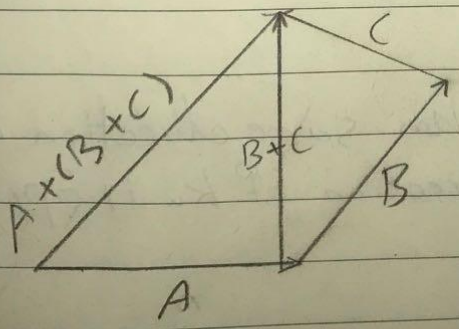
$$a + b = b + a$$ (parallelogram law)

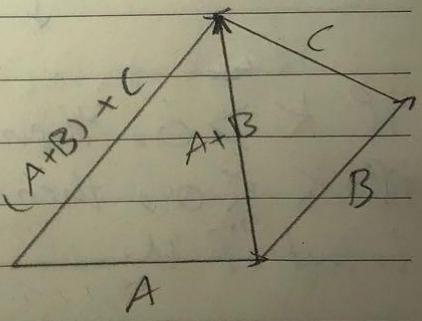
* both sums are equal to the same diagonal of the parallelogram.



② The associated law (the sum of three vectors does not depend on which pair of vectors added first:

$$A+(B+C) = (A+B)+C$$





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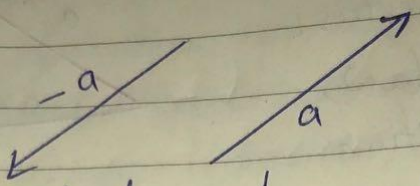
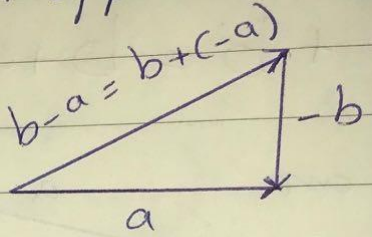
B- Vectors substraction



Same magnitude but opposite direction.

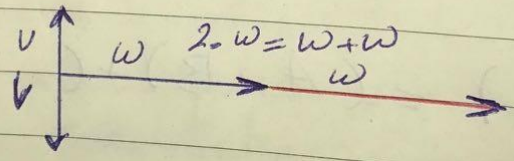
So, vectors substraction can be defined as $\Rightarrow$



C. "Scalar multiplication"

Let $u=(u_1, u_2, u_3)$, K = scalar

$\therefore K.u = (Ku_1, Ku_2, Ku_3)$



if $K > 0$, then Ku has same direction as u
if $K < 0$, then the direction of Ku is opposite to that of u.

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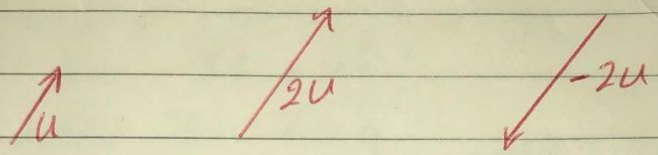
$$|ku| = \sqrt{(ku_1)^2 + (ku_2)^2 + (ku_3)^2} = \sqrt{k^2(u_1^2 + u_2^2 + u_3^2)}$$

$$= \sqrt{k^2} \cdot \sqrt{u_1^2 + u_2^2 + u_3^2} = |k| \cdot |u|$$



* Scalar multiplication satisfies many properties

- $s(a+b) = sa + sb$
- $(s+t)a = sa + ta$
- $1 * a = a$
- $(-1) * a = -a$
- $0 * a = 0$

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Exercise:

① let $U=(3,-2)$, $V=(-2,5)$ Find

a- the component form

b- The magnitude of vector, if eq. of

$=2U-3V$

Sol:-

(a) $V_1 = x_2 - x_1$

$V_2 = y_2 - y_1$, but $2U-3V \Rightarrow$ eq of

$\therefore 2U = (2(3), 2(-2)) = (6,-4)$

$3V = (3(-2), 3(5)) = (-6,15)$

$\therefore 2U-3V = ((-6-6), (15-(-4))) = (-12,19)$

(b)

$\sqrt{(-12)^2+(19)^2} = \sqrt{505} = 22.47$

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Vectors operations:-

(2) if $a = -2i + 4j$

$b = 3i - 2j$

$c = 4i + 5j$, then $a + b + c$ is ,

$a + b + c = 5i + 7j$

(2) if $u = -2i + 4j$

$v = 3i + 2j$

$w = 4i + 6j$, then $|u + v + w|$ is,

$|u + v + w| = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{169} = 13$

H.W :- P.(860) (1→5) (10→15)

(1) if $u = -i + 3j$ and $v = i + 2j$, then $ku + v$ is parallel to $w = -i + 4j$ if k is ?

(2) Let $u = (3, -2)$ and $v = (-2, 5)$ Find

(a) the component form

(b) The magnitud of vetor , if eq. of comp. is $(-2v)$

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