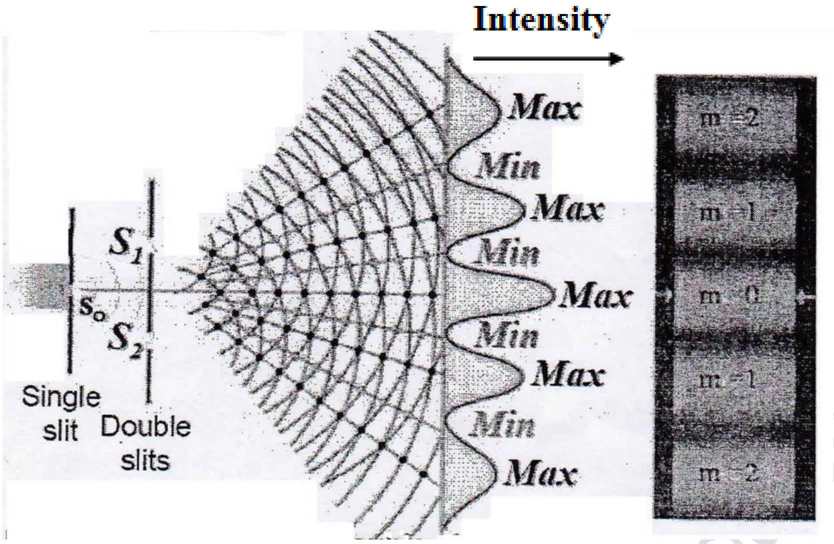
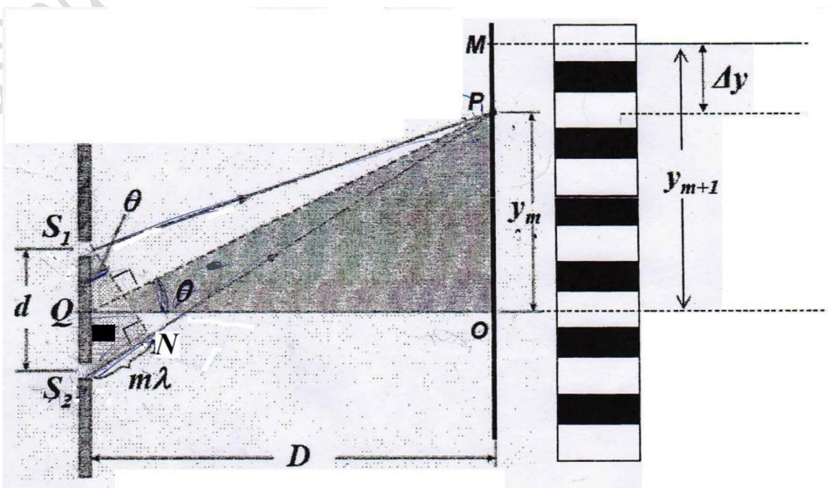
9.6 Young's double-slit experiment

- Figure below shows schematic diagram of Young's double-slit experiment



- Explanation of Young's double-slit experiment by using Huygens' principle
- ✓ Wave front from light source falls on S_o and diffraction occurs.
- ✓ Every point on wave front that falls on S_o acts as source of secondary wavelets that will produce a new wave front that propagate to slits S_1 and S_2
- ✓ S_1 and S_2 are two new sources of coherent waves in phase because they originate from the same source S_o.
- ✓ An interference pattern is formed on the screen.

10.6 Equation of Young's double-slit experiment



- Suppose P as shown in figure is the m^th^ bright fringe, so that

$$\boldsymbol{S_2P - S_1P = m\lambda}$$

- Let OP = y_m = distance from P to O.
- If $NP = S_1P$ *then* $S_2N = S_2P - NP = m\lambda$.
- In practice d is very small (<1 mm) and D>>d then S1N meets PQ at right angle. Therefore

angle PQO = angle $S_2S_1N = \theta$

- From the figure

$\Delta S_2S_1N \Longrightarrow \sin\theta = \dfrac{S_2N}{S_2S_1} = \dfrac{m\lambda}{d}$

$\Delta PQO \Longrightarrow \tan\theta = \dfrac{PO}{QO} = \dfrac{y_m}{D}$

Since θ is small, $\tan\theta = \sin\theta$

$$\frac{y_m}{D} = \frac{m\lambda}{d}$$

- Therefore, the separation between central bright fringe with m^th^ and (m+1)^th^ bright fringe is given by

For the **m^th^ bright fringe**:

$$y_m = \frac{m\lambda D}{d}$$

For the **(m+1)^th^ bright fringe:**

$$y_{m+1} = \frac{(m+1)\lambda D}{d}$$

- The separation between **successive (consecutive) bright or dark fringes, Δy** is given by

$$\Delta y = y_{m+1} - y_m = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}$$

$$\Delta y = \frac{\lambda D}{d}$$

Where:
m: order = 0,1,2,.....
λ: wavelength
D: distance between double-slits and screen
d: separation between double-slit

- The separation between **m^th dark fringe and central bright fringe** is given by

$$x_m = (m + \frac{1}{2})\frac{\lambda D}{d}$$

where
m: order = 0,1,2,....

- From the equation below,

$$\Delta y = \frac{\lambda D}{d}$$

- Δy depends on:
  i. the wavelength of light, λ.
  ii. the distance apart, d of the double slits.
  iii. distance between slits and screen, D.

- Explanation for the above factors:
  a) if λ is short and hence Δy decreases for fixed D and d. The interference fringes are closer to each other and vice-versa.
  b) if the distance apart of the slits diminished, Δy increased for fixed D and λ and vice-versa.
  c) if D increases Δy also increases for fixed λ and vice-versa.
  d) if the source S_o is winded the fringes gradually disappear. The slit So then equivalent to large number of narrow slits, each producing its own fringe system at different places. The bright and dark fringes of different systems therefore overlap, giving rise to a different illumination.
  e) if one of the slit, S_1 or S_2 is covered up, the fringes disappear.
  f) if the source slit S_o is moved nearer the double slits, Δy is unaffected but their intensity increases.
  g) if the experiment is carried out in different medium, for examples water, the fringe separation Δy decreased or increased depending on the wavelength, λ of the medium.
  h) if white light is used the central bright is white, and the fringes on either side are coloured. Blue is the colour nearer to the central fringe and red is the farther away as shown in figure below.

White

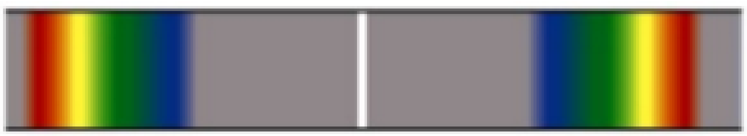


Table below shows the range of wavelength for colours of visible light

Colour	Range of λ (nm)
Violet	400 – 450
Blue	450 – 520
Green	520 – 560
Yellow	560 – 600
Orange	600 – 625
Red	625 – 700

Example 1:

In a young's double experiment, the slits separation is 1.0 mm. The distance between the slits and the screen is 1.0 m. The wavelength of the sodium light used is 5.9×10^{-5} cm.

(a) Calculate the separation between two consecutive fringes.

(b) If the sodium light is replaced with a blue light, what is the change to the interference pattern or the screen?

Solution: $d = 1\times 10^{-3}$ m, $D = 1.0$ m, $\lambda = 5.9 \times 10^{-7}$ m.

a. By applying the formula below

$$\Delta y = \frac{\lambda D}{d}$$

$$\Delta y = 5.9 \times 10^{-4} \text{ m}$$

b. Sodium light is yellow

$\lambda_{blue} < \lambda_{yellow}$ and $\Delta y = \frac{\lambda D}{d}$, where D and d are constant

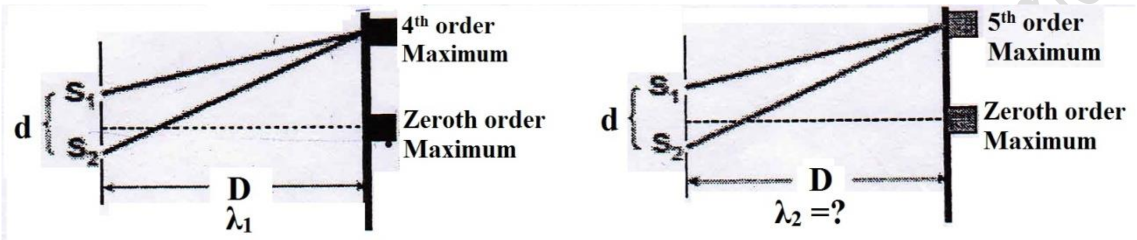
$\Delta y_{blue} < \Delta y_{yellow}$

hence, the fringes get closer to each other.

Example 2:

A monochromatic light of wavelength 600 nm falls on a system of double-slits of unknown slit separation. At the same time, the double-slit is illuminated by a monochromatic light of unknown wavelength. It was observed that the 4th order maximum of known wavelength light overlapped with 5th order maximum of the unknown wavelength light. Find the wavelength of the unknown wavelength light.

Solution: $\lambda 1 = 600 \times 10^{-9}$m



By applying the separation from central bright for maximum (bright) fringe, thus

$$y_m = \frac{mD\lambda}{d}$$

For 4th order maximum: For 5th order maximum:

$$y_4 = \frac{4D\lambda_1}{d} \qquad y_5 = \frac{4D\lambda_2}{d}$$

Because the fringe is overlap: thus

$$y_4 = y_5$$

$$\frac{4D\lambda_1}{d} = \frac{5D\lambda_2}{d}$$

$$\lambda 2 = 4.8 \times 10^{-7}\text{m or 480 nm.}$$

Example 3: H.W

Young's double-slit experiment is performed with 589 nm light and a distance of 2 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum.

Determine the spacing of the slits?

Ans: 1.54 mm

Example 4: H.W

A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.5 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

Ans: 515 nm