(14)

① Finding the projection of vectors

ايجاد مسقط متجه على متجه آخر باستخدام الضرب النقطي.

② Finding Scalar components

كلاهما يمكن ايجادهما باستخدام الضرب النقطي.

① vector projection of u on v :-

$$\boxed{\text{Proj}_v\, u = \left(\frac{u \cdot v}{|v|^2}\right) v} \rightarrow$$ قانون الاسقاط (حفظ)

$$\Rightarrow u = \frac{u \cdot v}{v \cdot v} v$$

② Scalar component of u in direction of v :

$$\boxed{|u| \cos\theta = \frac{u \cdot v}{|v|} = u \cdot \frac{v}{|v|}} \rightarrow$$ قانون Scalar (حفظ).

Ex :- Find the vector projection of $(u = 6i + 3j + 2k)$ onto $(v = i - 2j - 2k)$ and scalar component of (u) in the direction of (v) ??

Sol :-

① $$\text{Proj}_v\, u = \frac{u \cdot v}{v \cdot v} v = \frac{6 - 6 - 4}{1 + 4 + 4}(i - 2j - 2k)$$

$$= -\frac{4}{9}(i - 2j - 2k)$$

$$= -\frac{4}{9} i + \frac{8}{9} j + \frac{8}{9} k$$

② Scalar component of (U) in direction of (V): 15

$$|U| \cos\theta = U \cdot \frac{V}{|V|}$$

$$|V| = \sqrt{1^2 + (-2)^2 + (-2)^2} = 3$$

$$|U| \cos\theta = (6i + 3j + 2k) \cdot \left(\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}k\right)$$

$$= 2 - 2 - \frac{4}{3} = -\frac{4}{3}$$

Ex: Find the the vector projection of a force $F = 5i + 2j$ onto $V = i - 3j$ and scalar component of F on the direction of V?

Sol:-

① $$\text{proj}_V F = \left(\frac{F \cdot V}{|V|^2}\right) V$$

$$= \frac{5-6}{1+9}(i - 3j) = -\frac{1}{10}(i - 3j)$$

$$= -\frac{1}{10} i + \frac{3}{10} j$$

② The scalar component of F in the direction of V is:

$$|F| \cos\theta = \frac{F \cdot V}{|V|} = \frac{5-6}{\sqrt{1+9}} = \frac{-1}{\sqrt{10}}$$

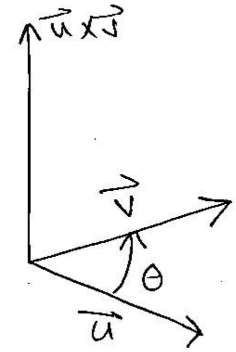
(1)

Lec. 6 : Cross Product

If we consider that $\vec{u}$ & $\vec{v}$ are two vectors, the cross product for them is:

$$\vec{u} \times \vec{v} = |u||v|\sin\theta$$

where $(\vec{u} \times \vec{v})$ orthogonal to both $(\vec{u}$ & $\vec{v})$ and the plane of them.



* properties of cross product:-

1- $(ru) \times (sv) = (rs)(u \times v)$

2- $u \times (v+w) = u \times v + u w$

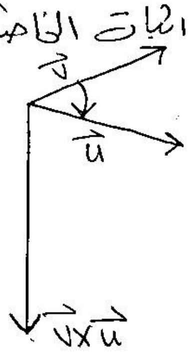
3- $(v+w) \times u = v \times u + w \times u$

4- $v \times u = -(u \times v)$

5- $u \times 0 = 0$

6- $(\vec{a} \times \vec{c}) \times \vec{b} \neq \vec{a} \times (\vec{b} \times \vec{c})$.

يمكن اثبات الخاصية رقم (4) من خلال الرسم:



(2)

ملاحظات مهمة :-

① ناتج عملية الضرب الاتجاهي هو متجه (vector) وليس scalar كما في حالة الضرب النقطي

② المتجه الناتج من عملية الضرب الاتجاهي يكون عمودي على كلا المتجهين وعمودي على المستوي الذي يحتويهما.

③ لإيجاد الحل لطريقة الضرب الاتجاهي هناك طريقتين :-

① $\vec{a} \times \vec{b} = |a||b| \sin\theta$

② using matrix (المصفوفة).

EX1:- $\vec{a} = i + 3j + 4k$, $\vec{b} = 2i + 7j - 5k$ find the cross product?

Sol:-

$$\begin{array}{cccc} & \oplus & \ominus & \oplus \\ \vec{a} \times \vec{b} = & \multicolumn{3}{c}{\begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix}} \end{array}$$

$$\vec{a} \times \vec{b} = i\begin{vmatrix} 3 & 4 \\ 7 & -5 \end{vmatrix} - j\begin{vmatrix} 1 & 4 \\ 2 & -5 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = (3 \times -5 - 4 \times 7)i - (1 \times -5 - 4 \times 2)j + (1 \times 7 - 3 \times 2)k$$

$$= (-15 - 28)i - (-5 - 8)j + (7 - 6)k$$

$$\therefore \vec{a} \times \vec{b} = -43i + 13j + k$$

3

Ex2:- Find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ when:

$\vec{u} = 2i + j + k$

$\vec{v} = -4i + 3j + k$?

Sol:-

①

$$\vec{u} \times \vec{v} = \begin{array}{ccc} + & - & + \end{array}$$
$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$$

هنا نكتب (u) هو الأول
و (v) هو الثاني

$$= i\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix}$$

$$= (1-3)i - (2+4)j + (6+4)k$$

$$\therefore \boxed{\vec{u} \times \vec{v} = -2i - 6j + 10k}$$

②

$$\vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ -4 & 3 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

بالحالة الثانية نكتب $(\vec{v})$ هو الأول
و ال $(\vec{u})$ هو الثاني

$$\vec{v} \times \vec{u} = i\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} -4 & 1 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} -4 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= (3-1)i - (-4-2)j + (-4-6)k$$

$$\therefore \boxed{\vec{v} \times \vec{u} = 2i + 6j - 10k}$$

نلاحظ ان الفرق بالنواتج
هو الاشارة فقط
إذن $(\vec{v} \times \vec{u} = -\vec{u} \times \vec{v})$

5

② عند ضرب متجه X نفسه وكما في المثال التالي

② When the vector is by itself.

$a = 4i - 2j + 5k$, find $\vec{a} \times \vec{a}$?

$$\vec{a} \times \vec{a} = \begin{vmatrix} \oplus & \ominus & \oplus \\ i & j & k \\ 4 & \bar{2} & 5 \\ 4 & \bar{2} & 5 \end{vmatrix}$$

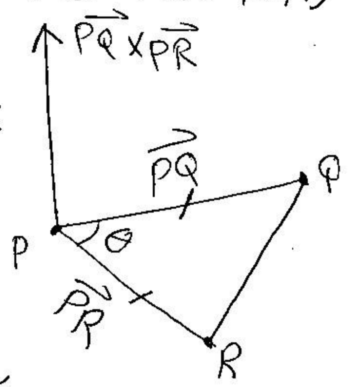
$$\vec{a} \times \vec{a} = i \begin{vmatrix} \bar{2} & 5 \\ \bar{2} & 5 \end{vmatrix} - j \begin{vmatrix} 4 & 4 \\ 5 & 5 \end{vmatrix} + k \begin{vmatrix} 4 & \bar{2} \\ 4 & \bar{2} \end{vmatrix}$$

$$= (-10 + 10)i - (20 - 20)j + (-8 + 8)k$$

$$\therefore \vec{a} \times \vec{a} = 0i + 0j + 0k$$

EX5:- Find a unit vector of the vector Perpendicular to the plane constructed by $P(1, \bar{1}, 2)$, $Q(2, 0, \bar{1})$ & $R(0, 2, 1)$?

Sol:-



$$\overrightarrow{PQ} = (2-1)i + (0-(-1))j + (-1+2)k$$

$$\therefore \boxed{\overrightarrow{PQ} = i + j - 3k}$$

$$\overrightarrow{PR} = (0-1)i + (2-(-1))j + (1-2)k$$

$$\therefore \boxed{\overrightarrow{PR} = -i + 3j - k}$$

(4)

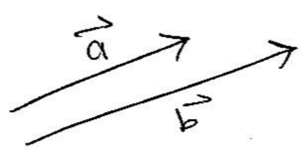
س/ متى يكون حاصل الضرب الاتجاهي = صفر

When the cross product = zero ?

(1) عندما يكون المتجهان متوازيان وكما في المثال التالي

(1) When the two vectors are parallel, i.e. ($\theta = 0°, 180°$)

EX3:- Find the cross product for the following vectors ?



Solg—

θ between $\vec{a}$ & $\vec{b} = 0°$

so, $\vec{a} \times \vec{b} = |a| . |b| . \sin\theta$

$\vec{a} \times \vec{b} = |a| |b| \sin 0$

$\vec{a} \times \vec{b} = |a| |b| *$ zero

$$\boxed{\begin{array}{l} \sin 0 = 0 \\ \sin 180 = 0 \end{array}}$$

$\therefore$ $\vec{a} \times \vec{b} =$ zero because $\vec{a}$ & $\vec{b}$ are Parallel vectors and θ between them $= 0°$

5

② عند ضرب متجه X نفسه وكما في المثال التالي

② When the vector is by itself.

$a = 4i - 2j + 5k$, find $\vec{a} \times \vec{a}$?

$$\vec{a} \times \vec{a} = \begin{vmatrix} \oplus & \ominus & \oplus \\ i & j & k \\ 4 & -2 & 5 \\ 4 & -2 & 5 \end{vmatrix}$$

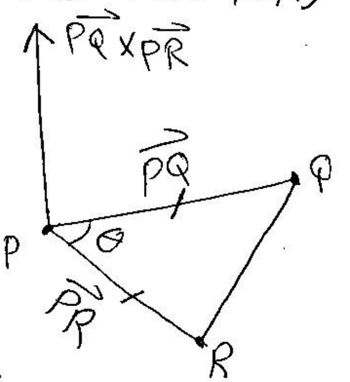
$$\vec{a} \times \vec{a} = i \begin{vmatrix} -2 & 5 \\ -2 & 5 \end{vmatrix} - j \begin{vmatrix} 4 & 4 \\ 5 & 5 \end{vmatrix} + k \begin{vmatrix} 4 & -2 \\ 4 & -2 \end{vmatrix}$$

$$= (-10 + 10)i - (20 - 20)j + (-8 + 8)k$$

$\therefore \vec{a} \times \vec{a} = 0i + 0j + 0k$

EX5:- Find a unit vector of the vector Perpendicular to the plane constructed by $P(1, -1, 2)$, $Q(2, 0, -1)$ & $R(0, 2, 1)$?

Sol:-



$\vec{PQ} = (2-1)i + (0-(-1))j + (-1+2)k$

$\therefore \boxed{\vec{PQ} = i + j - 3k}$

$\vec{PR} = (0-1)i + (2-(-1))j + (1-2)k$

$\therefore \boxed{\vec{PR} = -i + 3j - k}$

6

$$\vec{PQ} \times \vec{PR} = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} \end{array}$$

$$= i\begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}$$

$$= (-1+9)i - (-1-3)j + (3+1)k$$

$$\therefore \boxed{\vec{PQ} \times \vec{PR} = 8i + 4j + 4k}$$

$$\text{unit vector} = \frac{\vec{PQ} \times \vec{PR}}{|PQ \times PR|} = \frac{8i+4j+4k}{\sqrt{8^2+4^2+4^2}}$$

$$u.v. = \frac{8i+4j+4k}{\sqrt{96}}$$